Same Speed Railways

Introduction.

The Same Speed Railway is a theoretically ideal model of a railway with certain properties and characteristics such that its behaviour and performance can be predicted and mathematically analysed. Various types of physical railway do or can be made to approximate to the ideal, with very desirable consequences. I recognise three categories of Same Speed Railway:

- High Speed Railways
- Medium Speed Railways
- Low Speed Railways

An alternative classification is between (pure) Metros (where all trains stop at all stations) and Semi-Metros, which allow a mixture of stopping and non-stop trains, with overtaking at stations. Metros and semi-metros are available in all three speed ranges.

Metro systems have been around for a long time, and their performance is well understood. High Speed railways are now very familiar, but it is my contention that their performance is **not** well understood. The Medium Speed range is foreseen as for conversion of classic route to the Same Speed model, though there are none as yet, and also for the outer ranges of metro systems. The model would also be suitable for dedicated freight lines, though, again, there are none as yet.

The fundamental property of a Same Speed Railway is that the traffic is **homogeneous** in performance, consisting of trains whose dynamic performance, specifically their speed range and acceleration and deceleration rates, are identical, within a very narrow range. These trains all travel on the line at the same speed**, (**hence the name,) the **line speed**. (Strictly speaking, the trains do not need to be identical, but their dynamic characteristics must be such that they are all able to **perform** identically, as regards acceleration, deceleration and speed. This allows new trains, of improved performance, to be introduced. Initially they will run at the same performance as the existing trains, until these have been progressively withdrawn, when the improved characteristics of the new stock can be taken advantage of.)

This homogeneity of performance has a most important consequence: Same Speed Railways **do** have a precise value of line capacity, for a particular line speed: this value varies as the line speed itself varies, and this can be analysed mathematically and its characteristics deduced.

The precise meaning of line capacity is how many trains the line can accommodate, specifically how many trains can pass a particular point in a particular time (in a particular direction!), generally stated as trains per hour (tph). The faster they're going, the more get past, right? Wrong! What matters is not how fast you can go, but how fast you can (in a controlled manner) stop. Also, while it is true that line capacity increases linearly with speed at very low values, this quickly ceases to be the case as the speed increases further. A maximum value of line capacity is reached, at a surprisingly low speed, and thereafter, further increase in speed causes causes line capacity to decrease, albeit slowly.

Line capacity is a rather amorphous concept for conventional, mixed-traffic lines. The idea is clear enough, but such a line has no fixed value of line capacity. The value is heavily dependent on the traffic mix, different types of train, of different levels of performance, travelling at different and varying speeds. This can change several times every day, as the traffic-mix changes. Railwaymen over many generations have developed reliable but essentially rule-of-thumb methods for determining what traffic can be scheduled on the line. But the principles underlying Same Speed Railways ensure that line capacity is a **property**, with a specific, definite value, dependent only on the line speed (there **are** other influences, but these are second-order, compared with the line speed).

While the line speed (and therefore the line capacity) of a Same Speed Railway has been stated above to be constant, strictly speaking, this is true only for individual sections. It is perfectly possible for adjacent sections, (separated by some feature, typically a station at which at least some, possibly all, trains stop, but it could equally well be a junction, or a permanent speed restriction, such as across the Forth Bridge,) to have different values of line speed. The important point is that line speed is fixed within a section, but the value may undergo a step-change between sections. The overall line capacity is that of the lowest capacity section, which, for High and Medium Speed Railways, will be that or those with the highest value of line speed.

A same speed, non-stop train accelerates, at a prescribed rate, from its originating station up to the line speed, then travels the entire journey at that speed, until it decelerates, at a prescribed rate, to a stop at its destination. This behaviour applies similarly for the sections between intermediate stations, for stopping trains. All trains take exactly the same time for a specific journey, and for all sections thereof. This mode of operation has the coincidental benefit of making it very easy to calculate journey times. Note that I say ‘calculate’ rather than ‘estimate’. An estimate is a simplified calculation, which omits certain less important features – a quick and dirty calculation in fact. But these are precise and exact results for the Same Speed Railway model. They become estimates only when quoted for (usually) a High Speed line, whose realisation of that ideal is at best only approximate (but they are still **very good** estimates nonetheless).

This article started life as a description and explanation of High Speed railways. It included Appendix A, which demonstrates the theoretical existence of a maximum line capacity. (At that time I didn’t know how to calculate line capacities – that came much later – but the argument is still valid, so I leave it in.) Over time and article versions, the awareness gradually grew that there is a fundamentally different model of railway involved here, a new paradigm in fact, (in the current jargon,) the Same Speed model. This bears little resemblance to conventional, mixed traffic railways. The introduction is now recast to start from the Same Speed model, and to derive everything from that.

The original article now follows (omitting statements that I now find embarrassing by their manifest lack of knowledge). Appendix A remains, mainly from sentiment.

Appendix B deals with line capacity. It is focused on High Speed lines because of their innate importance and because they have certain characteristics not present with other categories. These are a consequence of the necessity to decelerate on the main line before diverging at a junction, because of speed limitations of the points. Medium Speed Railways share the same properties, except the need for diverging trains to decelerate on the main line. Both categories allow for a mixture of non-stop and stopping trains, with appropriate provision for overtaking. This imposes some necessary constraints, including, (astonishingly when first encountered, but natural and inevitable once understood,) the limitation of the line speed to only a small set of specific values. Metros are also dealt with. These have seemingly very different properties, since all trains stop at all stations, but the same underlying principles still apply, just applied rather differently. Finally, some account is given of resilience, and here, overtaking railways and metros do differ fundamentally.

Appendix C deals with calculating journey times. This is actually fairly trivial, but there are so many special cases to take into account. I particularly recommend the elegant concept of Propinquant Junctions. Fortunately, all this stuff is very readily automated in spreadsheets.

Appendix D also deals with calculating journey times. It lists all values for Adjacent Stations and Propinquant Junctions, for all the individual HS lines planned. These are values which have to be input explicitly into the journey time spreadsheets. New values are added as they are discovered (though the list must by now be essentially complete).

Appendix E documents this article's version history. This is a story interesting in itself, recording how various ideas developed.

Appendix F is an entirely theoretical account of the effect of varying the deceleration rate, that being the only other way (other than by line speed, that is) to affect capacity and journey times. This is a very difficult thing to do in practice, except for very small amounts. Is is also very much limited by what passengers would tolerate!

The Purpose

What this article seeks to do is to define what a Same Speed Railway is, (as compared with other types,) the circumstances in which it is appropriate to deploy it, (thus what we intend to achieve by it,) and recommendations on its deployment – a set of guidelines.

My article 'Towards a High Speed **Network**' makes the case for developing a network plan for all the HS routes which will eventually be needed, as opposed to the free-standing, isolated approach which characterises the HS2 proposal. The title is a reasonable shorthand, but really there is no such thing as a high speed network; there is only the **railway network**, certain lines of which happen to be high speed, but all of which are intimately connected and work together. This may seem an obvious point, but I contend that many countries which have developed high speed lines have developed them as a separate, stand-alone system, and any interfaces with the existing ('classic') railway have been an afterthought. Such an approach works to an extent, and does clearly have some benefits, but I contend that it loses the very significant benefits of synergy, and results in an overall rail system, parts of which are good, but the rest of which is disregarded, shabby and starved of investment.

That the UK is so far behind much of the rest of the world in developing HS lines does give us the opportunity to learn from and avoid the mistakes that others have made (just as the rest of the world learnt from our mistakes, as the original developers of railways and so, for example, went for much more generous loading gauges). This is a sweet irony.

The Nature of a High Speed Railway

A HS railway is a Same Speed railway designed and built to the best of modern standards. With the superb alignments which modern construction techniques make possible, the quality of modern trackwork, the power and flexibility of modern signalling and control technologies (especially dynamic block working and automatic train control), and the performance of modern trains, it isn't surprising that the trains can go a lot faster than on classic routes. But there's a lot more to it than that.

A HS railway is not a mixed-traffic railway, it is a dedicated, express passenger railway. Certain types of freight traffic, with similar performance characteristics to passenger traffic, such as mail and parcels, can also be accommodated. Theoretically, one could envisage a HS line being used for general freight traffic overnight, when there is no passenger traffic, but in many if not most cases, the types of alignment suited to passenger trains, involving (in classic terms) quite severe gradients, make HS lines unsuitable for heavy freight. (That said, a few lines in special circumstances **are** foreseen as accommodating freight also, in particular over parts of the Felixstowe – West Midlands freight trunk.)

A HS railway is, very importantly, a High Capacity railway, at least in comparison with a mixed-traffic railway, offering the maximum possible capacity **at a given line speed**. That last qualification is crucial; they are not high capacity *per se*, and in fact the higher the line speed, the lower the capacity. (Appendix B of the present article expounds the line speed / capacity relationship in detail.) All the traffic shares similar performance characteristics, in particular all travelling within the same, narrow speed band (which varies, obviously, between different locations, but all traffic has essentially the same speed at any particular location). This is the essential condition for enabling maximum throughput. Dynamic block and automatic train control further ensure that this maximum is at as high a value as possible.

HS trains never stop **on the main line**, or they **all** stop, at a particular location, i.e. a station. A (non-terminal) HS station generally consists of two island platforms, i.e. two platform faces in each direction. If not all services stop at the station, then the main line must continue unobstructed through the centre of the alignment, without adjacent platforms, (or in tunnel, underneath, or on viaduct, overhead,) and the stopping lines must diverge some distance either side of the station, using high speed point-work, so that trains diverge at high speed, then have adequate braking distance (mainly regenerative) to come to a stand at the station platform, then have adequate distance to accelerate back up to full line speed before re-joining the main line. (Note that this behaviour is **exactly** analogous to motorway driving: vehicles do not slow down on the motorway before diverging at a junction; they travel at full speed onto the slip road, and slow down there. Likewise they accelerate to full traffic speed before joining the motorway. ‘Slip-lines’ are, of course, rather longer than slip-roads.) The fundamental point is that nothing must prevent a train from travelling at full line speed anywhere on the main line. In certain locations (such as Nottingham), it is more convenient to have the HS station on a loop, away from the main line, so the main line bypasses the station completely.

The above explanation is valid in its essentials, and good enough for a high-level understanding, but it **is** very much simplified. I am careful, above, **not** to state that trains diverge from the main line at full line speed, because they don’t. The true situation is considerably more complicated and more subtle. Technology junkies are referred to appendix B, which contains the full story, and references to the original source articles, for those who **must** have the really hard stuff.

Some people think a HS line must have virtually no intermediate stations. This a misconception; the defining characteristic, as explained in the previous paragraph, is that a non-stop train is never **impeded** by the presence of stations, but can travel at uninterrupted line speed for as far as necessary. Nor is it impeded by other trains stopping at the stations, as these get out of its way in a timely fashion. All trains travel on the main line at full line speed; a train decelerates from line speed only when it has already left the main line, and is on a station line or loop, and accelerates on the station line or loop back up to line speed before rejoining the main line. Within reason, a HS line could have any number of intermediate stations, and some trains could stop at some or all of them without impeding those not stopping. (My proposals for HS3 envisage two categories of service, the HS Metro services to York and Preston, which stop at all intermediate stations, and the long-distance UHS – Ultra HS – services to the NE and Scotland, which travel non-stop to South Yorkshire or York.) Of course, the more intermediate stops a particular train makes, the lower is its overall end-to-end average speed, even though on the main line it travels at line speed. The long distance UHS services travel for long distances at line speed, without stopping, so their overall average speed is high. (And it is these long-distances-without-stopping UHS services that really make use of and justify the 360kph maximum.) In fact, given careful timetabling, a HS line can accommodate a mixture of stopping and non-stop services, but there is nevertheless a cost, which is explained in appendix B, in the section describing the effect of stations.

The next paragraph was valid at the time it was written, but has now been completely overtaken by events. Embarrassing as it now is, I leave it in place to show that I don't have a crystal ball.

HS lines are built to the GC-standard loading gauge. This is a practical rather than an essential characteristic, and is only really relevant to the UK, where the loading gauges of the classic lines are so restricted (in some countries, everything is GC gauge or even better anyway). This means that two types of train run on (UK) HS lines – GC-gauge (or 'captive') trains, which can only run on HS lines (or, in a few cases, extensions therefrom of GC-gauge on classic lines), and Classic-Compatible trains, which are built to UK standard loading gauge, and can run on both HS and classic lines (even sharing the same, variable platforms with GC-gauge trains – see Appendix B of my 'Network' article for an explanation of variable platforms).

The next paragraph is even more embarrassing, but again, I leave it in place as it illustrates the fatuity of the conventional wisdom.

Some people think it ridiculous to have trains which cannot run on all lines, but are restricted to a relatively small part of the network. Other things being equal, this argument has some merit. but other things are not equal – the increase in capacity offered by GC-gauge is profound. In particular, GC-gauge readily accommodates double-deck trains, with plenty of room inside them. As a rule of thumb, a double-deck train offers the same passenger capacity in two thirds of the length of an equivalent single-decker, or alternately, a single-decker has to be half as long again to offer the same capacity. This offers very serious savings in platform length and thus station area. In any case, I think the above is a defeatist attitude, which accepts the restricted UK gauges for ever. With GC gauge for HS lines, we have an important and growing proportion of the overall network which can accept the high-capacity, including double-deck, GC-gauge trains.

*What is really wrong with the previous paragraph is something which is not actually stated. GC-gauge, in allowing for larger trains does indeed increase **passenger capacity per train**. In particular, double-deck trains allow for higher passenger capacity in shorter train length, thus economising in platform length and thus station area. What GC-gauge does **not** do (and, of its nature, **cannot** do,) is increase **line capacity**, i.e. the number of trains per hour that the line can carry. There are two meanings of 'capacity' here, and an argument that really applies only to one of them is assumed to apply to the other, because the distinction is not made.*

*The conventional wisdom is that high speed railways 'increase capacity'. They don't (except in comparison with mixed-traffic lines). They reduce it. **Line capacity**, that is. The maximum theoretical capacity of a same-speed railway (homogeneity of traffic being an essential precondition of the very existence of a theoretical maximum) occurs at an astonishingly low speed of about 59mph (26.5m/s). Above that speed, the faster you go, the less capacity you have. A line speed of 100mph has **twice** the (theoretical) capacity of a line speed of 225mph.*

The main body of this article was written long before I knew how to perform the capacity vs speed calculation, which is what appendix B is all about. When I wrote it, I too believed, as I had been told by seemingly reliable authorities in the specialist railway press, and by statements from politicians and the DfT, repeating what (they believed) HS2 Ltd. had told them, that the really important justification for high speed was 'capacity'. This is mendacious drivel.

Appendix B gives the full story. But it is a stiff read. I have produced another article, written for a general audience, intelligent people but not technically minded. 'Line Capacity vs. Speed for Same Speed Railways' (also available on the website although written to different standards from the other articles,) describes the results, with a little background and some further elucidation. It refers back to appendix B for the full story, but stresses that such detail ***is not necessary in order to understand the results****, and their significance. In fact, anyone proposing to read appendix B is advised to read 'Line Capacity vs. Speed for High Speed Railways' first, as an introduction.*

When to Deploy HS Lines

There is only one fundamental, **deciding** reason to deploy a new HS line: when an existing classic route is overloaded and significant additional capacity is required. (Thus a HS line is always associated with a particular classic route.) The fact that trains can travel much faster on these lines is a **reinforcing** reason, not a deciding one.

HS-Antis and other romantic mediaevalists try to argue that upgrading the existing, classic line to expand its capacity would be cheaper and less disruptive. It is strange that anyone feels they can make this argument seriously, after the experience of the WCML upgrade, which was **monstrously** expensive, **hugely** disruptive over a **prolonged** period, and after all that didn't even deliver the goods, and needs further work now, a few years later. Of course, there are changes that could advantageously be made and should be made to classic routes – the odd flyover, discreet extra tracks here and there – but these are, however worthwhile in themselves, mere ameliorations, when what is needed is a quantum leap, and this requires new infrastructure.

There is, however, a more fundamental argument against trying to increase the capacity of an existing classic line beyond its reasonable limit. These are all mixed-traffic routes with, usually, several intermediate stations. This very fact severely restricts the available capacity, as compared with what the same infrastructure could accommodate if the traffic were homogeneous, as on a metro, for example. The requirement is usually to increase capacity between the end points, typically between a major regional centre and a London terminus, Manchester – Euston, for example. Beyond the reasonable capacity limit of the route, the only way to get additional capacity out of the existing infrastructure is to increase the priority of one type of traffic at the expense of the others. In the extreme case this would be just an express service between the end points, serving nowhere in between, and no other traffic, and so no longer a mixed-traffic route – in fact it has been converted into a same-speed railway of sorts. But even short of this extreme, it would involve severe degradation in the service offered to intermediate locations. Even the most simple-minded HS scheme, serving just the end points, significantly reduces the loading of the associated classic route, and allows a decent traffic mix there, even improving the service to intermediate locations; thus the HS line benefits people who don't even use it.

Guidelines to HS Deployment

Guideline 1: No location should suffer a worse service as a consequence of a HS line opening. Self-evidently true, surely? Yet many places, most infamously Stoke-on-Trent, will suffer a worse service when HS2 phase 2 opens, according to current plans.

The problem arises because express services on a classic trunk route between a major regional centre and London (Manchester – Euston, for example, again) typically have a number of stops at the regional end, to pick up traffic from lesser but still important locations in the originating region (the 'secondaries', say – in the present example Stockport, Macclesfield and Stoke-on-Trent), then a long non-stop (or just one or two stops) run to London. The bulk of the traffic is from the first station (Manchester Piccadilly). A HS line links the endpoints of the associated classic route, and would reasonably be expected to take over all the end-to-end traffic from the classic route. It may serve other intermediate locations, but will not directly serve the secondaries, which thus could face a worse service than previously. The way to solve this dilemma is to run a classic-compatible service along the initial section of the classic route, serving all the secondaries (and ideally a few more secondary-type locations, to help fill it), and then to leave the classic route and join the HS route at an intermediate junction. If the traffic is no longer sufficient to fill the classic-compatible train adequately, use a shorter formation. We thus have:

Guideline 2: There should be at least one intermediate junction to the HS route from the associated classic route, to allow classic compatible services to run serving those regional secondary locations served by the original classic express service, but not served directly by the HS route (and perhaps additional secondary locations on the classic route), joining the HS route at this intermediate junction, then high speed thereafter. This intermediate junction can also take other classic-compatible services from locations beyond the associated classic route (in the Manchester – London example, services such as from Preston and Liverpool). Indeed **all** services on the associated classic route from before the HS junction (which may originate on other classic routes which join it) are candidates to become classic-compatible services, freeing up slots on the classic route beyond that junction.

Guideline 3: Terminal HS stations in locations of high traffic demand are a very bad idea, as they need to be disproportionately large to provide the necessary capacity (trains terminating, being serviced in situ, then forming a service in the reverse location make prolonged demands on platforms). This applies especially to London locations. It is far better for such locations to have through stations (of the standard double-island model, with all services stopping), with the HS route subsequently branching to serve several terminal destinations, each individually needing only moderate capacity. A prime example of this is the proposed Euston Cross, with services travelling on to HS1 and Kent / East Sussex, and terminating at Maidstone, Hastings or Dover. This also provides excellent inter-regional facilities.

Guideline 4: Services on the associated classic route change, as soon as the HS route opens, to the Regional Metro pattern. This consists of two groups of (passenger) services, semi-fast and stopping. The semi-fast services are regular interval, over the whole or portions of the route, stopping at all traffic sources of reasonable size (i.e. towns / large villages or parkway-type locations with a sizeable drive-in area). The stopping services are generally hourly, stopping everywhere on a particular section of the route, and connecting into or out of the semi-fast service at each end. At all appropriate stations served by both HS and semi-fast regional metro trains, the regional metro trains are timetabled to make interchange connections into and out of the HS trains, and have similar frequencies. (Note that the HS trains mentioned in the preceding sentence could of course be classic compatibles, running on the initial section of the classic route.)

Guideline 5: (expanding on guideline 3). HS railways should be designed on the roots – trunk – branches model. Multiple services from different origins – the roots – progressively merge into a single trunk, and travel for the bulk of their journeys at high speed on the trunk. They then progressively diverge from the trunk – the branches, to reach their destinations.

Low Speed Railways

The fundamental characteristic of high speed railways, that all trains travel at the same line speed, anywhere on the main line, has nothing to do with high speed as such, but is the defining characteristic of a Same Speed Railway, whatever that speed actually is.

Obviously, for dedicated express passenger railways, the speed should be as high as practicable, taking into account the characteristics of the traffic (such as the average length of travel without stopping, thus whether ultra-high speed is appropriate or whether a lower maximum would give the same benefits, with a significant saving in construction costs). But the same principle would also apply very advantageously to a dedicated freight route – a Low Speed Railway. A same speed railway could thus be any railway where the traffic is all of the same type, specifically with the same performance characteristics and thus capable of travelling at the same speed. Low speed railways could indeed carry passenger traffic, but this would have to travel at the same line speed as the freight traffic – if this is in the range 50-70mph then that needn't be a problem on a secondary passenger route.

It is important to stress that a low speed railway is not simply a freight line as currently understood. Those aspects of same speed railways which **enable** all traffic to have the same speed – specifically the location and type of pointwork – apply in just the same way as already explained for high speed lines. If a low speed line has a passenger service, then the station platforms must be on passing loops – there are never platforms adjacent to the main line, because there is always the requirement for overtaking, by freight trains – and the stopping lines must diverge some distance either side of the station, to allow stopping trains to diverge at full line speed and then decelerate to come to a stand at the station platform, likewise to accelerate back up to full line speed before rejoining the main line. The pointwork required is obviously less demanding than for the high speed case, and the stopping lines shorter. Note that exactly the same considerations apply to freight trains, diverging into sidings in goods yards, but also that the stopping distances for freight trains will be considerably longer than those for passenger trains decelerating from the same line speeds.

For all that, a lot of classic routes for which freight is the dominant traffic could readily be enhanced to (low) same speed standards at moderate cost. Examples which spring to mind include Felixstowe – Peterborough – Leicester – Nuneaton – West Midlands (multiple destinations), GN/GE line Peterborough – Spalding – Lincoln – Doncaster and the Settle and Carlisle line north of Skipton (followed by the GSW route on to Glasgow).

It is worth repeating yet again that the whole purpose and justification of same speed lines is to maximise capacity, **at the chosen line speed**, whatever that is. Other technologies, principally signalling and control, determine the actual maximum value, but it is the fact that all traffic has the same speed which enables a maximum to be achieved at all.

Appendix A – Theoretical Maximum Line Capacity

Assume a same speed line with the characteristics:

λ = (maximum) length of train
δ = (minimum) permissible distance between trains
ν = line speed – speed of every train
Train Envelope = Length of train + separation distance to following train
= $\lambda + \delta$

A given train, and thus a train envelope, travels a distance νt in time t,
so the number of trains passing a given point in time t is $\nu t / (\lambda + \delta)$,
thus the capacity of the line is $\nu / (\lambda + \delta)$ trains per unit of time.

For traditional fixed-block working, δ has a constant value, so the capacity is linearly proportional to line speed. This is actually correct, but of course in traditional working, a maximum speed was selected, and the block length determined as the braking distance of a typical train travelling at that maximum (or possibly vice versa). In this situation, maximum capacity is indeed achieved when all trains travel at that maximum speed.

I am not sufficiently familiar with the technical details of dynamic block working to know how the distance to be maintained between trains is determined, but for illustrative purposes I assume it is proportional to the square of the speed, so that it relates to the kinetic energy of the train, which seems plausible. So take $\delta = \kappa\nu^2$. So the line capacity c is:
$c = \nu / (\lambda + \kappa\nu^2)$

$$\begin{aligned}\partial c/\partial\nu &= \{(\lambda + \kappa\nu^2) – \nu(2\kappa\nu)\} / (\lambda + \kappa\nu^2)^2 \\ &= (\lambda - \kappa\nu^2) / (\lambda + \kappa\nu^2)^2 \\ &= 0 \text{ when } \nu^2 = \lambda/\kappa\end{aligned}$$

So maximum capacity $c_{max} = \surd(\lambda/\kappa) / (\lambda + \kappa(\lambda/\kappa)) = 1/(2\surd(\kappa\lambda))$

[Also
$$\begin{aligned}\partial^2 c/\partial\nu^2 &= \{-2\kappa\nu(\lambda + \kappa\nu^2)^2 - (\lambda - \kappa\nu^2)(4\kappa\nu(\lambda + \kappa\nu^2))\} / ((\lambda + \kappa\nu^2)^2)^2 \\ &= \{-2\kappa\nu(\lambda + \kappa\nu^2) - 4\kappa\nu\,(\lambda - \kappa\nu^2)\} / (\lambda + \kappa\nu^2)^3 \\ &= \{2\kappa^2\nu^3 - 6\kappa\nu\lambda\} / (\lambda + \kappa\nu^2)^3 \\ &= 2\kappa\nu(\kappa\nu^2 - 3\lambda) / (\lambda + \kappa\nu^2)^3 \\ &= 2\kappa\nu\{(\lambda + \kappa\nu^2) - 4\lambda\} / (\lambda + \kappa\nu^2)^3\end{aligned}$$
When $\nu^2 = \lambda/\kappa$ then $\partial^2 c/\partial\nu^2 = -\surd(\kappa\lambda) / (2\lambda^2)$, i.e. $\partial^2 c/\partial\nu^2 < 0$ so the above extreme value is indeed a maximum. (I would have been profoundly shocked had it turned out to be a minimum!)]

(I admit I didn't actually **remember** the formula for differentiating a quotient – not at any time in the last 50 years! – but the excellent website 'Paul's Online Math Notes', at
http://tutorial.math.lamar.edu/Classes/CalcI/ProductQuotientRule.aspx
reminded me.)

This appendix is purely theoretical, but does usefully demonstrate that there does exist an actual maximum capacity at an actual optimum speed. At the time this was written, I didn't know precisely how the distance between trains should be determined. Appendix B, following, (a much later addition,) remedies this deficiency, and gives the real life stuff, for technology junkies.

Appendix B – Actual High Speed Line Capacity

I am indebted to Piers Connor of PRC Rail Consulting Ltd. for the information on which the results derived in this appendix are based. PRC Rail Consulting Ltd. publishes a series of occasional articles 'Technical Web Pages', at:

http://www.railway-technical.com/prcrailpage.shtml

Two articles are particularly relevant. The first is '(Rules for) High Speed Line Capacity' v3, 26 August 2011, at:

http://www.railway-technical.com/Infopaper%203%20High%20Speed%20Line%20Capacity%20v3.pdf

which is in the Technical Web Pages series, and treats the subject as a series of 10 rules.

The article: 'High Speed Railway Capacity' (not sure of the date, but probably 2014) at:

http://www.railway-technical.com/High%20Speed%20Railway%20Capacity%20v13%20conf.pdf

seems more of a working paper, and serves as the background to a presentation at Birmingham University in December 2014, at:

http://www.railway-technical.com/HSR%20Presentation%20Piers%20Connor%20v1.pdf

All of these are thoroughly interesting, and contain essential information; technology junkies will, I am sure, love them.

The reason I refer to both papers titled 'High Speed Rail Capacity', is that they use different bases; the 2011 article considers a top speed of 300kph, whereas the later paper, which covers the same ground but in rather more detail, takes a top speed of 360kph. I need both, since my projected routes are either Ultra High Speed, UHS – HS2, HS3 and HS4 over most of the distance – for which 360kph is a good top speed, or HS Metro, where trains stop at all stations, for which 300kph is entirely satisfactory. These values apply to new infrastructure. Where a section of classic line is incorporated within the HS main line, this will be enhanced to a line speed of 225kph if possible, or 200kph if not. (No classic sections will, in general, be incorporated, except on the immediate approach to stations, which are not capable of at least 200kph.)

This appendix originally dealt with two principal, overall topics, line capacity and estimated journey times, but with the (fairly) new appendix C dealing in exhaustive detail with the techniques for estimating journey times, the (relatively superficial) journey time sections of the present appendix have (at v.4.7) been shifted across and integrated with appendix C, which is clearly where they really belong,.

The following table summarises the relevant values taken from the above sources, and various results derived from them. (The sources use metric units, and while I perform the calculations in these units, I usually quote the results in imperial units also, as I like them considerably better, as probably do most people of my generation.)

Fundamental Dynamic Values					
Line Speed (kph)	200	225	300	360	400
Line Speed (mph)	125	140	187.5	225	250
Line Speed (m/s)	55.6	62.5	83.3	100.0	111.1
Train/Platform Length (m)	400	400	400	400	400
Train/Platform Length (ft)	1312	1312	1312	1312	1312
Buffer Zone (m)	300	300	300	300	300
Buffer Zone (ft)	984	984	984	984	984
Average Acceleration (m/s**2)	0.3	0.3	0.3	0.3	0.3
Average Deceleration (m/s**2)	0.5	0.5	0.5	0.5	0.5
Acceleration Time (s)	185	208	278	333	370
Acceleration Time (min:sec)	03:05	03:28	04:37	05:33	06:10
Acceleration Distance (km)	5.1	6.5	11.6	16.7	20.6
Acceleration Distance (mile)	3.2	4.0	7.2	10.4	12.8
Service Brake Time (s)	111	125	167	200	222
Service Brake Time (min:sec)	01:51	02:05	02:47	03:20	03:42
Service Brake Distance (km)	3.1	3.9	6.9	10.0	12.3
Service Brake Distance (mile)	1.9	2.4	4.3	6.2	7.7
Turnout Limit Speed (kph)	None	None	230	230	230
Turnout Limit Speed (mph)	(line	(line	143.8	143.8	143.8
Turnout Limit Speed (m/s)	speed)	speed)	63.89	63.89	63.89
(Basic) Train Separation Distance (km)	3.8	4.6	7.6	10.7	13.0
(Basic) Train Separation Distance (mile)	2.4	2.9	4.7	6.6	8.1
Extended Train Separation Distance (km)	3.8	4.6	8.1	12.2	15.6
Extended Train Separation Distance (mile)	2.4	2.9	5.1	7.6	9.7

Interpretation and Consequences of the above Values

The above values, (for average acceleration and deceleration, train length, buffer zone and turnout line speed,) are direct quotes from the two articles. Acceleration distance and time, and service brake distance and time, are calculated values, derived as explained in the next section. They represent the distance required to accelerate from stationary to line speed, and the time taken, or to decelerate under normal operating conditions, (thus not an **emergency** brake application,) from line speed to stationary, and the time taken to do so. (Piers Connor gives values for line speeds of 300 and 360kph, which do indeed agree exactly with the derived values, above.)

The (basic) train separation distance is the minimum distance which must be maintained between the front of one train and the front of the following train; it is composed of the braking distance, the buffer zone and the train length. (The buffer zone seems rather an arbitrary, 'rule of thumb' component. Piers Connor uses a value of only 100m, in the earlier article, for a line speed of 300kph. I have taken his later, more conservative value, throughout.) It is thus the distance which must be maintained between trains for a train to be able to stop without hitting the preceding train in the extreme case that that preceding train suddenly stopped dead (so to speak – what it means is if the preceding train suffered a catastrophic accident). It is assumed that train control and monitoring technology is so good that each train 'knows' its precise distance behind the preceding train at all times, and will automatically brake accordingly if this falls below the separation distance for any reason. (Piers Connor also includes a response time in one such calculation; this seems to imply a degree of manual control. I do not include this; interested readers should refer to the original articles for further details.)

The turnout limit speed is the maximum speed allowed when diverging or joining at a junction. The value of 230kph reflects the best available now, (or as it was at 2014,) for new installations. This is where the simplified explanation of HS operation given earlier (trains diverge from the main line at full line speed) really falls down; there are no points currently available which would allow for a turnout speed of 360kph or even 300kph, nor likely to be in the near future. (This is not a problem at the lower line speeds considered, of 200 and 225kph.) What this means in practice is that diverging trains have to slow down on the main line, to reduce their speed to the turnout limit by the point at which the turnout is reached. (Likewise trains joining the main line accelerate up to the turnout limit, by the time they reach the junction, and then continue to accelerate on the main line until they reach full line speed.) This critically determines the locations of the turnout / rejoin junctions either side of a station. The **extended** train separation distance in the above table includes an extra component to cover divergence at a junction. The calculation of this enhanced component is explained in the section 'The Effect of Junctions: Advanced Aspects'. By adopting the extended separation distance, the statement that stopping trains do not delay non-stop ones because they get out of their way in a timely fashion is literally true. A stopping train slowing down before the diverging junction for a station stop, or, of course, a diverging train at a route (as opposed to a station) junction, does not delay an immediately following non-stop / straight-ahead train because the enhanced separation distance is such that the following train, travelling at full line speed, gets no closer to the diverging train than the **train** separation distance, at that precise point where the diverging train actually diverges at the junction, and is then out of the path of the following train..

Of course, the above station penalty applies only where some trains are non-stop (UHS) over most of the distance. For HS Metro routes – all except HS2, HS3 and HS4 – where every train stops at every station, there is simply no problem, nor any need for station loops and fancy point work at stations. This is of course still required at genuine route junctions – the only ones left.

Derivation of Necessary and Useful Results; Line Capacity

For those of us who last used calculus regularly some time ago (50 years in my case) a short crib is in order.

If s, v, a and t are distance, speed, acceleration and time, then:

$v = ds/dt$ $\quad$ $a = dv/dt$ $\quad$ so, assuming constant acceleration a:

$v = \int a\, dt = at$ $\quad$ $s = \int v\, dt = at^2/2$ $\quad$ thus, for **definite** integrals:

$v - v_0 = a(t - t_0)$ so $v = v_0 + a(t - t_0)$

$s - s_0 = \int v\, dt = \int [v_0 + a(t - t_0)]\, dt = v_0(t - t_0) + a(t^2 - t_0^2)/2 - at_0(t - t_0)$

$\quad = (t - t_0)[v_0 + a(t + t_0)/2 - at_0] = (t - t_0)[v_0 + a(t - t_0)/2]$

$s = s_0 + v_0(t - t_0) + a(t - t_0)^2/2$

Notice how involved it gets when we are dealing with definite integrals, and a double integration (acceleration to speed, speed to distance) is involved. (It took me a **long** while to work it out!) If $s_0 = v_0 = t_0 = 0$ then it simplifies enormously, which is why it is very much easier to calculate the results to or from a standstill, and take differences to obtain intermediate results. (The above formula for s will calculate **directly** the deceleration distance to the junction; s_0 and t_0 are zero, but v_0 **isn't** – and don't forget $a<0$!).

First of all I will consider line capacity as a function of speed. Be warned that this is a very simple-minded approach, (refer back to Piers Connor's original articles for a full account of the many variables involved,) but the results are, I feel, very interesting and quite surprising.

Train Separation Distance (I call this 'Train Envelope' in appendix A) = Service Brake Distance + Train Length + Buffer

Service Brake Distance = distance required to come to a standstill (under normal, not emergency braking conditions)

$v = at$, so $t = v/a$. $s = at^2/2, = v^2/2a$.

So Train Envelope = $v^2/2a + 400 + 300 = v^2/2a + 700$

Capacity = number of trains passing a given point per unit time, = speed / envelope

Thus **$c = v/(v^2/2a + 700)$ trains per second, $= 3600v/(v^2/2a + 700)$ tph**

$$dc/dv = [(v^2/2a + 700)*3600 - 3600v* v/a]/(v^2 + 700)^2 = 3600(700 - v^2/2a)/(v^2/2a + 700)^2$$
$$= \mathbf{0 \text{ when } v^2 = 1400a;\ v = \sqrt{(1400a)}, = 26.46m/s \text{ given } a = 0.5m/s^2}$$

$$d^2c/dv^2 = [-(v^2/2a + 700)^2 * 3600v/a - 3600(700 - v^2/2a) * 2(v^2/2a + 700) * v/a]/ (v^2/2a + 700)^4]$$
$$= (3600v/a) * [-(v^4/4a^2 + 1400v^2/2a + 490000) + 2(v^2/2a - 700) *(v^2/2a + 700)]/(v^2/2a + 700)^4]$$
$$= (3600v/a) * [-v^4/4a^2 - 1400v^2/2a - 490000 + 2(v^4/4a^2 - 490000)]/(v^2/2a + 700)^4]$$
$$= (3600v/a) * [v^4/4a^2 - 1400 v^2/2a - 1470000)/(v^2/2a + 700)^4]$$

When $v^2/2a = 700$ then $v^4/4a^2 - 1400 v^2/2a - 1470000 = 490000 - 980000 - 1470000 = -637000$, in other words the second derivative of the capacity is negative at the extremum value of v, confirming that this extremum is in fact a maximum.

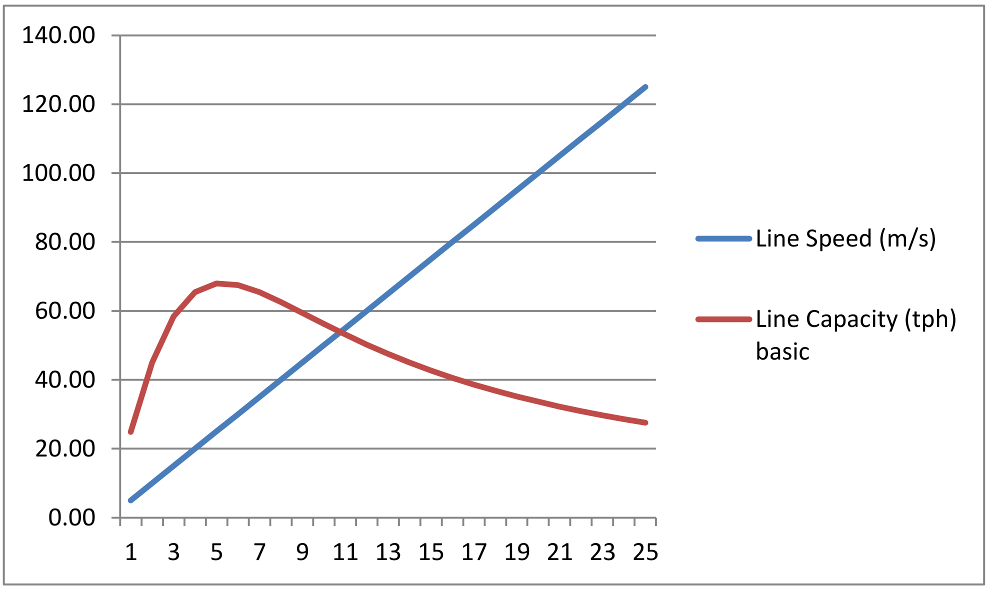
Thus, for the deceleration rate $0.5m/s^2$, the maximum capacity of the line,
c_{max}, = 68.0336tph, at a line speed, v_{cmax} = 26.4575m/s, = 95.2471kph = 59.1855mph.

This is, as stressed, a very simplistic argument, nonetheless, it is rather surprising that the maximum capacity occurs at such an astonishingly low speed. On the other hand, it may not be surprising that the highest capacity values occur in the speed range of the typical metro system.

It does, I regret to say, make the argument that HS railways are, more importantly, high capacity railways, look rather sick. They **are** of course high capacity, but only in comparison with mixed-traffic railways; they derive their capacity benefit from the traffic being homogeneous in its performance characteristics, just like a metro. (A heterogeneous traffic mix is lethal for capacity.) One may thus say that a HS railway has the maximum available capacity **for a particular line speed**, and one must now never omit the latter qualification. Of course, whereas maximum capacity at a line speed of 60mph is just fine for metros, there would be very few takers for long distance travel at that speed, no matter how many trains per hour the line could carry. The maximum speed to aim for is thus a business decision, not a technical one.

The full set of results, with graph, follows:

Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph) basic
5.00	18.00	11.19	24.83
10.00	36.00	22.37	45.00
15.00	54.00	33.56	58.38
20.00	72.00	44.75	65.45
25.00	90.00	55.94	67.92
30.00	108.00	67.12	67.50
35.00	126.00	78.31	65.45
40.00	144.00	89.50	62.61
45.00	162.00	100.68	59.45
50.00	180.00	111.87	56.25
55.00	198.00	123.06	53.15
60.00	216.00	134.24	50.23
65.00	234.00	145.43	47.51
70.00	252.00	156.62	45.00
75.00	270.00	167.81	42.69
80.00	288.00	178.99	40.56
85.00	306.00	190.18	38.61
90.00	324.00	201.37	36.82
95.00	342.00	212.55	35.17
100.00	360.00	223.74	33.64
105.00	378.00	234.93	32.24
110.00	396.00	246.12	30.94
115.00	414.00	257.30	29.73
120.00	432.00	268.49	28.61
125.00	450.00	279.68	27.57



(This calculation will be repeated in the next section but one, using an extended train separation distance. But first, the effect of junctions must be investigated.)

The Effect of Junctions

Two distinct cases need to be considered, the pure route junction, where routes diverge (for different destinations) or converge, and the double junctions required either side of a station, where some services are non-stop. The calculation is illustrated for line speed 360kph.

Consider first the diverging case:

The diverging train must decelerate to the turnout speed limit, by the time that it reaches the junction. (As noted earlier, the calculation is most easily performed by taking the decelerations to zero, then taking the differences.) Thus, for line speed 360kph and turnout limit speed 230kph:

1. 230kph to zero:
   $v = 0$ $v_0 = 63.89$ $a = -0.5$ $s0 = 0$ $t_0 = 0$ so:
   $63.89 = 0.5t$ so $t = 127.78$sec
   $s = 0.5t^2/2 = 127.78^2/4 = 4082$metres
2. 360kph to zero:
   $v = 0$ $v_0 = 100.00$ $a = -0.5$ $s_0 = 0$ $t_0 = 0$ so:
   $100.00 = 0.5t$ so $t = 200.00$sec
   $s = 0.5t^2/2 = 200.00^2/4 = 10000$metres

so the diverging train decelerates from 360kph to 230kph at the junction in a distance of (10000 – 4082) = 5918metres, 5.92km, (3.68miles,) and in a time of (200.00 – 127.78) = 72 secs.

Now consider the converging case (imagine that the train accelerates from a standstill, reaching the turnout / turnin speed at the junction):

1. Zero to 230kph:
   $v = 63.89$ $v_0 = 0$ $a = 0.3$ $s0 = 0$ $t_0 = 0$ so:
   $63.89 = 0.3t$ so $t = 213$sec
   $s = 0.3t^2/2 = 0.15 * 148^2 = 6803$metres
2. Zero to 360kph:
   $v = 100.00$ $v_0 = 0$ $a = 0.3$ $s_0 = 0$ $t_0 = 0$ so:
   $100.00 = 0.3t$ so $t = 333$sec
   $s = 0.3t^2/2 = 0.15 * 333^2 = 16667$metres

so the converging train accelerates from 230kph at the junction to 360kph in a distance of (16667 – 6803) = 9864metres, 9.86km, (6.13miles,) and in a time of (333 – 213) = 120 secs.

For the case of a train diverging at a route junction, as opposed to stopping at a station, it must first decelerate from 360kph to 230kph, then travel through the junction at 230kph, (the train length being 400m, this takes t = s/v = 400/63.89 = 6.26sec,) then accelerates back up to 360kph. Thus the total time decelerating and reaccelerating is 72 (decelerating) + 6 (diverging) + 120 (accelerating = 198sec, likewise, the distance travelled is 5918 (decelerating) + 400 (diverging) + 9864 (accelerating) = 16182m. The time it would take to travel 16182m at line speed, 360kph, is 16182/100 = 161.82 = 162sec. So the time penalty imposed by a route junction on a diverging train is 198 – 162 = 37sec (rounded!!! – the actual values, to 6 decimal places, are: 198.853463 – 161.818930 = 37.034533). Exactly the same values apply for a converging train. So the time penalty imposed by a route junction is 37sec for all diverging or converging trains, with no penalty at all for trains passing straight through on the main line.

The situation around a station requires very little further calculation. The total distance between the beginning of the deceleration before the station and completion of acceleration after it is the sum of the deceleration distance to a full stop at the station (10000metres in 200secs) and the acceleration distance from stationary after it (16667metres in 333secs). Thus we have a total distance affected by the presence of the station of 10000 + 16667 = 26667metres = 26.7km, and a total deceleration / acceleration time of 200 + 333 = 533secs. This distance travelled at 360kph would take 26667/100 = 267secs, so the penalty time for stopping at the station is 533 – 267 = 266secs = 4min26secs, plus whatever the waiting time is at the station, ideally about 3 minutes, so the total time penalty of a station stop is 7 minutes, let’s say, for a line speed of 360kph.

The above exposition uses line speed 360kph for illustration. The results for all the line speeds of interest are given in the table below:

Junction Effects					
Line Speed (kph)	200	225	300	360	400
Line Speed (m/s)	55.6	62.5	83.3	100.0	111.1
Turnout Limit Speed (kph)	200	225	230	230	230
Turnout Limit Speed (m/s)	55.6	62.5	63.9	63.9	63.9
Average Acceleration (m/s**2)	0.3	0.3	0.3	0.3	0.3
Average Deceleration (m/s**2)	0.5	0.5	0.5	0.5	0.5
Station Decelerating Time, Total (s)	111	125	167	200	222
Station Decelerating Distance, Total (km)	3.1	3.9	6.9	10.0	12.3
Decelerating Time on Station Loop (s)	111	125	128	128	128
Decelerating Distance on Station Loop (km)	3.1	3.9	4.1	4.1	4.1
Decelerating Time on Main Line (s)	0	0	39	72	94
Decelerating Distance on Main Line (km)	0.0	0.0	2.9	5.9	8.3
Station Accelerating Time, Total (s)	185	208	278	333	370
Station Accelerating Distance, Total (km)	5.1	6.5	11.6	16.7	20.6
Accelerating Time on Ststion Loop (s)	185	208	213	213	213
Accelerating Distance on Station Loop (km)	5.1	6.5	6.8	6.8	6.8
Accelerating Time on Main Line (s)	0	0	65	120	157
Accelerating Distance on Main Line (km)	0.0	0.0	4.8	9.9	13.8
Time to Travel Across Route Junction (s) (*)	7	6	6	6	6
Route Junction Time Penalty (s)	0	0	14	37	56
Station Stop Time Penalty (mins)	5.5	5.8	6.7	7.4	7.9

(*) This is for a route junction which is also a track junction, which is not invariably the case.

Say c.7 minutes for a station stop time penalty, which is close enough for both 360 and 300kph.

The Effect of Junctions: Advanced Capacity Aspects; Extended TSD

As explained earlier, the (basic) train separation distance is the (minimum) distance which must be maintained at all times between a given train and the one immediately preceding it (when the trains are both in motion, of course). It is assumed that train monitoring and control is so good that this actual, dynamic value is always known (to the following train – it’s of little interest to the preceding train). In other words, we assume that each train 'knows', at each instant, its precise distance behind the preceding train.

If the preceding train is due to diverge at a junction, then (for all line speeds above the turnout limit speed of the junction, 230kph in the present context,) it must begin to decelerate on the main line, before reaching the junction, to reduce its speed to the turnout limit speed by the time it actually arrives at the junction. If services on the route were scheduled to maintain only the basic train separation distance (to achieve maximum line capacity at the given line speed) then, as soon as the train began to decelerate, the train immediately following would very quickly detect that it was gaining on the preceding train, in that the separation distance had fallen below the minimum, and would itself automatically begin to decelerate, to bring the separation distance back up to the required value. (Remember that the major component of the actual, current train separation distance varies as the square on the actual, current speed, so the effect would be that the following train is always travelling very slightly faster than the diverging train, and getting closer to it, since the dynamic separation distance is itself reducing with the speed.) And the next following train would likewise detect that the separation distance had reduced, and begin to decelerate, and the one behind that, and so on all the way back down the line. So the one train due to diverge at the junction would, as soon as it began to decelerate, cause every following train to decelerate likewise. Clearly, this is a ludicrous situation, so some other strategy must be developed.

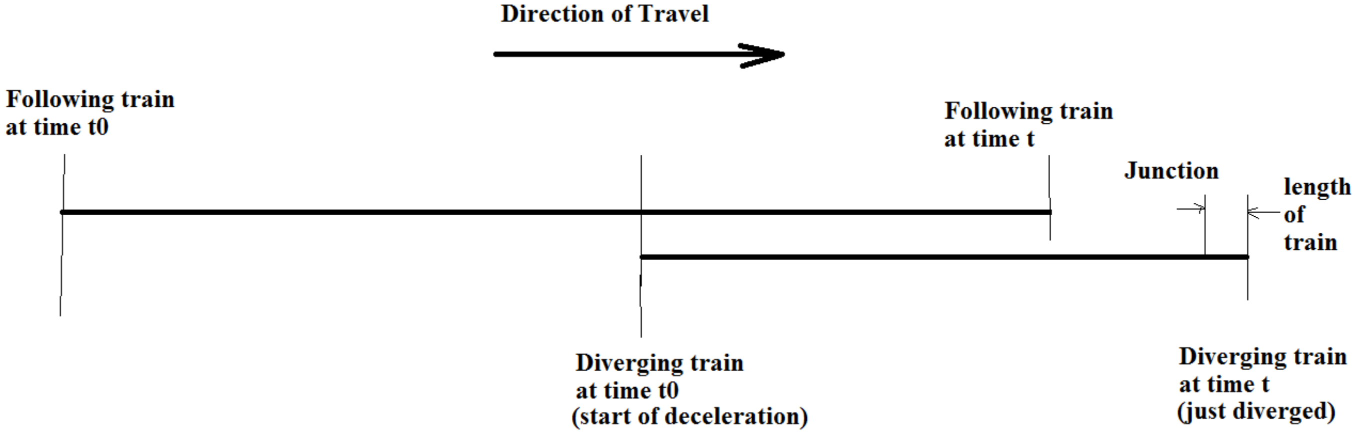
I first describe this alternative strategy, then derive the new value of the separation distance, the **Extended Train Separation Distance**.

The fundamental requirement is that the dynamic separation distance between trains never falls below the basic train separation distance, but of course it doesn't matter at all if it exceeds that value (except insofar as it reduces the line capacity somewhat). So the idea is that trains, running at full line speed, maintain an increased separation distance such that, if one train is scheduled to diverge at a junction, and the immediately following train does not diverge, then that following train may continue to run at full line speed, all the time getting closer to the diverging train, but only getting as close to it as the basic train separation distance at the point when the diverging train has actually just diverged, i.e. when it has **completely** run through the junction and the entire train is now on the diverged track, and thus no longer in the way of the following train. The extended train separation distance is the distance between trains (front of first train to front of second) at which the above is precisely true. This is the smallest possible value for which the following train need not decelerate at all, so, truly, 'stopping trains do not obstruct non-stop trains because they get out of their way in a timely fashion'. Likewise for diverging and non-diverging trains at a route junction as opposed to a station junction.

The diverging train decelerates to the turnout limit speed in a distance $s = v_l t + at^2/2$, and $v_t = v_l + at$ where v_l is the line speed and v_t the turnout limit speed (both of which are known), t is the deceleration time and a the acceleration (negative value, of course). (These are from the definite integral formulae derived in the calculus crib on p.11.) So $t = (v_t – v_l)/a$ and $s = v_l(v_t – v_l)/a + a((v_t – v_l)^2/a^2)/2$. These values are for when the train actually reaches the junction. It also has to travel through the junction. This involves an extra distance of 400m (the length of the train), travelled at speed v_t thus in a time of $400/v_t$. (Yes, this is an approximation. The train is still decelerating, so assuming it crosses the junction at the constant turnout limit speed gives a slight underestimate of the time taken. But since this quantity (the time to cross the junction at constant speed v_t) is itself tiny anyway – 6.26s – the inaccuracy is minute.

Thus, in time $t = (v_l – v_t)/a + 400/v_t$ secs, the diverging train decelerates from v_l to (slightly less than) v_t in a distance $s = v_l(v_t – v_l)/a + a((v_t – v_l)^2/a^2)/2 + 400, = (v_l^2 - v_t^2)/2a + 400$.
In the same time, the following train travels a distance $v_l t = v_l((v_l - v_t)/a + 400/v_t)$ at line speed v_l.

In the following line diagram, s_b, the basic train separation distance, = $v_l^2/2a$ + const, (const being the constant stuff included, the train length and buffer zone,) and s_e, the extended train separation distance, is the distance between the trains at time t0, as s_b is the distance between them at time t. Thus:



$\leftarrow$-------------------------- $v_l((v_l – v_t)/a + 400/v_t)$ -------------------------------$\rightarrow\leftarrow$----- s_b ------$\rightarrow$

$\leftarrow$--------------------- s_e ------------------------$\rightarrow\leftarrow$------------- $(v_l^2 - v_t^2)/2a + 400$ ----------------$\rightarrow$

$s_e = v_l((v_l – v_t)/a + 400/v_t) + s_b - ((v_l^2 - v_t^2)/2a + 400)$
$\quad = v_l(v_l – v_t)/a + (v_l^2/2a + \text{const}) – (v_l^2 - v_t^2)/2a - 400(1 – v_l/v_t)$

So $\mathbf{s_e = [v_l^2 + (v_l – v_t)^2]/2a + const - 400(1 – v_l/v_t)}$ **and** $\mathbf{s_b = v_l^2/2a + const}$
Capacity = speed/envelope $\mathbf{c = v/[[v_l^2 + (v_l – v_t)^2]/2a + const - 400(1 – v_l/v_t)]}$ tps, *3600 for tph
(Note that const = 400 + 300 = 700m. The separately quoted quantity 400 refers to crossing the junction.)
Thus $\mathbf{(s_e – s_b) = (v_l – v_t)^2/2a - 400(1 – v_l/v_t)}$

Thus, to allow the following train to proceed at line speed all the way, we need an **extra** distance between trains of $(v_l – v_t)^2/2a - 400(1 – v_l/v_t)$

These are completely general results. Applying the particular values of interest (the separately quoted 700m is of course the constant stuff – train length and buffer zone, and deceleration a is $0.5m/s^2$):

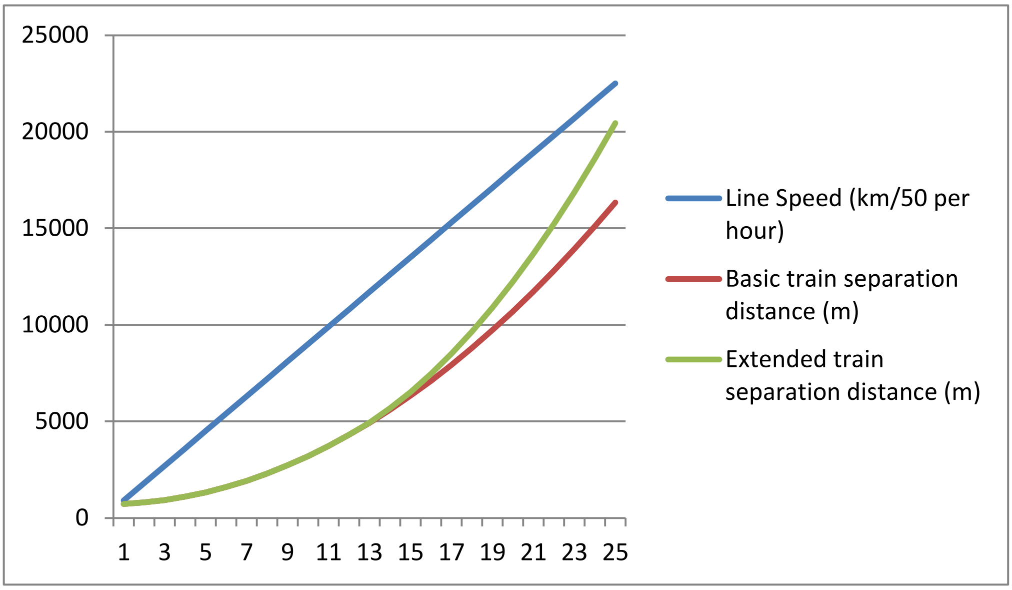
1. **v_l = 300kph (v_t = 230kph) s_e = 7437 + 700 s_b = 6939 + 700 ($s_e – s_b$) = 498**
2. **v_l = 360kph (v_t = 230kph) s_e = 11529 + 700 s_b = 10000 + 700 ($s_e – s_b$) = 1529**
3. **v_l = 400kph (v_t = 230kph) s_e = 14870 + 700 s_b = 12345 + 700 ($s_e – s_b$) = 2525**

Thus for v_l = 300kph, s_e adds 6.5% to s_b, for v_l = 360kph it adds 14.3% and for v_l = 400kph it adds 19.4%.

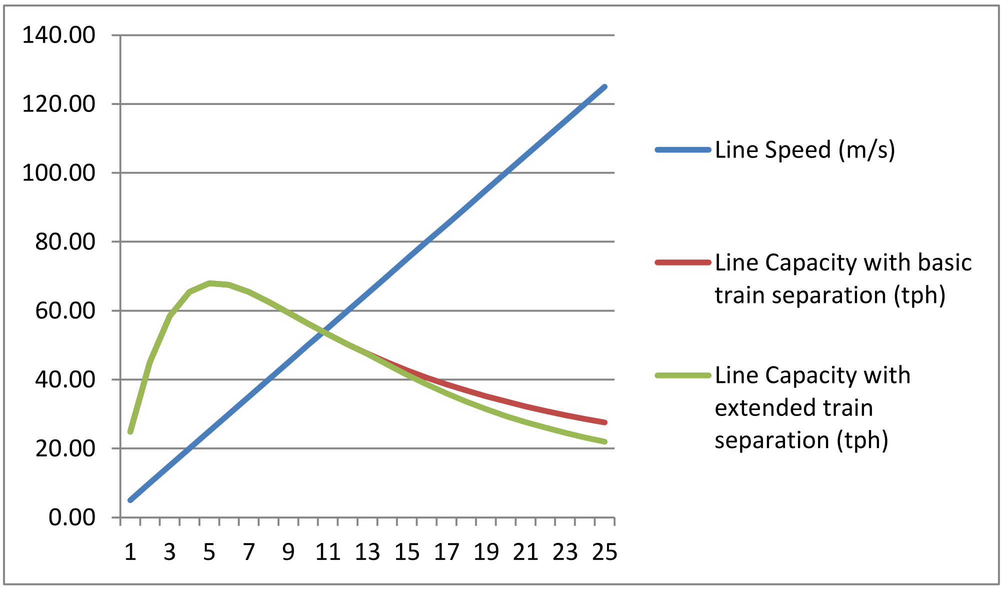
Basic and extended train separation distances have been derived in a spreadsheet, and plotted on a line chart. My apologies for the truly weird unit used for line speed (50ths of a km per hour, i.e. the number of 20metre units per hour!) – it is of course purely to get this variable to use the full area of the chart – otherwise it's stuck right at the bottom, with a gradient of near zero.

Then follows a spreadsheet of line capacity, giving the results for both basic and extended train separation distances.

Advanced Junction Effects				
Line Speed (m/s)	Basic TSD (m)	Extended TSD (m)	Basic - Extended (m)	% of Basic
5	725	725	0	0
10	800	800	0	0
15	925	925	0	0
20	1100	1100	0	0
25	1325	1325	0	0
30	1600	1600	0	0
35	1925	1925	0	0
40	2300	2300	0	0
45	2725	2725	0	0
50	3200	3200	0	0
55	3725	3725	0	0
60	4300	4300	0	0
65	4925	4933	8	0.2
70	5600	5676	76	1.4
75	6325	6518	193	3.1
80	7100	7460	360	5.1
85	7925	8503	578	7.3
90	8800	9645	845	9.6
95	9725	10888	1163	12.0
100	10700	12230	1530	14.3
105	11725	13673	1948	16.6
110	12800	15215	2415	18.9
115	13925	16857	2932	21.1
120	15100	18600	3500	23.2
125	16325	20442	4117	25.2



Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph) basic	Line Capacity (tph) ext.
5.00	18.00	11.19	24.83	24.83
10.00	36.00	22.37	45.00	45.00
15.00	54.00	33.56	58.38	58.38
20.00	72.00	44.75	65.45	65.45
25.00	90.00	55.94	67.92	67.92
30.00	108.00	67.12	67.50	67.50
35.00	126.00	78.31	65.45	65.45
40.00	144.00	89.50	62.61	62.61
45.00	162.00	100.68	59.45	59.45
50.00	180.00	111.87	56.25	56.25
55.00	198.00	123.06	53.15	53.15
60.00	216.00	134.24	50.23	50.23
65.00	234.00	145.43	47.51	47.43
70.00	252.00	156.62	45.00	44.40
75.00	270.00	167.81	42.69	41.42
80.00	288.00	178.99	40.56	38.60
85.00	306.00	190.18	38.61	35.99
90.00	324.00	201.37	36.82	33.59
95.00	342.00	212.55	35.17	31.41
100.00	360.00	223.74	33.64	29.44
105.00	378.00	234.93	32.24	27.65
110.00	396.00	246.12	30.94	26.03
115.00	414.00	257.30	29.73	24.56
120.00	432.00	268.49	28.61	23.23
125.00	450.00	279.68	27.57	22.01



The Effect of Junctions: Even More Advanced Aspects; Converging Trains

As well as considering the effect of trains diverging at junctions, we also need to consider trains joining at junctions. This is surprisingly difficult, even to describe, let alone to analyse from first principles. (At least, that's what I find; you may be cleverer than me.) It's strange that it is so much easier to envisage and analyse a stream of traffic from which certain trains diverge, than the reverse case where trains join an existing stream (in which spaces have been reserved for them).

The main thing to recognise about the divergence pattern, taking the extended train separation distance to ensure that a train diverging at a junction has no effect on the following train which proceeds straight ahead on the main line, is that the results are (slightly) over-pessimistic (not a bad thing from a safety perspective, of course). The only case where the extended separation distance is actually required is precisely that, where a diverging train is directly followed by a non-diverging one. When two adjacent trains both proceed directly along the main line, the basic separation distance between them would be adequate. If two adjacent trains are both diverging, then the distance between them needs to be a little greater that the basic value, but not as much as the extended value, since the second train has already begun its own deceleration for the junction long before it gets too close (at line speed) to the preceding one. (There's no need to calculate this; the mere perception that it is less than the extended value suffices.) In fact, it is quickly clear that the worst, i.e. lowest, capacity occurs if trains are alternately diverging and straight ahead, when the separation values are (i.e. need to be at least) basic and extended separation distances, alternately. Thus, if s_b and s_e are respectively the basic and extended separation values, then the separation distance between any two adjacent trains **of the same type,** (diverging or straight ahead, but with a train of the **other** type between them,) is (s_b + s_e), so the **actual worst minimum** capacity at line speed v_l is $2v_l/(s_b+s_e)$ – an astonishingly simple result. Note precisely what I'm saying here: in the worst case, when diverging and straight-ahead trains alternate, the above formula gives the best possible value for capacity. In any other traffic mix, a (very slightly) higher capacity value would (at least in theory) be possible (by holding individual train pairs to the minimum separation value that they actually require). In fact we'd never bother even to attempt it: the gains would be minute and the extra complication considerable. But it does give reassurance that the extended train separation distance is in fact a good, conservative, indeed slightly pessimistic standard.

The spreadsheet and graph of capacity vs line speed is reworked to display the three results, for basic and extended separations, and a 'mixed' value, from $c = 2v_l/(s_b+s_e)$, showing the actual, theoretical worst case.

The above considerations of the case of diverging trains give the necessary clue to the best way of analysing the case of converging trains. The requirement is to calculate the minimum separation distance which must be maintained between two adjacent trains travelling at full line speed on the main line, which will allow a converging train to be inserted between them at a junction. An argument. identical in its essentials to the one above, shows that the absolute worst service pattern, from the point of view of line capacity, is when trains alternate between converging and straight ahead. Any other service pattern could (in theory, though it wouldn't be worth doing in practice, to add so much complexity for so little gain,) give a very slightly higher line capacity. Calculating the capacity on the usual formula capacity = line speed / separation distance, using the value above for separation distance, then doubling the result to include the converging trains, give the absolute minimum value of capacity.

A converging train joins the main line at a junction at the precise instant where a train on the main line which has just run through the junction, travelling at full line speed, has reached a point a new separation distance – the **convergence** separation distance s_c – beyond the junction. This new distance $s_c = v_t^2/2a_d$, where v_t is (as usual) the turnout limit speed for the junction, which is the speed at which the converging train is actually travelling at that point, and a_d is the average deceleration value, because, of course, the separation distance is always determined by the need to bring the following train to a standstill. This distance is of course significantly smaller than the basic separation distance, which is for stopping from the full line speed. (s_c = 4083 + 700 = 4783m and s_b = 10000 + 700 = 10700m, well over twice as much, for a line speed of 360kph and turnout speed of 230kph.)

The converging train, of length 400m as usual, travels through the junction at the turnout limit speed, thus taking a time of 400 / 63.9 = 6s. It then accelerates up to full line speed in 120s and a distance of 9900m (these values are from the table of junction effects, on 14). But in this total time of 126s, the preceding train on the main line, travelling at full line speed, has travelled 12600m. The train separation distance is thus now 12600 – 9900 = 2700m. This is in fact very much less than the basic train separation distance of 10700m at this line speed. Thus the basic train separation distance, never mind the extended one, is very much more than adequate to accommodate converging trains also. Of course, that's not what we want; we want the separation distance between the preceding train and the converged train to be the extended separation distance. The point of this argument is to demonstrate that there is plenty of scope for this, i.e. the case of a converging train is very much less demanding than the diverging case.

We must also consider the following train on the main line. This is also travelling at full line speed, such that, at the point that the converging train itself reaches line speed, the following train has closed the distance between them to precisely the extended train separation distance. As explained in the previous paragraph, the converging train easily slots onto the extended train separation distance behind the first train, being, when it has accelerated up to line speed, precisely the extended separation distance behind the preceding train, and also the same distance ahead of the following train. Thus a separation distance of twice the extended value, between adjacent trains on the main line, provides very ample scope for a converging train to slot in between them at the junction, such that, when it has accelerated to line speed, it is exactly the extended separation distance behind the first train and the same ahead of the second.

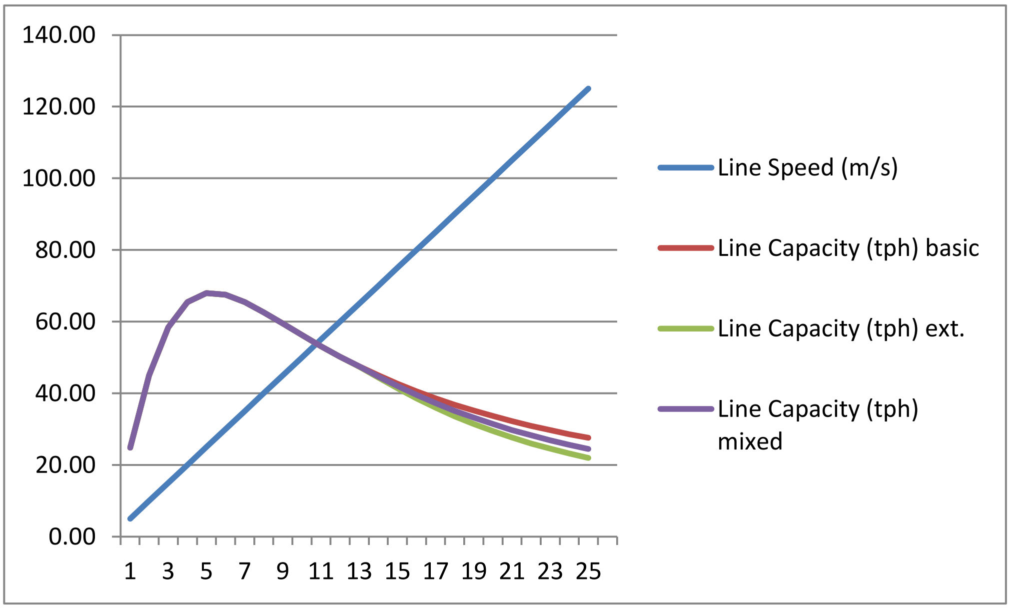
This surprising result (it surprised me, anyway,) means that the extended train separation distance is valid in all cases, for a mix of diverging, converging and straight-ahead trains. It is, indeed, a reassuringly pessimistic standard. This has been an unavoidably intricate argument. See also the section immediately after the next, for a much later, alternative derivation, which is, I believe, much easier to follow.

There follows the spreadsheet and graph of capacity vs line speed for all traffic mixes.

On the basis of these capacity values, I feel justified in using 24tph as the new maximum capacity, this being slightly less than the value for a line speed of 400kph (250mph). This does of course assume absolute reliability in timekeeping, through full automatic train control, so is idealistically optimistic. But it is, nonetheless, a reasonable target to aim for, particularly if we actually go, long term, for a line speed of only(!) 360kph (225mph).

Capacity vs. Line Speed for Traffic Flows including Diverging (and Converging) Trains

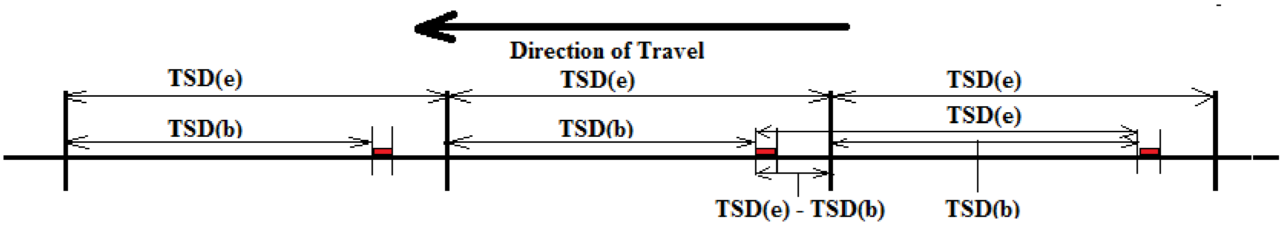
Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph) basic	Line Capacity (tph) ext.	Line Capacity (tph) mixed
5.00	18.00	11.19	24.83	24.83	24.83
10.00	36.00	22.37	45.00	45.00	45.00
15.00	54.00	33.56	58.38	58.38	58.38
20.00	72.00	44.75	65.45	65.45	65.45
25.00	90.00	55.94	67.92	67.92	67.92
30.00	108.00	67.12	67.50	67.50	67.50
35.00	126.00	78.31	65.45	65.45	65.45
40.00	144.00	89.50	62.61	62.61	62.61
45.00	162.00	100.68	59.45	59.45	59.45
50.00	180.00	111.87	56.25	56.25	56.25
55.00	198.00	123.06	53.15	53.15	53.15
60.00	216.00	134.24	50.23	50.23	50.23
65.00	234.00	145.43	47.51	47.43	47.47
70.00	252.00	156.62	45.00	44.40	44.70
75.00	270.00	167.81	42.69	41.42	42.05
80.00	288.00	178.99	40.56	38.60	39.56
85.00	306.00	190.18	38.61	35.99	37.25
90.00	324.00	201.37	36.82	33.59	35.13
95.00	342.00	212.55	35.17	31.41	33.18
100.00	360.00	223.74	33.64	29.44	31.40
105.00	378.00	234.93	32.24	27.65	29.77
110.00	396.00	246.12	30.94	26.03	28.27
115.00	414.00	257.30	29.73	24.56	26.90
120.00	432.00	268.49	28.61	23.23	25.64
125.00	450.00	279.68	27.57	22.01	24.48



The Capacity-Slot Model

The previous section but one dealt with trains converging onto the main line at a junction, and demonstrated that this situation was also covered, (more than adequately,) by the Extended Train Separation standard. Appendix B was added to this article at version 3.0 in February 2016, and that section, in precisely that form, was part of the original appendix. As noted, the case of converging trains is surprisingly difficult to describe (with precision), let alone analyse. I now return to the topic, nearly two years later (January 2018), approaching it from the precise behaviour of trains in the neighbourhood of a station, where some of them stop, and others overtake. This topic forms the next section, and uses the Capacity-Slot model. It is worth first devoting attention to the model itself, to demonstrate its rigour. It covers, inter alia, trains both diverging from and converging onto the main line, and does so in a way which is, I believe, considerably easier to understand than my previous effort.

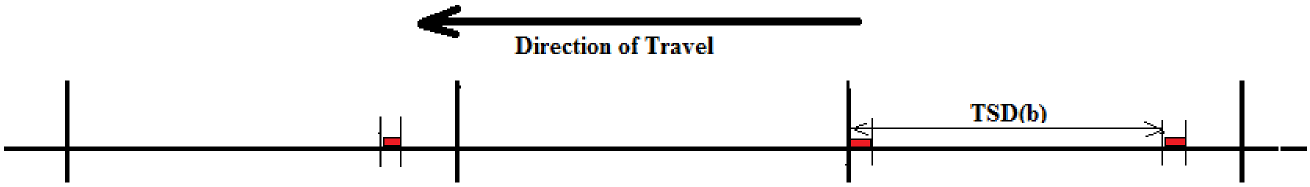
The Capacity-Slot model considers the main line to be occupied by a continuous sequence of slots, moving at the constant line speed. Each of these slots may be occupied by a single train (travelling at constant line speed, obviously). These trains are separated by the Extended Train Separation Distance, TSD(e). (This notation comes from the Line Capacity vs. Speed article.) The trains all occupy the same position within their slot, the slot size thus also being TSD(e). This steady-state condition is now depicted:



The little red oblongs represent the trains. The diagram is not quite to scale; the trains should be further back in the slot – the Basic Train Separation Distance, TSD(b), would in practice extend over 86% of the slot. Also they are drawn much too large; the actual train length would only be c.3% of the slot length.

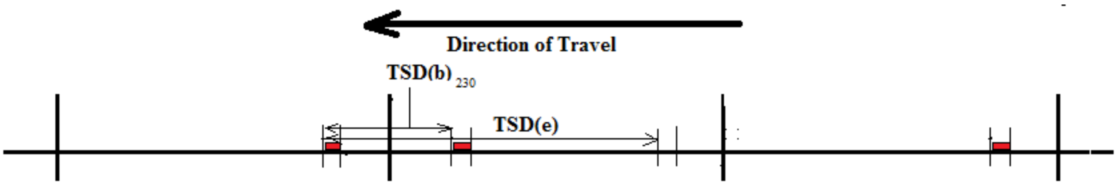
Just to recapitulate, TSD(b) = stopping distance from line speed + train length (400m) + buffer (300m), = 10.7km at 360kph and 7.6km at 300kph (and 4.8km at 230kph, which value will be needed shortly). Likewise TSD(e) is TSD(b) plus the difference between (the distance taken to decelerate from line speed to 230kph plus the train length) and the distance travelled at line speed in the same time (subtracting the former from the latter, of course. The train length (400m) is added to the deceleration distance because the diverging train must have completely diverged at the junction, and be (just) out of the path of the following train; it is of quite a different purpose from the train length in TSD(b).
TSD(e) = 12.3km at 360kph and 8.0km at 300kph. (Note that all these diagrams assume, in their scaling, a line speed of 360kph.)

The next diagram illustrates what happens when the middle train diverges.



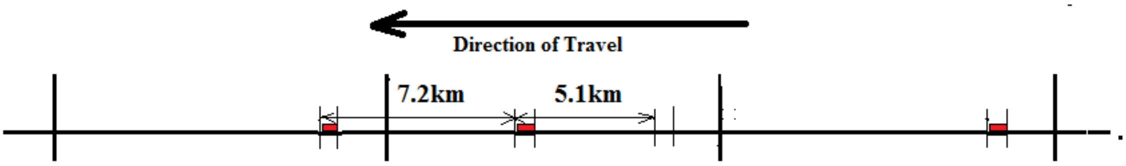
It decelerates to 230kph on the main line, during which time it moves towards the back of its slot, and the following train therefore gets closer. It then traverses the junction at 230kph. The diagram illustrates the situation when the train has just completely traversed the junction. It is, instantaneously, right at the back of its slot, whereupon it gives up the slot. Note in the diagram that all distances are measured to or from the **front** of the train. (This is always the case in all journey time calculations.) Despite appearances, it is right at the back of its own slot, and has **not** joined the following train in **its** slot! The distance between it and the following, straight-ahead train is now, instantaneously, TSD(b), the irreducible minimum at that line speed.

The next two diagrams illustrate what happens when a train converges onto the main line:



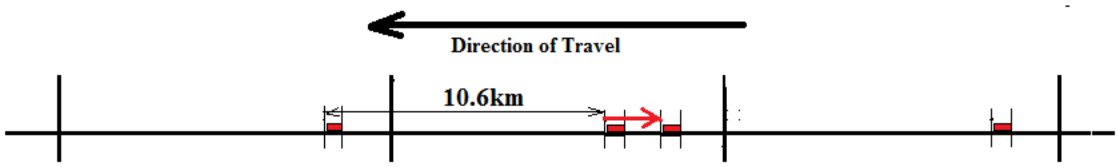
The converging train occupies the vacant slot, a distance $TSD(b)_{230}$ behind the preceding train. (This value was quoted above.) It only needs $TSD(b)_{230}$, rather than $TSD(e)_{230}$, because the preceding train is travelling (much) faster, so is getting further away rather than closer (which is what the extended distance is concerned with). The distance **between** the trains is depicted fairly accurately in the above diagram.

Note that the distance $TSD(b)_{230}$ is measured at the instant the front of the converging train reaches the main line; that is also the instant at which it takes over the slot. The rest of the train (400m) must then traverse the junction at a constant 230kph before the train can begin to accelerate up to line speed. The distance required to accelerate from 230kph to 360kph = 16.67 – 6.80 = 9.87km, to which is added 0.4km, for the train traversing the junction at 230kph, thus 10.27km in total. This it does in a time of 333.3 – 213.0 = 120.3 sec, to which is added 6.3 sec for traversing the junction, thus 126.6 sec in total. In this time the preceding train travels 12.66km at 360kph (100m/sec). the preceding train is thus now 7.17km in front. This is much too close; it should be TSD(e) = 12.3km. What this means is that the converging train has taken up a slot position 12.3 – 7.2 = 5.1km in advance of where it should be.



There are two ways in which this could be corrected (as it **must** be corrected): either the train delays its convergence until the latest possible moment, when the above procedure will ensure that it reaches line speed at exactly the correct position within its slot, or it converges at some earlier point, but initially travels at a steady 230kph, until it is just the right distance behind the preceding train, to accelerate up to line speed and reach exactly the correct position within its slot. (I suppose there is a third possibility: it could converge at an earlier point and then accelerate up to line speed, but at a lower rate of acceleration, so that, again, it reached line speed at exactly the correct location, but this strikes me as unnecessarily complicated; too clever by half!)

Consider convergence as late as possible, i.e. the converging train reaches the main line a distance s_c behind the preceding train , accelerating to line speed at exactly the correct position within its slot. The distance to accelerate up to line speed is 10.27km in a time of 126.6 sec, during which time the preceding train travels 12.66km, all these as before. Thus the preceding train is now 12.66 – 10.27 + s_c, = 12.3km, i.e. TSD(e). Thus s_c = 10.63km. The following diagram illustrates:



The red arrow indicates that the train moves further back in its slot to the precise position required, during its acceleration.

We thus have a **distance window** of 5.8km, between 4.8 and 10.6km behind the preceding train, during which the converging train may join the main line and occupy its slot. Since the preceding train is travelling at line speed, 360kph, (100m/sec,) that translates into a time window of 58 sec, between 48 and 106 sec behind the preceding train.

The way the convergence would actually be handled is that if a train's arrival time at the junction, as determined by its scheduled departure time from a station, or by its schedule from a converging route, falls within the time window for the (empty!) slot, then that is the time taken, and whatever distance necessary is travelled at 230kph after convergence to ensure precise arrival at its standard slot position. This ensures that the train holds to its schedule, If this is not possible, the train is delayed to join the first available, i.e. empty slot, at the earliest possible time, i.e. 48 sec behind the train position in the preceding slot; I state it precisely like that, because it is entirely possible that the preceding slot is empty, but our train may not join it because it has already missed the window for that slot. Having joined a slot at an arbitrary position in the slot window, the train travels at 230kph until it is 10.6km behind the preceding train or the preceding train slot position, and then accelerates up to line speed,

Summarising:

Slot Window for line speed 360kph: 5.8km, between 4.8 and 10.6km behind preceding train, or
58 sec, between 48 and 106 sec behind preceding train.

Slot Window for line speed 300kph: 3.0km, between 4.4 and 7.4kmbehind preceding train, and
36 sec, between 53 and 89 sec behind preceding train.

I think that the above treatment elucidates the precise behaviour very adequately. But I'm still very pleased with my first effort, which is correct in all its essentials. The only new result that the capacity slot treatment has added is the concept of the slot window, and how a converging train locates precisely the right position. The earlier treatment was, in any case, not interested in such matters, and was only aiming to confirm that the converging case was also covered by the Extended Train Separation Distance, which it did.

Anyone worrying about where the capacity slots come from and where they go to should imagine that, at the destination end of each line, they instantly reverse direction and come back along the other line, thus a continuous, endless cycle, a virtual conveyor belt, in effect. This may seem a humorous concept, but the consequences are definitely non-trivial.

We need to consider how, precisely, trains initially join a slot. This is superficially similar in concept to a train gaining a new slot on departure from an intermediate station, as already elucidated, but quite different in detail. The difference is that, whereas on re-starting from an intermediate station, the train accelerates up to turnout limit speed on the station loop before joining an available slot on the main line, at a minimum distance of TSD(b)$_{230}$ behind the preceding (non-stopping) train. To do this, it has to start well before its preceding main-line train passes through the station (non-platform line) without stopping. This is possible because they are on different tracks. This is not the case starting from the origin (and the concept of a non-stop train is meaningless there).

In fact the solution in this case is trivially simple. The trains depart from the originating station at a regular interval equal to the time it takes to travel TSD(e) at line speed (=TSD(b) for line speed < 230kph of course). Thus, when they have reached line speed, they are exactly TSD(e) apart. Thus the departure interval for line speed 300kph is 91.2 secs, for 360kph is 107 secs and for 400kph is 117.4 secs. The result is a surprise the first time you see it, thereafter it is obvious, given that that capacity = speed / train envelope, i.e. TSD.

The Effect of Stations

The previous sections have considered in detail the effect of intermediate stations, where some trains are non-stop and need to be able to overtake those stopping at the station. It is in fact quite straightforward to enable this capability, by means of station loops and the extended train separation distance, so that a train stopping at a station has absolutely no impact on a following train not stopping at the station.

But there is still a penalty, and it may be serious. It concerns line capacity, but doesn't directly affect the overall capacity value itself (for the precise meaning of this statement, see below). The Capacity-Slot model is described in the previous section, and its operation explained. Any train in motion **on the main line** occupies one capacity slot. If a train stops at an intermediate station, it gives up its capacity slot, on diverging from the main line onto the station loop, and requires another one to be available for it to occupy when it re-joins the main line at the other end of the station loop. Thus if it makes *n* intermediate station stops, it uses *n+1* slots in total, albeit only one at a time. The slot given up when diverging for a station stop immediately becomes available for re-use by another train, either joining the main line, (from another route,) or re-joining the main line after calling at a later station. **It is always possible for a slot to be re-used**, several times maybe.

The problem here is that, at the time a train wishes to restart from an intermediate station, a free slot may not immediately be available for it, and it must therefore wait (i.e. delay its departure from the station) for the next free slot. Also, as explained in the previous section, there is a slot window, expressed as either distance or time, only during which a converging train may enter the slot, since, although a train always **could** enter a slot, only by entering during the slot window would it be able to reach its required position within the slot. It may well be, if the main-line loading is high, that several capacity slots in a row are occupied, before the next free slot occurs. Given a slot time of c.2 minutes, that could impose a severe time penalty on a station stop, in addition to the unavoidable c.7 minutes. So, while this model **will always work** – the capacity is still there, though the dynamic distribution of it may not be optimal – for **good** performance, it requires some very neat scheduling, and this may not always be practicable. This scheduling has two aspects:

1. to draw up the optimum timetable, so that the (dynamic) slot distribution in normal service minimises the (probably unavoidable) extra time penalties, and
2. to perform dynamic scheduling in real time, in particular, when a train, through lax operating performance or following an unavoidable incident, misses its scheduled slot.

Readers of a philosophical inclination may wonder precisely why there is this extra time penalty for stopping trains; after all, there is no change to the line capacity. I suggest that it is because the fundamental requirement for maximum capacity, that the traffic be homogeneous has actually been breached, in that some trains make station stops which other trains don't. I think that a more philosophically satisfying answer than an excursion through queueing theory.

Timetabling Considerations and Sweet-Speeds.

Earlier versions of this article (prior to v7.0) concluded the previous section with, in effect, an admission that it was very difficult to make accurate predictions as to how behaviour at and around intermediate stations was to be determined and managed. It has indeed been an exceptionally difficult subject to elucidate, but the work has finally been done, and the results now follow. It is important to stress that the treatment in the previous sections of Appendix B remains totally valid. What follows is, to the best of my belief, completely new, in that I can find no trace of it in the literature published online. As to the contents of the previous sections, the Capacity Slot Model is a fundamental foundation of what now follows.

The fundamental aspect of the problem is that it obliges consideration of timetabling, of how the various services are to be scheduled. So far, all calculations have started with the line speed as the independent variable (which means it's the one to which values are assigned, and all the results are derived from that, simply by inserting the relevant speeds into formulae). But timetabling requires us to look at things from the other end, starting from the capacity, strictly the inter-train times, but these relate directly to capacity. Although line speed is still the independent variable, it is now itself also the desired result. Solution of this problem has to be numeric, by an iterative process. Effectively this involves trying different values of line speed until we get the desired capacity value, to any desired degree of precision. (This is a well-known but tedious and time-consuming process, but spreadsheets are a terrific help in performing it.)

The calculation process is:

For a series of line speeds, calculate the capacity slot size, (which is TSD(e) for High Speed Railways or TSD(b) for other categories, for that speed,) and from that calculate the capacity slot time (which is the time taken to travel a distance equal to the slot size at line speed. The number of capacity slot times per hour is the line capacity in tph. What is required is the line speed corresponding to desired values of capacity.

Pages 30 and 31 list a spread-sheet produced by this process, titled 'Capacity Slot Timetabling'. (The spread-sheets have all been post-processed using M/S Paint©, with the intention of keeping the printed version to manageable size, in particular so that all the rows can be held on a single page, but sectioning by column over several pages is of course unavoidable.) The spread-sheet highlights the solution rows, by red borders and enlarged text. At present we're concerned only with the first six columns; the meaning of the others will become clear shortly. Note that the spread-sheets have been listed starting with an even page number. This is so that the printed version of this article presents the results in pairs of pages to maximise the convenience of reading them, starting always with an even page, to ensure that the first two pages can always be viewed together,

Having spent a lot of time and effort deriving the results in this manner, it was then realised that it could be started from the other end, with line capacity as the independent variable, and obtain the corresponding line speed from that, simply by solving a quadratic equation. The process is expounded on the next page, and the spread-sheet resulting is listed on pages 32-35 (it contains a lot more results and thus columns).

Let v_l (m/s) be the line speed and v_b the Buffer-end Speed (= 57.02m/s / 205.29kph / 127.48mph), which is the boundary between medium and high speeds. Let b be the buffer zone (= 830m) , which is the static component of the Train Separation Distance (basic or extended). Let c_t (s) be the capacity slot time. Finally, let c_l (tph) be the line capacity and a_d be the deceleration rate (= $0.5m/s^2$).

Medium (and Low) Speeds (<=v_b):

Capacity slot length = $TSD(b) = v_l^2/(2*a_d) + b = v_l c_t$, so $c_t = [v_l^2/(2*a_d) + b]/v_l$

Alternatively: $v_l^2 – 2*a_d*c_t*v_l + 2*a_d* b = 0$

By the standard solution for a quadratic:

$$v_l = [2*a_d*c_t \pm \sqrt{[(2*a_d*c_t)^2 – 8*a_d* b \}]} / 2$$

or: $$\mathbf{v_l = a_d*c_t \pm \sqrt{[(a_d*c_t)^2 – 2*a_d*b]}}$$

We know that $c_t = 3600 / c_l$, so we have known values for every quantity in the solution.

High Speeds (>=v_b):

Capacity slot length = $TSD(e) = v_l^2/(2*a_d) + b + e$ (where $e = (v_l – v_b)^2/(2*a_d)$)

$$= [\{ v_l^2 + (v_l – v_b)^2 \} / 2*a_d + b]$$

so $c_t = [\{ v_l^2 + (v_l – v_b)^2 \} / 2*a_d + b] / v_l$

Alternatively: $v_l^2 – (a_d*c_t + v_b)*v_l + (a_d*b + v_b^2/2) = 0$

By the standard solution for a quadratic:

$$\mathbf{v_l = [(a_d*c_t + v_b) \pm \sqrt{\{ (a_d*c_t + v_b)^2 – 4*(a_d*b + v_b^2/2) \}}] / 2}$$

We know that $c_t = 3600 / c_l$, so we have known values for every quantity in the solution.

It looks nasty, but spread-sheets eat this sort of stuff for breakfast.

The initial work is to obtain the line speeds corresponding to those line capacities, and for this, only the first three columns, are required. The 3rd column is merely the ‘b’ parameter of the quadratic equation, calculated in advance for the High Speed cases. The solution of the quadratic goes straight into column 4.

The spread-sheet highlights the solution rows, by red borders and enlarged text. The row of maximum capacity is now the penultimate row of the spread-sheet. (The bottom row gives a capacity value just above the maximum; the error message at the bottom of every column from the third onward means that the quantity whose square root is required in the above equation has just gone negative.)

Note a very significant difference to the presentation of the results. When line speed is the independent variable, all the results appear in a single spread-sheet. But when line capacity is the independent variable, this has a maximum value of very slightly over 68tph. Accordingly, the results have to be presented in two spread-sheets, corresponding to the ± values in the above equations, the plus values, following, corresponding to capacities in the High and Medium speed ranges, and the negative value to the Low Speed range (these results are given in the next section). At present we’re concerned only with the first seven columns; the meaning of the others will become clear shortly.

Line Speed (kph)	Line Speed mph)	Line Speed (m/s)	Capacity Slot Size (= TSD(e)) (km)	Capacity Slot Time (s)	Capacity Slots per hour (= Line Capacity (tph))	Acceleration Time Zero - Line Speed (s)	Acceleration Time in Capacity Slots	Acceleration Distance Zero - Line Speed (km)	Deceleration Time Line Speed - Zero (s)	Dec Cap
10	6.25	2.78	0.8377	301.58	11.94	9.26	0.030702724	0.0129	5.56	
17.048538	10.655336	4.735705	0.8524	180.00	20.000000	15.79	0.087698241	0.0374	9.47	(
20	12.50	5.56	0.8609	154.96	23.23	18.52	0.119508581	0.0514	11.11	
20.714621	12.946638	5.754061	0.8631	150.00	24.000000	19.18	0.127868033	0.0552	11.51	
21.654552	13.534095	6.015153	0.8662	144.00	25.000000	20.05	0.139239659	0.0603	12.03	(
26.529158	16.580724	7.369211	0.8843	120.00	30.000000	24.56	0.204700293	0.0905	14.74	(
28.576310	17.860194	7.937864	0.8930	112.50	32.000000	26.46	0.235195966	0.1050	15.88	(
30	18.75	8.33	0.8994	107.93	33.35	27.78	0.257360511	0.1157	16.67	
32.883719	20.552325	9.134367	0.9134	100.00	36.000000	30.45	0.304478884	0.1391	18.27	
37.552421	23.470263	10.431228	0.9388	90.00	40.000000	34.77	0.386341776	0.1814	20.86	(
40	25.00	11.11	0.9535	85.81	41.95	37.04	0.431611205	0.2058	22.22	
44.104054	27.565034	12.251126	0.9801	80.00	45.000000	40.84	0.51046359	0.2502	24.50	(
48.581252	30.363283	13.494792	1.0121	75.00	48.000000	44.98	0.599768544	0.3035	26.99	(
50	31.25	13.89	1.0229	73.65	48.88	46.30	0.628608211	0.3215	27.78	
51.886681	32.429175	14.412967	1.0377	72.00	50.000000	48.04	0.667266984	0.3462	28.83	
60	37.50	16.67	1.1078	66.47	54.16	55.56	0.835840856	0.4630	33.33	
70	43.75	19.44	1.2081	62.13	57.94	64.81	1.043210192	0.6301	38.89	
77.880239	48.675149	21.633400	1.2980	60.00	60.000000	72.11	1.201855541	0.7800	43.27	(
80	50.00	22.22	1.3238	59.57	60.43	74.07	1.243433119	0.8230	44.44	
90	56.25	25.00	1.4550	58.20	61.86	83.33	1.431844215	1.0417	50.00	
100	62.50	27.78	1.6016	57.66	62.44	92.59	1.605899432	1.2860	55.56	
103.7149999	64.82187495	28.8097222	1.6600	57.62	62.47891211	96.03	1.66666676	1.3833	57.62	
110	68.75	30.56	1.7636	57.72	62.37	101.85	1.764609802	1.5561	61.11	
120	75.00	33.33	1.9411	58.23	61.82	111.11	1.908032818	1.8519	66.67	
130	81.25	36.11	2.1340	59.10	60.92	120.37	2.03687098	2.1734	72.22	
138.119761	86.324851	38.366600	2.3020	60.00	60.000000	127.89	2.131477793	2.4533	76.73	1
140	87.50	38.89	2.3423	60.23	59.77	129.63	2.15218117	2.5206	77.78	
150	93.75	41.67	2.5661	61.59	58.45	138.89	2.255177888	2.8935	83.33	
160	100.00	44.44	2.8053	63.12	57.03	148.15	2.347107923	3.2922	88.89	
170	106.25	47.22	3.0599	64.80	55.56	157.41	2.429175661	3.7166	94.44	
180	112.50	50.00	3.3300	66.60	54.05	166.67	2.502502503	4.1667	100.00	
190	118.75	52.78	3.6155	68.50	52.55	175.93	2.568108222	4.6425	105.56	
200	125.00	55.56	3.9164	70.50	51.07	185.19	2.62690582	5.1440	111.11	
205.2880903	128.3050564	57.02446951	4.0818	71.58	50.29	190.08	2.65552615	5.4197	114.05	
205.2880903	128.3050564	57.02446951	4.0818	71.58	50.29	190.08	2.65552615	5.4197	114.05	
207.287592	129.55	57.58	4.1458	72.00	50.000000	191.93	2.665735497	5.5257	115.16	1
210	131.25	58.33	4.2345	72.59	49.59	194.44	2.678620135	5.6713	116.67	
220.134005	137.583753	61.148335	4.5861	75.00	48.000000	203.83	2.717703766	6.2319	122.30	
230	143.75	63.89	4.9589	77.62	46.38	212.96	2.743741275	6.8030	127.78	
238.287918	148.929948	66.191088	5.2953	80.00	45.000000	220.64	2.757962009	7.3021	132.38	1
240	150.00	66.67	5.3674	80.51	44.71	222.22	2.760138898	7.4074	133.33	
250	156.25	69.44	5.8068	83.62	43.05	231.48	2.76833021	8.0376	138.89	
260	162.50	72.22	6.2770	86.91	41.42	240.74	2.769917622	8.6934	144.44	
270	168.75	75.00	6.7781	90.37	39.83	250.00	2.766253893	9.3750	150.00	
280	175.00	77.78	7.3101	93.99	38.30	259.26	2.758465311	10.0823	155.56	
268.938449	168.086531	74.705125	6.7235	90.00	40.000000	249.02	2.76685647	9.3014	149.41	1
290	181.25	80.56	7.8729	97.73	36.84	268.52	2.74747961	10.8153	161.11	
295.899690	184.937306	82.194358	8.2194	100.00	36.000000	273.98	2.739811941	11.2599	164.39	1
300	187.50	83.33	8.4666	101.60	35.43	277.78	2.734054528	11.5741	166.67	
310	193.75	86.11	9.0912	105.57	34.10	287.04	2.718804705	12.3585	172.22	
320	200.00	88.89	9.7466	109.65	32.83	296.30	2.7022258	13.1687	177.78	
326.868765	204.292978	90.796879	10.2146	112.50	32.000000	302.66	2.690277898	13.7401	181.59	1
330	206.25	91.67	10.4329	113.81	31.63	305.56	2.68471544	14.0046	183.33	
340	212.50	94.44	11.1500	118.06	30.49	314.81	2.666591031	14.8663	188.89	
344.513062	215.320664	95.698073	11.4838	120.00	30.000000	318.99	2.658279801	15.2635	191.40	1
350	218.75	97.22	11.8980	122.38	29.42	324.07	2.648104654	15.7536	194.44	
360	225.00	100.00	12.6769	126.77	28.40	333.33	2.629455409	16.6667	200.00	
370	231.25	102.78	13.4866	131.22	27.43	342.59	2.61079955	17.6055	205.56	
380	237.50	105.56	14.3272	135.73	26.52	351.85	2.592258767	18.5700	211.11	
390	243.75	108.33	15.1987	140.30	25.66	361.11	2.573926918	19.5602	216.67	
398.036978	248.773111	110.565827	15.9215	144.00	25.000000	368.55	2.559394147	20.3747	221.13	1
400	250.00	111.11	16.1010	144.91	24.84	370.37	2.555875498	20.5761	222.22	
410	256.25	113.89	17.0342	149.57	24.07	379.63	2.538158066	21.6178	227.78	
410.920391	256.825245	114.144553	17.1217	150.00	24.000000	380.48	2.536545626	21.7150	228.29	1
420	262.50	116.67	17.9983	154.27	23.34	388.89	2.520813818	22.6852	233.33	
430	268.75	119.44	18.9932	159.01	22.64	398.15	2.50387048	23.7783	238.89	
440	275.00	122.22	20.0190	163.79	21.98	407.41	2.487346646	24.8971	244.44	
450	281.25	125.00	21.0757	168.61	21.35	416.67	2.471253656	26.0417	250.00	

ed	Deceleration Time in Capacity Slots	Deceleration Distance Line Speed - Zero (km)	Station Stop Travelling Time (decel'n + accel'n but no wait) (s)	Station Stop Travelling Time (in Capacity Slots)	Station Stop Distance (decel'n + accel'n) (km)	Station Stop Distance in Capacity Slots	Corrected Slot Stream Advance (in integral Ca[pacity Slots)	Corrected Station Wait Time (s)	Corrected Slot Stream Advance (with 1 extra slot)	Corrected Station Wait Time (s)
.56	0.018421634	0.0077	14.81	0.049124358	0.0206	0.024562179	1	294.17	2	595.75
47	0.052618945	0.0224	25.26	0.140317186	0.0598	0.07015859	1	167.37	2	347.37
.11	0.071705148	0.0309	29.63	0.191213729	0.0823	0.095606865	1	140.14	2	295.10
51	0.07672082	0.0331	30.69	0.204588853	0.0883	0.10229443	1	134.66	2	284.66
03	0.083543795	0.0362	32.08	0.222783454	0.0965	0.11139173	1	127.96	2	271.96
74	0.122820176	0.0543	39.30	0.327520468	0.1448	0.16376023	1	100.35	2	220.35
88	0.141117579	0.0630	42.34	0.376313545	0.1680	0.18815677	1	91.33	2	203.83
.67	0.154416306	0.0694	44.44	0.411776817	0.1852	0.205888408	1	85.71	2	193.64
27	0.18268733	0.0834	48.72	0.487166214	0.2225	0.24358311	1	75.64	2	175.64
86	0.231805066	0.1088	55.63	0.618146842	0.2902	0.30907342	1	62.18	2	152.18
.22	0.258966723	0.1235	59.26	0.690577927	0.3292	0.345288964	1	56.18	2	141.99
50	0.306278154	0.1501	65.34	0.816741743	0.4002	0.40837087	1	47.33	2	127.33
99	0.359861126	0.1821	71.97	0.95962967	0.4856	0.47981483	1	39.01	2	114.01
.78	0.377164927	0.1929	74.07	1.005773138	0.5144	0.502886569	1	36.61	2	110.26
83	0.40036019	0.2077	76.87	1.067627174	0.5540	0.53381359	1	33.57	2	105.57
.33	0.501504514	0.2778	88.89	1.337345369	0.7407	0.668672685	1	22.02	2	88.49
.89	0.625926115	0.3781	103.70	1.669136307	1.0082	0.834568154	1	10.28	2	72.41
27	0.721113324	0.4680	115.38	1.922968865	1.2480	0.96148443	1	2.31	2	62.31
.44	0.746059871	0.4938	118.52	1.98949299	1.3169	0.994746495	1	0.31	2	59.89
.00	0.859106529	0.6250	133.33	2.290950745	1.6667	1.145475372	2	49.73	3	107.93
.56	0.963539659	0.7716	148.15	2.569439091	2.0576	1.284719546	2	41.24	3	98.90
.62	1.000000056	0.8300	153.65	2.666666816	2.2133	1.333333408	2	38.41	3	96.03
.11	1.058765881	0.9336	162.96	2.823375684	2.4897	1.411687842	2	33.96	3	91.68
.67	1.144819691	1.1111	177.78	3.052852509	2.9630	1.526426255	2	27.58	3	85.81
.22	1.222122588	1.3040	192.59	3.258993569	3.4774	1.629496784	2	21.90	3	80.99
73	1.278886676	1.4720	204.62	3.410364468	3.9253	1.70518223	2	17.69	3	77.69
.78	1.291308702	1.5123	207.41	3.443489872	4.0329	1.721744936	2	16.76	3	76.99
.33	1.353106733	1.7361	222.22	3.608284621	4.6296	1.804142311	2	12.06	3	73.65
.89	1.408264754	1.9753	237.04	3.755372677	5.2675	1.877686338	2	7.72	3	70.84
.44	1.457505396	2.2299	251.85	3.886681057	5.9465	1.943340528	2	3.67	3	68.47
.00	1.501501502	2.5000	266.67	4.004004004	6.6667	2.002002002	3	66.47	4	133.07
.56	1.540864933	2.7855	281.48	4.108973155	7.4280	2.054486578	3	64.77	4	133.28
.11	1.576143492	3.0864	296.30	4.203049312	8.2305	2.101524656	3	63.34	4	133.83
.05	1.59331569	3.2518	304.13	4.24884184	8.6714	2.12442092	3	62.67	4	134.25
.05	1.59331569	3.2518	304.13	4.24884184	8.6714	2.12442092	3	62.67	4	134.25
16	1.599441298	3.3154	307.09	4.265176796	8.8412	2.1325884	3	62.45	4	134.45
.67	1.607172081	3.4028	311.11	4.285792216	9.0741	2.142896108	3	62.22	4	134.81
30	1.63062226	3.7391	326.12	4.348326025	9.9710	2.17416301	3	61.94	4	136.94
.78	1.646244765	4.0818	340.74	4.389986039	10.8848	2.19499302	3	62.48	4	140.10
38	1.654777205	4.3813	353.0191371	4.412739214	11.6834	2.20636961	3	63.49	4	143.49
.33	1.656083339	4.4444	355.56	4.416222236	11.8519	2.208111118	3	63.76	4	144.27
.89	1.660998126	4.8225	370.37	4.429328335	12.8601	2.214664168	3	65.67	4	149.29
.44	1.661950573	5.2160	385.19	4.431868195	13.9095	2.215934098	3	68.15	4	155.06
.00	1.659752336	5.6250	400.00	4.42600623	15.0000	2.213003115	3	71.12	4	161.50
.56	1.655079187	6.0494	414.81	4.413544498	16.1317	2.206772249	3	74.55	4	168.54
41	1.660113882	5.5809	398.43	4.426970352	14.8823	2.21348518	3	70.79	4	160.79
.11	1.648487766	6.4892	429.63	4.395967376	17.3045	2.197983688	3	78.38	4	176.12
39	1.643887165	6.7559	438.37	4.383699106	18.0158	2.19184955	3	80.82	4	180.82
.67	1.640432717	6.9444	444.44	4.374487246	18.5185	2.187243623	3	82.58	4	184.17
.22	1.631282823	7.4151	459.26	4.350087529	19.7737	2.175043764	3	87.09	4	192.67
.78	1.62133548	7.9012	474.07	4.32356128	21.0700	2.16178064	3	91.91	4	201.56
59	1.614166739	8.2441	484.25	4.304444637	21.9842	2.15222232	3	95.37	4	207.87
.33	1.610829264	8.4028	488.89	4.295544705	22.4074	2.147772352	3	96.99	4	210.81
.89	1.599954619	8.9198	503.70	4.266545649	23.7860	2.133272825	3	102.32	4	220.38
40	1.594967881	9.1581	510.39	4.253247682	24.4217	2.12662384	3	104.81	4	224.81
.44	1.588862792	9.4522	518.52	4.236967446	25.2058	2.118483723	3	107.88	4	230.26
.00	1.577673245	10.0000	533.33	4.207128654	26.6667	2.103564327	3	113.64	4	240.41
.56	1.56647973	10.5633	548.15	4.17727928	28.1687	2.08863964	3	119.59	4	250.81
.11	1.55535526	11.1420	562.96	4.147614027	29.7119	2.073807013	3	125.71	4	261.45
.67	1.544356151	11.7361	577.78	4.118283068	31.2963	2.059141534	3	132.00	4	272.29
13	1.535636488	12.2248	589.68	4.095030635	32.5995	2.04751532	3	137.16	4	281.16
.22	1.533525299	12.3457	592.59	4.089400797	32.9218	2.044700399	3	138.43	4	283.34
.78	1.52289484	12.9707	607.41	4.061052906	34.5885	2.030526453	3	145.00	4	294.57
29	1.521927376	13.0290	608.77	4.058473002	34.7439	2.0292365	3	145.61	4	295.61
.33	1.512488291	13.6111	622.22	4.033302108	36.2963	2.016651054	3	151.70	4	305.97
.89	1.502322288	14.2670	637.04	4.006192768	38.0453	2.003096384	3	158.52	4	317.53
.44	1.492407988	14.9383	651.85	3.979754634	39.8354	1.989877317	2	1.66	3	165.45
.00	1.482752194	15.6250	666.67	3.95400585	41.6667	1.977002925	2	3.88	3	172.48

Line Capacity (tph)	Capacity Slot Time (s)	Quad Equationn Param b.	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Slot Length (km)	TSD(e) / TSD(b) (km)	Acceleration Time Zero - Line Speed (s)	Acceleration Time in Capacity Slots	Ac n I Ze Sp
10	360	237.0244695	228.08	821.07	510.21	82.1074	82.1074	760.25	2.111816339	8
11	327.2727273	220.6608332	210.99	759.56	471.98	69.0505	69.0505	703.29	2.148949659	
12	300	207.0244695	196.65	707.93	439.90	58.9938	58.9938	655.49	2.184954927	6
13	276.9230769	195.486008	184.42	663.91	412.55	51.0700	51.0700	614.73	2.219863297	
14	257.1428571	185.5958981	173.86	625.89	388.92	44.7061	44.7061	579.52	2.253701415	
15	240	177.0244695	164.63	592.66	368.27	39.5106	39.5106	548.76	2.286491873	4.
16	225	169.5244695	156.48	563.34	350.05	35.2085	35.2085	521.61	2.318253572	4
17	211.7647059	162.9068225	149.23	537.23	333.83	31.6018	31.6018	497.44	2.349002022	
18	200	157.0244695	142.72	513.81	319.28	28.5450	28.5450	475.75	2.378749575	3.
19	189.4736842	151.7613116	136.85	492.65	306.13	25.9290	25.9290	456.16	2.407505601	
20	180	147.0244695	131.50	473.42	294.18	23.6709	23.6709	438.35	2.435276625	2
21	171.4285714	142.7387552	126.62	455.83	283.25	21.7064	21.7064	422.07	2.462066417	
22	163.6363636	138.8426513	122.13	439.68	273.21	19.9853	19.9853	407.11	2.487876046	
23	156.5217391	135.2853391	117.99	424.76	263.94	18.4677	18.4677	393.29	2.512703893	
24	150	132.0244695	114.14	410.92	255.34	17.1217	17.1217	380.48	2.536545626	2
25	144	129.0244695	110.57	398.04	247.34	15.9215	15.9215	368.55	2.559394147	2
26	138.4615385	126.2552387	107.22	385.99	239.85	14.8459	14.8459	357.40	2.581239485	
27	133.3333333	123.6911362	104.08	374.70	232.83	13.8777	13.8777	346.94	2.60206866	
28	128.5714286	121.3101838	101.13	364.06	226.23	13.0023	13.0023	337.10	2.621865485	
29	124.137931	119.093435	98.34	354.02	219.99	12.2077	12.2077	327.80	2.640610329	
30	120	117.0244695	95.70	344.51	214.08	11.4838	11.4838	318.99	2.658279801	1.
31	116.1290323	115.0889856	93.19	335.48	208.46	10.8219	10.8219	310.63	2.674846373	
32	112.5	113.2744695	90.80	326.87	203.11	10.2146	10.2146	302.66	2.690277898	1.
33	109.0909091	111.5699241	88.51	318.64	198.00	9.6559	9.6559	295.04	2.704537027	
34	105.8823529	109.965646	86.32	310.76	193.10	9.1401	9.1401	287.74	2.71758048	
35	102.8571429	108.4530409	84.22	303.19	188.40	8.6626	8.6626	280.73	2.729358144	
36	100	107.0244695	82.19	295.90	183.87	8.2194	8.2194	273.98	2.739811941	1
37	97.2972973	105.6731182	80.24	288.85	179.49	7.8069	7.8069	267.46	2.748874408	
38	94.73684211	104.3928906	78.34	282.03	175.25	7.4218	7.4218	261.14	2.756466882	
39	92.30769231	103.1783157	76.50	275.40	171.13	7.0615	7.0615	255.00	2.762497164	
40	90	102.0244695	74.71	268.94	167.12	6.7235	6.7235	249.02	2.76685647	
41	87.80487805	100.9269085	72.95	262.62	163.19	6.4054	6.4054	243.17	2.769415382	
42	85.71428571	99.88161237	71.23	256.42	159.34	6.1053	6.1053	237.43	2.77001837	
43	83.72093023	98.88493463	69.53	250.32	155.55	5.8214	5.8214	231.78	2.768476226	
44	81.81818182	97.93356042	67.86	244.29	151.80	5.5520	5.5520	226.19	2.764555335	
45	80	97.02446951	66.19	238.29	148.07	5.2953	5.2953	220.64	2.757962009	
46	78.26086957	96.1549043	64.53	232.29	144.34	5.0498	5.0498	215.09	2.748318776	
46	78.26086957	96.1549043	64.53	232.29	144.34	5.0498	5.0498	215.09	2.748318776	
47	76.59574468	95.32234186	62.85	226.26	140.59	4.8140	4.8140	209.50	2.735126889	
48	75	94.52446951	61.15	220.13	136.79	4.5861	4.5861	203.83	2.717703766	
49	73.46938776	93.75916339	59.40	213.85	132.88	4.3642	4.3642	198.01	2.6950711	
50	72	93.02446951	57.58	207.29	128.81	4.1458	4.1458	191.93	2.665735497	
50.29364177	71.57962464	92.81428184	57.02	205.29	127.56	4.0818	4.0818	190.08	2.65552615	
50.29364177	71.57962464		57.02	205.29	127.56	4.0818	4.0818	190.08	2.65552615	
51	70.58823529		55.68	200.46	124.56	3.9305	3.9305	185.61	2.629438193	
52	69.23076923		53.80	193.70	120.36	3.7249	3.7249	179.35	2.590590428	
53	67.9245283		51.95	187.01	116.20	3.5284	3.5284	173.16	2.549230001	
54	66.66666667		50.10	180.36	112.07	3.3400	3.3400	167.00	2.504985089	
55	65.45454545		48.25	173.71	107.94	3.1584	3.1584	160.85	2.457371627	
56	64.28571429		46.40	167.03	103.79	2.9826	2.9826	154.65	2.405738862	
57	63.15789474		44.51	160.24	99.57	2.8112	2.8112	148.37	2.349175202	
58	62.06896552		42.57	153.26	95.24	2.6425	2.6425	141.91	2.286331827	
59	61.01694915		40.55	145.97	90.70	2.4740	2.4740	135.16	2.215054064	
60	60		38.37	138.12	85.83	2.3020	2.3020	127.89	2.131477793	
61	59.01639344		35.89	129.21	80.29	2.1181	2.1181	119.63	2.027148074	
62	58.06451613		32.62	117.43	72.97	1.8941	1.8941	108.73	1.872630559	
62.47891211	57.61944116		28.8097222	103.7149999	64.44734856	1.6600	1.6600	96.03	1.66666676	
62.47891211	57.61944116		#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	

Overtaking (i.e. High Speed and Medium Speed) Lines, Section 1

in	Acceleration Distance Zero - Line Speed (km)	Deceleration Time Line Speed - Zero (s)	Deceleration Time in Capacity Slots	Deceleration Distance Line Speed - Zero (km)	Station Stop Travelling Time (decel'n + accel'n but no wait) (s)	Station Stop Travelling Time in Capacity Slots	Station Stop Distance (decel'n + accel'n) (km)	Station Stop Distance in Capacity Slots	Corrected Slot Stream Advance (in integral Capacity Slots)	Corrected Station Wait Time (s)	Corr Strea Adva 1 ext
339	86.6979	456.15	1.267089803	52.0187	1216.41	3.378906142	138.7166	1.689453071	2	111.80	
9659	74.1931	421.98	1.289369795	44.5158	1125.27	3.438319455	118.7089	1.719159727	2	91.91	
927	64.4494	393.29	1.310972956	38.6696	1048.78	3.495927882	103.1190	1.747963941	2	75.61	
3297	56.6842	368.84	1.331917978	34.0105	983.57	3.551781275	90.6947	1.775890637	2	62.06	
1415	50.3771	347.71	1.352220849	30.2262	927.24	3.605922265	80.6033	1.802961132	2	50.67	
373	45.1703	329.25	1.371895124	27.1022	878.01	3.658386997	72.2725	1.829193498	2	40.99	
772	40.8111	312.96	1.390952143	24.4867	834.57	3.709205715	65.2977	1.854602857	2	32.71	
2022	37.1163	298.46	1.409401213	22.2698	795.90	3.758403236	59.3862	1.879201618	2	25.58	
575	33.9507	285.45	1.427249745	20.3704	761.20	3.80599932	54.3211	1.90299966	2	19.40	
5601	31.2121	273.70	1.44450336	18.7273	729.85	3.852008961	49.9394	1.926004481	2	14.02	
525	28.8226	263.01	1.461165975	17.2935	701.36	3.896442599	46.1161	1.9482213	2	9.32	
5417	26.7213	253.24	1.47723985	16.0328	675.31	3.939306267	42.7540	1.969653134	2	5.20	
5046	24.8604	244.26	1.492725628	14.9162	651.37	3.980601674	39.7767	1.990300837	2	1.59	
3893	23.2019	235.98	1.507622336	13.9211	629.27	4.020326228	37.1230	2.010163114	3	154.93	
526	21.7150	228.29	1.521927376	13.0290	608.77	4.058473002	34.7439	2.029236501	3	145.61	
147	20.3747	221.13	1.535636488	12.2248	589.68	4.095030635	32.5995	2.047515317	3	137.16	
9485	19.1605	214.44	1.548743691	11.4963	571.84	4.129983177	30.6568	2.064991588	3	129.46	
5866	18.0554	208.17	1.561241196	10.8332	555.11	4.163309856	28.8886	2.081654928	3	122.45	
5485	17.0452	202.26	1.573119291	10.2271	539.36	4.194984776	27.2723	2.097492388	3	116.04	
0329	16.1179	196.68	1.584366197	9.6707	524.48	4.224976526	25.7887	2.112488263	3	110.17	
301	15.2635	191.40	1.594967881	9.1581	510.39	4.253247682	24.4217	2.126623841	3	104.81	
5373	14.4734	186.38	1.604907824	8.6840	497.00	4.279754197	23.1574	2.139877099	3	99.89	
398	13.7401	181.59	1.614166739	8.2441	484.25	4.304444637	21.9842	2.152222319	3	95.37	
7027	13.0573	177.02	1.622722216	7.8344	472.06	4.327259243	20.8917	2.163629622	3	91.24	
3048	12.4195	172.65	1.630548288	7.4517	460.39	4.348128768	19.8712	2.174064384	3	87.45	
3144	11.8217	168.44	1.637614886	7.0930	449.17	4.36697303	18.9148	2.183486515	3	83.98	
941	11.2599	164.39	1.643887165	6.7559	438.37	4.383699106	18.0158	2.191849553	3	80.82	
4408	10.7301	160.47	1.649324645	6.4380	427.93	4.398199053	17.1681	2.199099527	3	77.93	
5882	10.2290	156.68	1.653880129	6.1374	417.82	4.410347011	16.3665	2.205173506	5	264.77	
7164	9.7537	153.00	1.657498298	5.8522	408.00	4.419995462	15.6060	2.209997731	3	72.92	
547	9.3014	149.41	1.660113882	5.5809	398.43	4.426970352	14.8823	2.213485176	3	70.79	
5382	8.8696	145.90	1.661649229	5.3218	389.07	4.431064611	14.1914	2.215532305	3	68.88	
1837	8.4560	142.46	1.662011022	5.0736	379.89	4.432029392	13.5295	2.216014696	3	67.20	
5226	8.0583	139.07	1.661085736	4.8350	370.85	4.429561962	12.8932	2.214780981	3	65.74	
5335	7.6743	135.71	1.658733201	4.6046	361.91	4.423288535	12.2790	2.211644268	3	64.50	
009	7.3021	132.38	1.654777205	4.3813	353.02	4.412739214	11.6834	2.206369607	3	63.49	
3776	6.9393	129.05	1.648991266	4.1636	344.14	4.397310041	11.1029	2.198655021	3	62.71	
3776	6.9393	129.05	1.648991266	4.1636	344.14	4.397310041	11.1029	2.198655021	3	62.71	
5889	6.5835	125.70	1.641076134	3.9501	335.20	4.376203023	10.5336	2.188101511	3	62.19	
766	6.2319	122.30	1.63062226	3.7391	326.12	4.348326025	9.9710	2.174163013	3	61.94	
0711	5.8809	118.80	1.61704266	3.5285	316.81	4.312113759	9.4095	2.15605688	3	62.00	
497	5.5257	115.16	1.599441298	3.3154	307.09	4.265176796	8.8412	2.132588398	3	62.45	
2615	5.4197	114.05	1.59331569	3.2518	304.13	4.24884184	8.6714	2.12442092	3	62.67	
2615	5.4197	114.05	1.59331569	3.2518	304.13	4.24884184	8.6714	2.12442092	3	62.67	
3193	5.1675	111.36	1.577662916	3.1005	296.97	4.207101109	8.2680	2.103550555	3	63.28	
0428	4.8249	107.61	1.554354257	2.8949	286.96	4.144944685	7.7198	2.072472342	3	64.21	
0001	4.4974	103.89	1.529538	2.6984	277.05	4.078768001	7.1959	2.039384	3	65.25	
5089	4.1833	100.20	1.502991054	2.5100	267.20	4.007976143	6.6933	2.003988071	3	66.40	
1627	3.8807	96.51	1.474422976	2.3284	257.35	3.931794604	6.2092	1.965897302	2	2.23	
3862	3.5877	92.79	1.443443317	2.1526	247.45	3.849182178	5.7403	1.924591089	2	4.85	
5202	3.3020	89.02	1.409505121	1.9812	237.39	3.758680324	5.2832	1.879340162	2	7.62	
1827	3.0208	85.15	1.371799096	1.8125	227.06	3.658130923	4.8332	1.829065462	2	10.61	
4064	2.7401	81.09	1.329032439	1.6440	216.25	3.544086503	4.3841	1.772043252	2	13.91	
793	2.4533	76.73	1.278886676	1.4720	204.62	3.410364468	3.9253	1.705182234	2	17.69	
3074	2.1469	71.78	1.216288844	1.2881	191.42	3.243436918	3.4350	1.621718459	2	22.32	
0559	1.7734	65.24	1.123578335	1.0641	173.97	2.996208894	2.8375	1.498104447	2	29.14	
5676	1.3833	57.62	1.000000056	0.8300	153.65	2.666666816	2.2133	1.333333408	2	38.41	
	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#

Overtaking (i.e. High Speed and Medium Speed) Lines, Section 2

ted ime	Corrected Slot Stream Advance (with 1 extra slot)	Corrected Station Wait Time (s)	Corrected Slot Stream Advance (with 2 extra slots)	Corrected Station Wait Time (s)	Corrected Slot Stream Advance (with 3 extra slots)	Corrected Station Wait Time (s)	Corrected Slot Stream Advance (with 4 extra slots)	Corrected Station Wait Time (s)	Corrected Slot Stream Advance (with 5 extra slots)	Corrected Station Wait Time (s)	Corrected Slot Stream Advance (with 5 extra slots)	Corrected Station Wait Time (s)
80	3	471.80	4	831.80	5	1191.80	6	1551.80	7	1911.80	8	2271.80
.91	3	419.18	4	746.46	5	1073.73	6	1401.00	7	1728.27	8	2055.55
61	3	375.61	4	675.61	5	975.61	6	1275.61	7	1575.61	8	1875.61
.06	3	338.98	4	615.91	5	892.83	6	1169.75	7	1446.68	8	1723.60
.67	3	307.81	4	564.95	5	822.10	6	1079.24	7	1336.38	8	1593.52
99	3	280.99	4	520.99	5	760.99	6	1000.99	7	1240.99	8	1480.99
71	3	257.71	4	482.71	5	707.71	6	932.71	7	1157.71	8	1382.71
.58	3	237.35	4	449.11	5	660.87	6	872.64	7	1084.40	8	1296.17
40	3	219.40	4	419.40	5	619.40	6	819.40	7	1019.40	8	1219.40
.02	3	203.49	4	392.97	5	582.44	6	771.91	7	961.39	8	1150.86
32	3	189.32	4	369.32	5	549.32	6	729.32	7	909.32	8	1089.32
.20	3	176.63	4	348.06	5	519.49	6	690.92	7	862.35	8	1033.77
.59	3	165.22	4	328.86	5	492.50	6	656.13	7	819.77	8	983.41
.93	4	311.45	5	467.97	6	624.50	7	781.02	8	937.54	9	1094.06
61	4	295.61	5	445.61	6	595.61	7	745.61	8	895.61	9	1045.61
16	4	281.16	5	425.16	6	569.16	7	713.16	8	857.16	9	1001.16
.46	4	267.92	5	406.39	6	544.85	7	683.31	8	821.77	9	960.23
.45	4	255.78	5	389.11	6	522.45	7	655.78	8	789.11	9	922.45
.04	4	244.61	5	373.18	6	501.75	7	630.32	8	758.89	9	887.47
.17	4	234.31	5	358.45	6	482.59	7	606.73	8	730.86	9	855.00
81	4	224.81	5	344.81	6	464.81	7	584.81	8	704.81	9	824.81
.89	4	216.01	5	332.14	6	448.27	7	564.40	8	680.53	9	796.66
37	4	207.87	5	320.37	6	432.87	7	545.37	8	657.87	9	770.37
.24	4	200.33	5	309.42	6	418.51	7	527.60	8	636.69	9	745.79
.45	4	193.33	5	299.22	6	405.10	7	510.98	8	616.86	9	722.75
.98	4	186.84	5	289.70	6	392.56	7	495.41	8	598.27	9	701.13
82	4	180.82	5	280.82	6	380.82	7	480.82	8	580.82	9	680.82
.93	4	175.22	5	272.52	6	369.82	7	467.11	8	564.41	9	661.71
.77	6	359.51	7	454.25	8	548.98	9	643.72	10	738.46	11	833.19
.92	4	165.23	5	257.54	6	349.85	7	442.15	8	534.46	9	626.77
79	4	160.79	5	250.79	6	340.79	7	430.79	8	520.79	9	610.79
.88	4	156.68	5	244.49	6	332.29	7	420.10	8	507.90	9	595.71
.20	4	152.91	5	238.63	6	324.34	7	410.06	8	495.77	9	581.48
.74	4	149.46	5	233.18	6	316.90	7	400.62	8	484.34	9	568.06
.50	4	146.32	5	228.14	6	309.96	7	391.77	8	473.59	9	555.41
49	4	143.49	5	223.49	6	303.49	7	383.49	8	463.49	9	543.49
.71	4	140.97	5	219.24	6	297.50	7	375.76	8	454.02	9	532.28
.71	4	140.97	5	219.24	6	297.50	7	375.76	8	454.02	9	532.28
.19	4	138.78	5	215.38	6	291.98	7	368.57	8	445.17	9	521.76
94	4	136.94	5	211.94	6	286.94	7	361.94	8	436.94	9	511.94
.00	4	135.47	5	208.94	6	282.41	7	355.88	8	429.35	9	502.82
45	4	134.45	5	206.45	6	278.45	7	350.45	8	422.45	9	494.45
.67	4	134.25	5	205.83	6	277.41	7	348.99	8	420.57	9	492.15
.67	4	134.25	5	205.83	6	277.41	7	348.99	8	420.57	9	492.15
.28	4	133.87	5	204.46	6	275.04	7	345.63	8	416.22	9	486.81
.21	4	133.44	5	202.67	6	271.91	7	341.14	8	410.37	9	479.60
.25	4	133.17	5	201.10	6	269.02	7	336.95	8	404.87	9	472.80
.40	4	133.07	5	199.73	6	266.40	7	333.07	8	399.73	9	466.40
.23	3	67.69	4	133.14	5	198.60	6	264.05	7	329.50	8	394.96
.85	3	69.13	4	133.42	5	197.70	6	261.99	7	326.28	8	390.56
.62	3	70.78	4	133.94	5	197.09	6	260.25	7	323.41	8	386.57
.61	3	72.68	4	134.75	5	196.82	6	258.89	7	320.95	8	383.02
.91	3	74.93	4	135.94	5	196.96	6	257.98	7	318.99	8	380.01
69	3	77.69	4	137.69	5	197.69	6	257.69	7	317.69	8	377.69
.32	3	81.34	4	140.36	5	199.37	6	258.39	7	317.41	8	376.42
.14	3	87.21	4	145.27	5	203.34	6	261.40	7	319.46	8	377.53
.41	3	96.03	4	153.65	5	211.27	6	268.89	7	326.51	8	384.13
M!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!

The idea occurred to me some time ago that it could, in theory, be possible to fill the capacity slot left vacant when a stopping train diverges at a station, by another train which had stopped earlier at that station, when that empty slot reached the other end of the station loop. I say ‘station loop’, but, strictly speaking, (for the High Speed category only), it is that section of track on the main line from the location where a stopping train starts to decelerate, before it physically diverges onto the station loop, to that location where it finally reaches line speed, and its correct position within its new capacity slot, after physically re-joining the main line from the station loop (into its new slot but not in its final position within the slot) and completes its acceleration up to line speed, on the main line. (All of this is explained in exhaustive detail in the previous section but one, on the Capacity Slot Model.) From now on, I shall call this the **virtual** station loop, and the physical station loop, which diverges from the main line, and on which the station platforms are physically located, I shall call the physical station loop, explicitly. So there can no longer be any ambiguity about which I mean. (For the Medium and Low Speed categories, the virtual and physical station loops are the same.) The main implication of this idea is that it should, theoretically at least, be possible to operate a mixture of non-stop and stopping trains without any loss of line capacity, a very desirable outcome. Note that the distance is the same, going via the physical station loop or staying on the main line throughout.

It is, in fact, entirely possible. More than that, it is the **only** sensible and indeed the only practicable way to operate a Same Speed Railway (even a metro – see the next section ‘Stations on the Main Line: HS-Metros, Pure Metros and Semi-Metros’).

In order to be able to merge the timings of non-stop and stopping services, we need to have an integer multiple of capacity slots per hour to make the construction of a usable timetable possible at all. If this were not so, there would be no regular framework constant over time; the environment would change from hour to hour. The hourly number of capacity slots is of course simply the line capacity in trains per hour. So all that this is really saying is that the line capacity in tph must be an integer. For **any** line capacity, it is possible to determine the corresponding capacity slot time in seconds, simply by dividing 3600 by the line capacity in tph. So this condition, while clearly necessary, is nowhere near sufficient. But before determining what would make it sufficient, it is necessary to consider the details of the process by which a stopping train is overtaken.

The fundamental principle involved with stopping trains is:
Accelerating from zero, with an arbitrary but uniform rate of acceleration, up to an arbitrary speed, and then immediately decelerating back down to zero, with an arbitrary but uniform rate of deceleration, takes precisely twice the time required to travel the same distance at that arbitrary but constant speed. The same obviously applies to deceleration followed by acceleration.

By numerical demonstration, this is clearly always true, (that indeed is how I discovered it, by pure chance – serendipity again – I had never encountered it previously). In fact the underlying reality applies to both acceleration and deceleration portions individually. The formal proof is straightforward:

> An initially stationary object accelerates with uniform acceleration rate a up to speed v in time t, such that $v = at$
> in the same time, it travels a distance $s = at^2/2$.
> In the same amount of time, travelling at constant speed v, it would cover the distance
> $s' = vt = at^2, = 2s$.
> In other words, in the time it takes to accelerate to v, it could travel twice as far at constant speed v. So it takes twice the time it would take to cover the same distance at constant speed. Q.E.D.
> (Many thanks to Dr. David Sutherland, for the above neat exposition.)

What this means is that by the time the stopping train reaches the end of the (virtual) station loop, having decelerated to zero and re-accelerated back up to line speed, but without any wait time at the station, the capacity slot which it gave up on entering the (physical) station loop has travelled in the slot stream on the main line, at constant line speed, twice the distance of the (virtual) station loop length. In other words, it is now a distance equal to the (virtual) station loop length ahead of its former occupying train. I call this distance (or time, since the speed is constant,) expressed in slots, the **Slot Stream Advance**. This will not, automatically, be an integer multiple of the slot (length or time). For the train to be able to re-join the slot stream, and thus the main line, the slot stream advance must be **made to be** an integer multiple of slots. This ensures that the train, on reaching the end of the (virtual) station loop, coincides precisely in location and speed with the next (empty) slot, previously occupied and given up by the **next** (stopping) train. To achieve this, the train is held at the station for an equalisation or basic wait time equal to that fraction of a time slot which must be added to the slot stream advance, to make the (Corrected) Slot Stream Advance an integer. Incidentally, that determines the repeat frequency: a stopping train occurs every n slots where n = the slot stream advance.

Note that no numerical values at all have been mentioned. The above argument applies to **any** value of line capacity whatever. (It still applies if the capacity isn't even an integer.)

The value just described, the basic wait time, being a fraction of the slot time, is too small to be a usable station wait time; it is too low for any but a very small number of passengers to leave and join the train. An arbitrary same number of slot times may be added to the wait time and the slot stream advance, while still maintaining the ability of the train to re-join the slot stream. The slot stream divides logically into a number of virtual sub-streams, the same number as the slot stream advance. At least one sub-stream, but possibly more, will be a stopping sub-stream, and the rest will all be non-stop. An individual stopping sub-stream is associated with a particular set of stations, and a particular platform face at each station (though this last requirement is readily varied operationally). The actual traffic pattern on the main line consists of a train from each sub-stream in turn, the pattern repeating indefinitely.

The further condition which, together with the condition that the line capacity must be an integer number of trains per hour, ensures a viable, usable timetable, is now simply stated. The slot stream advance, and thus the number of capacity sub-streams, must be an integer sub-multiple of the line capacity, so that it repeats an integer number of times every hour. Furthermore it produces a clock-face timetable: the trains stop at the same times every hour, and the time interval between adjacent trains is always the same (not necessarily an integer number of minutes, but certainly an integer number of seconds). Thus, supposing the line capacity is 32tph, if there are 4 sub-streams of 8tph each, then the stop times at a station repeat every 7½ minutes, likewise a line capacity of 30tph with 5 sub-streams each of 6tph gives station stop times of every 10 minutes.

There are only a very few possible values of line speed which satisfy the above two necessary and, together, sufficient conditions. The table below contains the results, for the range of line speeds of interest, (with my selection of best choices in red).

Line Cap-acity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)_	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
60	60	38.37	138.12	85.83	3.92 / 2.44	4 / 5 / 6	138 / 198 / 258	4 / 5 / 6
50	72	57.58	207.29	128.81	8.84 / 5.49	5	206	6
48	75	61.15	220.13	136.79	9.97 / 6.19	4 / 6 / 8	212 / 287 / 362	5 / 7½ / 10
45	80	69.01	248.44	154.38	11.68 / 7.26	5	216	6⅔
40	90	74.71	268.94	167.12	14.88 / 9.24	5 / 8	251 / 521	7½ / 12
36	100	82.19	295.90	183.87	18.02 / 11.19	4 / 6	181 / 381	6⅔ / 10
32	112.5	90.80	326.87	203.11	21.98 / 13.65	4 / 8	208 / 433	7½ / 15
30	120	95.70	344.51	214.08	24.42 / 15.17	5 / 6	345 / 465	10 / 12
25	144	110.57	398.04	247.34	32.60 / 20.24	5	425	12
24	150	114.14	410.92	255.34	34.74 / 21.57	6 / 8	596 / 896	15 / 20

These are the only ones worth worrying about. In fact, if there is more than one option on offer, then, generally, only one is worth considering – the others offer too little – or even too much – wait time at the station. There are a few more, but at line speeds beyond even the dreams of HS2 Ltd. (The next one is 20tph, with a line speed of 294mph!)

Note that the values in the above table **are the only speeds at which it is sensible or even possible to operate a High (or Medium) Speed Railway**. I call them the **Sweet-Speeds**. In other words, **there is no unfettered free choice of line speed for a HS railway**, only those few sweet-speeds are available. Overtaking is possible for **any** line speed whatever, but very, very few speeds give a viable timetable.

Of the nine available Sweet-Speeds in the High Speed range, the first two (capacities 50 and 48tph are a bit too slow. Their line speeds are in the narrow range between the turnout limit and buffer-end speeds. They are genuine High Speed cases, in that they perform the first part of their deceleration while still in the path of the following train, But in both cases, they start their deceleration only when at least the front of the train has moved on to the physical station loop. (in the 50tph case, it starts decelerating only when the switch is already in the process of resetting back to the main line!). Irrespective of formal category, these two would in practice be used for Medium Speed applications. (the only genuine Medium-Sweet-Speed case is for capacity 60tph.)

The two highest speeds (capacities 25 and 24tph) give station wait times of 7 minutes and above, which are far too long. (They are also, in my opinion, rather too fast.). The remaining 5 all give very acceptable options, as I have highlighted. Of these, the one I personally like best is 32tph for the (to me) decisive reason that it offers the perfect clock-face timetable of a train every 7½ minutes, or the equally good one, (by leaving alternate slots empty – see the section on Resilience,) of every quarter of an hour. It also offers a very decent line speed of 203mph, not too fast and not too slow, and the best station wait time, of 3min28sec. My second choice would be 40tph, for its equally good timetable, though it may be considered, (by HS2 Ltd.,) as, at (only!) 167mph, a bit slow.

The technique alluded to above, of running a timetable with half the slots empty, is in fact a very useful, indeed essential, operational method. It is essential if the Same Speed Line divides into two routes, and the services divide equally, trains taking alternate routes in turn. The branching routes continue to be scheduled exactly as before; half the trains are phantoms, but their dynamic behaviour is identical to the real ones. Travelling in the opposite direction, all the (real) trains are perfectly aligned to merge the two branches' traffic.

Exactly the same considerations apply if the services divide but the line itself doesn't, e.g. if half the trains terminate at an intermediate station, while the rest continue further.

In fact, it makes very good sense to schedule an entire Same Speed route, (or even several, interconnected, cooperating routes, scheduled as a group,) to the same capacity-based timetable throughout. This serves the fundamental purpose of **imposing a uniform, capacity-slot-based time standard throughout**. Over much of the route, (at least) half of the (stopping) trains will be phantoms, but so what? The remaining real trains will be delivering exactly the service required at a particular location.

From the numerical results, it is seen that, for High Speed lines over the entire speed range of interest (and beyond), 108 – 264mph (55 – 23tph), the minimum slot stream advance is 3 slots. Outside that range, at both ends, it is 2. For Medium Speed lines, over their entire rather short speed range, between 64 and 127mph, (62 – 51+tph), the minimum slot stream advance is 2 slots.

Before considering an actual numerical example, something else is worth illustrating in general. More than one train will be on the (virtual) station loop at one time. Is there any possibility they could get in each other's way?

For uniform rates of acceleration and deceleration, from zero to and to zero from a given speed, both the times taken and the distances travelled are inversely proportional to those rates. This is very easily demonstrated:

$$v = a_a t_a = a_d t_d \quad \text{so } t_a/t_d = a_d/a_a$$

For the same times $\quad s_a = a_a t_a^2/2 \qquad s_d = a_d t_d^2/2$

so $\quad s_a/s_d = a_a t_a^2/\, a_d t_d^2 = (a_a/a_d)*(t_a/t_d)^2 \quad = (a_a/a_d)*(a_d/a_a)^2 = a_d/a_a \qquad$ Q.E.D.

Therefore, provided **only** that the deceleration rate is greater than the acceleration rate, (as, in practice, it always is,) the acceleration time and distance are greater than the deceleration time and distance. In the present context, the train reaches the station in a shorter time and distance than it then requires to re-accelerate back up to line speed. The empty slot given up by the train reaches (via the main line) the far end of the (virtual) station loop at the same time as the train has stopped at the station and waited there for a certain time. The empty slot must travel a certain distance further until it is an integer number of slots (time or distance) from the start of the (virtual) station loop, at which point the slot containing the next stopping train reaches the start of station loop. The train has waited at the station for the same amount of time, but since it did part of its waiting before its empty slot reached the end of station loop, by the time that the slot containing the next stopping train reaches the start of station loop, the current stopping train has already departed from the station. In other words, the train leaves the station before the next stopping train reaches the start of station loop. (In the impractical situation of deceleration and acceleration rates being equal, the train would depart the station precisely as the next stopping train entered the station loop. No time values have been specified, so this is always true. The argument is a bit cumbersome, but the result is clear and definitive.

This stuff is not easy to visualise, so here is an actual numerical example, using my favourite line capacity of 32tph.

Line capacity = 32tph. Slot time = 112.5sec. Slot length = 10.2146km.
Line speed = 90.80m/s =326.87kph = 203.11mph.
Deceleration time = 181.59sec = 1.6142 slots Deceleration distance = 8.2441km = 0.8071 slots

Acceleration time = 302.66sec = 2.6902 slots Acceleration distance = 13.7401km = 1.3451 slots
(Virtual) Station Loop Travelling time = 484.25sec = 4.3044 slots
Station Loop distance = 21.9842km = 2.1522 slots

By the time the stopping train has travelled the length of the station loop, its empty slot, which it gave up on entering the loop, has travelled 4.3044 slots (time or distance) on the main line. It is thus 2.1522 slots (time or distance) beyond the end of the (virtual) station loop. In order to make this 3 slots exactly, it must travel a further 0.8478 slots (time or distance). This implies that the train must wait for 0.8478 **time** slots = 95.38sec at the station.

We now follow the progress of slot stream and stopping train.

Time Slot 1:
The empty slot advances 1 slot along the main line.
The train decelerates for 1 time slot. It thus has 0.6142 time slots of deceleration still to do.

Time slot 2:
The empty slot advances a further 1 slot along the main line. It is now 2 slots beyond start of (virtual) loop.
The train completes its deceleration in 0.6142 time slots, and reaches the station. It waits there for 0.3858 time slots. It thus has 0.4620 time slots still to wait.

Time Slot 3:
The empty slot advances a further 1 slot along the main line. It is now 3 slots beyond start of loop. It has also passed the end of the (virtual) station loop; it is 0.8478 slots beyond end of loop. The slot containing the next stopping train has arrived at start of loop.
The train waits at the station for a further 0.4620 time slots. It then departs, performing the first 0.5380 time slots of its acceleration. It thus has 2.1522 time slots of acceleration still to do.

Time Slot 4:
The empty slot advances a further 1 slot along the main line. It is now 4 slots beyond start of loop, and 1.8478 slots beyond end of loop.
The second empty slot advances 1 slot along the main line.
The train performs 1 time slot of acceleration. It thus has 1.1522 slots of acceleration still to do.
The second stopping train decelerates for 1 time slot. It thus has 0.6142 time slots of deceleration still to do.

Time Slot 5:
The empty slot advances a further 1 slot along the main line. It is now 5 slots beyond start of loop, and 2.8478slots beyond end of loop.
The second empty slot advances a further 1 slot along the main line. It is now 2 slots beyond start of loop.
The train performs 1 time slot of acceleration. It thus has 0.1522 time slots of acceleration still to do.
The second train completes its deceleration in 0.6142 time slots, and reaches the station. It waits there for 0.3858 time slots. It thus has 0.4620 time slots still to wait.

During Time Slot 6:
The empty slot advances a further 0.1522 slots along the main line, to 5.1522 slots from start of loop and the (corrected) slot stream advance is **3 slots exactly beyond end of loop.**
The second empty slot advances a further 0.1522 slots along the main line. It is now 2.1522 slots beyond start of loop. **It therefore coincides precisely with end of loop.**

The train performs its final 0.1522 time slots of acceleration. **It therefore coincides precisely with end of loop and is travelling at line speed.**
(What the second stopping train does is immaterial in this context.)

In practice, this line speed requires a slot stream advance value of 4, thus also 4 slot sub-streams. An extra time slot must therefore be added to the station wait time also, giving 1.7813 time slots = 207.88sec.

Readers may legitimately wonder why I am so careful to specify **time** slots on every occasion for the train, but only 'slots' for the slot stream. This is because the slot stream moves at constant speed – the line speed – throughout, so time and distance slots are in permanent 1:1 correspondence. The **only** constant speed experienced by the train is zero, as it waits at the station. But its motion (or lack of it!) is still accurately measured in time slots.

This section has, in a sense, addressed a superficial problem, that of overtaking, but, in order to solve it, had to address the fundamental, underlying problem, which is:
how to remove individual trains from the slot stream, on the main line, and then put them back again, into the slot stream, on the main line, later. (This is, invariably, in order to stop at station.)

If the main line bypasses the station platforms, then non-stop trains are able to overtake. But this does not have to be the case. It could be that the main line simply divides into a number of tracks, each one of which serves a station platform, so **all** the trains stop there. This is the definition of a pure metro. The slot stream simply continues on its way, and the various platform branches merge to re-form the main line, which re-joins the slot stream, (**very** much that way round!).

The treatment given in the present section exactly follows the way I originally discovered it. The notion of a station loop, on which a stopping train physically calls at a station, while all the overtaking trains, still in their capacity slots in the slot stream on the main line (i.e. the non-platform track) overtake it, reflects what actually, physically happens, and forms a clear picture in everyone's mind. But it just won't do for the next section, which deals with metros.

So I now introduce the concept of the **station calling section**. This is the section of track on and over which a train calls at a station. Its composition is formally identical to the (virtual) station loop, dealt with previously, for a train which is to be overtaken. There will in all cases be a unique portion of track associated with a specific station platform, but the rest of the section, covering the deceleration and acceleration tracks, **may** be unique, (for a physical station loop,) or **may not be** unique, (for the divisions and subdivisions of the main line into platform tracks, at a station where all trains stop, and there are no through, non-platform tracks). In the former case, the slot stream has an imaginable reality, but in the latter, it doesn't. It just passes through the station area without any perceptible presence or location, but when the platform tracks, on leaving the station merge and coalesce to re-form the main line, it is re-associated with it. It has thus become what it truly is, a purely virtual time standard, to which the behaviours of all trains participating in the service refer. The only point that the reader really needs to take away from this is that the calculations are, in all essentials, identical for both cases.

Stations on the Main Line: HS-Metros, Pure Metros and Semi-Metros

A route, or section thereof, where all trains call at all stations, **is a metro**, a **pure** metro, in fact. This is the case, irrespective of the line speed.

The traditional metro has been around for a long time, developing its characteristic properties and operational methods over a long period. It is, in fact, the earliest form of a Same Speed Railway. But it was only by considering the characteristics of High Speed Railways that I originally developed the **concept** of Same Speed Railways. The realisation that metros shared many of the same properties led to the idea of the Same Speed Railway as an underlying paradigm, of which High Speed Railways and (Low Speed) metros were manifestations or categories. I do claim this as an original perception, but cannot possibly, and do not suppose that no-one else has ever thought of it. But if they have, they don't seem to have made anything of it. I first documented the idea in the initial version of the 'Same Speed Railways' article, which goes back possibly as far as late 2013, but was first published (at v2.1) in the earliest version of my website, in May 2015. (It was published earlier in a magazine article, but I can't now say precisely when.)

The concept of the High Speed Metro was originally developed and proposed as an alternative way of constructing and operating High Speed Railways. Unfortunately it was based on a misperception that, since no overtaking was required, and therefore no station loops (true), then no high-speed point-work was required at stations. This last is false. A train cannot simply begin decelerating at an arbitrary point on the main line, or its non-decelerating or, rather, not-yet-decelerating successor would begin to close the already-minimum separation (once that had dccreased to TSD(b), if necessary). So the main line must **divide into two** at the beginning of the station calling section, (see final paragraph of previous section,) and alternate trains take alternate tracks in approaching the station. The fact of two approach tracks determines that the number of platform faces at the station must be even – this is not absolutely essential but it is more convenient and operationally sanitary to have each platform face correspond to a particular approach track.

As noted above, the all-trains-stop-at all-stations, or no-overtaking model is a defining characteristic of a pure metro. A conventional, Low Speed metro may appear very different from a High or Medium Speed railway, but appearances are deceptive; the same theory applies to **all** Same Speed Railways. But it is convenient to retain the name HS-Metro, in recognition of the vast difference in perception. And there are indeed differences of detail, if not of essence.

A **semi-metro** is a metro which allows (some) overtaking. So how does that differ from a High Speed Railway with overtaking, as detailed on the previous section? Stated simply, there's less overtaking in a semi-metro. In practice, it's easy to tell the difference (easier than to define it, in fact, like defining an elephant). For the sort of intermediate stations along a High Speed Railway, a stopping service of 4tph is likely to be entirely adequate (and highly satisfactory to the populations served). The stopping service will typically be by a single, 8tph sub-stream, half of them phantoms. The other sub-streams (typically three) will all be non-stop, and may or may not contain phantoms. A semi-metro in the Low Speed range will typically be of 2 sub-streams, one stopping at every station and the other non-stop. The stopping sub-stream will itself very likely be 50% phantom.

The calculations for the Medium and Low speed ranges are detailed in the previous section, on p.29. The first equation applies in the present context, since TSD(b) applies in this range, and also (for Low Speed) the negative value for the square root. Pages 42-44 give the spread-sheet of the results.

Line Capacity (tph)	Capacity Slot Time (s)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Slot Length (km)	Acceleration Time Zero - Line Speed (s)	Acceleration Time (in Capacity Slots)	Acceleration Distance Zero - Line Speed (km)	Acceleration Distance (in Capacity Slots)	De Sp
10	360	2.32	8.35	5.19	0.8354	7.74	0.0214862	0.0090	0.01074312	
11	327.2727273	2.56	9.20	5.72	0.8365	8.52	0.026034094	0.0109	0.013017047	
12	300	2.79	10.05	6.25	0.8378	9.31	0.0310296	0.0130	0.0155148	
13	276.9230769	3.03	10.91	6.78	0.8392	10.10	0.036476843	0.0153	0.018238421	
14	257.1428571	3.27	11.77	7.31	0.8407	10.90	0.042380393	0.0178	0.021190197	
15	240	3.51	12.63	7.85	0.8423	11.70	0.0487452	0.0205	0.02437262	
16	225	3.75	13.51	8.39	0.8441	12.50	0.0555768	0.0235	0.027788421	
17	211.7647059	3.99	14.38	8.94	0.8460	13.32	0.062881171	0.0266	0.03144059	
18	200	4.24	15.26	9.48	0.8480	14.13	0.0706647	0.0300	0.03533236	
19	189.4736842	4.49	16.15	10.04	0.8501	14.96	0.078934527	0.0336	0.039467264	
20	180	4.74	17.05	10.59	0.8524	15.79	0.0876982	0.0374	0.04384912	
21	171.4285714	4.99	17.95	11.16	0.8549	16.62	0.096964131	0.0414	0.048482066	
22	163.6363636	5.24	18.86	11.72	0.8575	17.47	0.106741147	0.0458	0.053370574	
23	156.5217391	5.50	19.78	12.29	0.8602	18.32	0.117038962	0.0503	0.058519481	
24	150	5.75	20.71	12.87	0.8631	19.18	0.127868	0.0552	0.06393402	
25	144	6.02	21.65	13.46	0.8662	20.05	0.1392397	0.0603	0.06961983	
26	138.4615385	6.28	22.61	14.05	0.8694	20.93	0.151166052	0.0657	0.075583026	
27	133.3333333	6.55	23.57	14.64	0.8729	21.82	0.16366042	0.0714	0.08183021	
28	128.5714286	6.82	24.54	15.25	0.8765	22.72	0.176737051	0.0775	0.088368525	
29	124.137931	7.09	25.53	15.86	0.8803	23.64	0.190411417	0.0838	0.095205709	
30	120	7.37	26.53	16.48	0.8843	24.56	0.2047003	0.0905	0.10235015	
31	116.1290323	7.65	27.54	17.12	0.8885	25.50	0.21962188	0.0976	0.10981094	
32	112.5	7.94	28.58	17.76	0.8930	26.46	0.235196	0.1050	0.11759798	
33	109.0909091	8.23	29.62	18.41	0.8977	27.43	0.251444091	0.1129	0.125722046	
34	105.8823529	8.53	30.69	19.07	0.9027	28.42	0.268389753	0.1211	0.134194877	
35	102.8571429	8.83	31.78	19.75	0.9079	29.42	0.286058637	0.1299	0.143029318	
36	100	9.13	32.88	20.43	0.9134	30.45	0.3044789	0.1391	0.15223944	
37	97.2972973	9.45	34.01	21.14	0.9193	31.49	0.323681411	0.1488	0.161840706	
38	94.73684211	9.77	35.17	21.85	0.9254	32.56	0.343700282	0.1590	0.171850141	
39	92.30769231	10.10	36.35	22.58	0.9319	33.65	0.364573148	0.1699	0.182286574	
40	90	10.43	37.55	23.33	0.9388	34.77	0.3863418	0.1814	0.19317089	
41	87.80487805	10.78	38.79	24.10	0.9461	35.92	0.409052682	0.1935	0.204526341	
42	85.71428571	11.13	40.06	24.89	0.9538	37.09	0.432757892	0.2064	0.216378946	
43	83.72093023	11.49	41.37	25.71	0.9620	38.30	0.457515886	0.2201	0.228757943	
44	81.81818182	11.87	42.71	26.54	0.9708	39.55	0.483392745	0.2346	0.241696372	
45	80	12.25	44.10	27.41	0.9801	40.84	0.5104636	0.2502	0.25523179	
46	78.26086957	12.65	45.54	28.30	0.9900	42.17	0.538814392	0.2667	0.269407196	
47	76.59574468	13.06	47.03	29.23	1.0007	43.55	0.568544282	0.2845	0.284272141	
48	75	13.49	48.58	30.19	1.0121	44.98	0.5997685	0.3035	0.29988427	
49	73.46938776	13.94	50.20	31.19	1.0244	46.48	0.63262254	0.3240	0.31631127	
50	72	14.41	51.89	32.24	1.0377	48.04	0.667267	0.3462	0.33363349	
51	70.58823529	14.91	53.66	33.34	1.0522	49.69	0.70389514	0.3703	0.35194757	
52	69.23076923	15.43	55.53	34.51	1.0680	51.42	0.742742905	0.3966	0.371371453	
53	67.9245283	15.98	57.52	35.74	1.0853	53.26	0.784103333	0.4255	0.392051666	
54	66.66666667	16.57	59.64	37.06	1.1045	55.22	0.828348244	0.4574	0.414174122	
55	65.45454545	17.20	61.92	38.48	1.1259	57.34	0.875961706	0.4931	0.437980853	
56	64.28571429	17.89	64.40	40.02	1.1500	59.63	0.927594472	0.5334	0.463797236	
57	63.15789474	18.65	67.13	41.71	1.1777	62.16	0.984158131	0.5795	0.492079066	
58	62.06896552	19.50	70.19	43.61	1.2101	64.99	1.047001506	0.6335	0.523500753	
59	61.01694915	20.47	73.69	45.79	1.2490	68.23	1.118279269	0.6984	0.559139634	
60	60	21.63	77.88	48.39	1.2980	72.11	1.2018555	0.7800	0.60092777	
61	59.01639344	23.13	83.25	51.73	1.3648	77.09	1.306185259	0.8913	0.65309263	
62	58.06451613	25.44	91.60	56.92	1.4774	84.81	1.460702775	1.0790	0.730351387	
62.47891211	57.61944116	28.81	103.71	64.45	1.6600	96.03	1.666666573	1.3833	0.833333287	
62.47891211	57.61944116	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	

	Deceleration Time Line Speed - Zero (s)	Deceleration Time (in Capacity Slots)	Deceleration Distance Line Speed - Zero (km)	Deceleration Distance (in Capacity Slots)	Station Stop Travelling Time (decel'n + accel'n but no wait) (s)	Station Stop Travelling Time (in Capacity Slots)	Station Stop Distance (decel'n + accel'n) (km)	Station Stop Distance (in Caocity Slots)	Cor Stre (in i Cap
12	4.64	0.0128917	0.0054	0.0064	12.38	0.03	0.0144	0.0172	
047	5.11	0.015620456	0.0065	0.0078102	13.63	0.04165455	0.0174	0.02082727	
48	5.59	0.0186178	0.0078	0.0093	14.89	0.049647346	0.0208	0.02482	
421	6.06	0.021886106	0.0092	0.0109431	16.16	0.058362949	0.0245	0.02918147	
197	6.54	0.025428236	0.0107	0.0127141	17.44	0.067808629	0.0285	0.03390431	
62	7.02	0.0292471	0.0123	0.0146	18.72	0.077992379	0.0328	0.039	
421	7.50	0.0333461	0.0141	0.0167	20.01	0.088922946	0.0375	0.04446	
59	7.99	0.037728703	0.0160	0.0188644	21.31	0.100609874	0.0426	0.05030494	
36	8.48	0.0423988	0.0180	0.0212	22.61	0.113063548	0.0479	0.05653	
264	8.97	0.047360716	0.0201	0.0236804	23.93	0.126295243	0.0537	0.06314762	
12	9.47	0.0526189	0.0224	0.0263	25.26	0.140317186	0.0598	0.07016	
066	9.97	0.058178479	0.0249	0.0290892	26.60	0.15514261	0.0663	0.07757131	
574	10.48	0.064044688	0.0275	0.0320223	27.95	0.170785835	0.0732	0.08539292	
481	10.99	0.070223377	0.0302	0.0351117	29.31	0.18726234	0.0805	0.09363117	
02	11.51	0.0767208	0.0331	0.0384	30.69	0.204588853	0.0883	0.10229	
83	12.03	0.0835438	0.0362	0.0418	32.08	0.222783454	0.0965	0.11139	
026	12.56	0.090699631	0.0394	0.0453498	33.49	0.241865684	0.1051	0.12093284	
021	13.09	0.098196252	0.0429	0.0490981	34.91	0.261856672	0.1143	0.13092834	
525	13.63	0.10604223	0.0465	0.0530211	36.36	0.282779281	0.1239	0.14138964	
709	14.18	0.11424685	0.0503	0.0571234	37.82	0.304658268	0.1341	0.15232913	
15	14.74	0.1228202	0.0543	0.0614	39.30	0.327520468	0.1448	0.16376	
094	15.30	0.131773128	0.0585	0.0658866	40.81	0.351395008	0.1561	0.1756975	
98	15.88	0.1411176	0.0630	0.0706	42.34	0.376313545	0.1680	0.18816	
046	16.46	0.150866455	0.0677	0.0754332	43.89	0.402310546	0.1806	0.20115527	
877	17.05	0.161033852	0.0727	0.0805169	45.47	0.429423605	0.1938	0.2147118	
318	17.65	0.171635182	0.0779	0.0858176	47.08	0.457693819	0.2078	0.22884691	
44	18.27	0.18268733	0.0834	0.0913	48.72	0.487166214	0.2225	0.24358	
706	18.90	0.194208847	0.0893	0.0971044	50.39	0.517890258	0.2380	0.25894513	
141	19.54	0.206220169	0.0954	0.1031101	52.10	0.549920451	0.2545	0.27496023	
574	20.19	0.218743889	0.1019	0.1093719	53.84	0.583317037	0.2718	0.29165852	
89	20.86	0.231805066	0.1088	0.1159	55.63	0.618146842	0.2902	0.30907	
341	21.55	0.245431609	0.1161	0.1227158	57.47	0.654484291	0.3096	0.32724215	
946	22.26	0.259654735	0.1238	0.1298274	59.35	0.692412627	0.3302	0.34620631	
943	22.98	0.274509532	0.1320	0.1372548	61.29	0.732025418	0.3521	0.36601271	
372	23.73	0.290035647	0.1408	0.1450178	63.28	0.773428392	0.3754	0.3867142	
79	24.50	0.306278154	0.1501	0.1531	65.34	0.816741743	0.4002	0.40837	
196	25.30	0.323288635	0.1600	0.1616443	67.47	0.862103026	0.4268	0.43105151	
141	26.13	0.341126569	0.1707	0.1705633	69.68	0.909670852	0.4551	0.45483543	
27	26.99	0.359861126	0.1821	0.1799	71.97	0.95962967	0.4856	0.47981	
127	27.89	0.379573524	0.1944	0.1897868	74.37	1.012196064	0.5185	0.50609803	
49	28.83	0.4003602	0.2077	0.2002	76.87	1.067627174	0.5540	0.53381	
757	29.81	0.422337084	0.2222	0.2111685	79.50	1.126232224	0.5925	0.56311611	
453	30.85	0.445645743	0.2380	0.2228229	82.27	1.188388649	0.6346	0.59419432	
566	31.96	0.470462	0.2553	0.235231	85.22	1.254565332	0.6808	0.62728267	
122	33.13	0.497008946	0.2745	0.2485045	88.36	1.32535719	0.7319	0.6626786	
853	34.40	0.525577024	0.2959	0.2627885	91.74	1.40153873	0.7890	0.70076936	
236	35.78	0.556556683	0.3200	0.2782783	95.41	1.484151155	0.8534	0.74207558	
066	37.29	0.590494879	0.3477	0.2952474	99.45	1.57465301	0.9272	0.7873265	
753	38.99	0.628200904	0.3801	0.3141005	103.98	1.67520241	1.0136	0.83760121	
634	40.94	0.670967561	0.4190	0.3354838	109.17	1.78924683	1.1174	0.89462342	
77	43.27	0.7211133	0.4680	0.3606	115.38	1.922968865	1.2480	0.96148	
263	46.25	0.783711156	0.5348	0.3918556	123.34	2.089896415	1.4262	1.04494821	
387	50.89	0.876421665	0.6474	0.4382108	135.70	2.337124439	1.7265	1.16856222	
287	57.62	0.999999944	0.8300	0.5	153.65	2.666666517	2.2133	1.33333326	
	#NUM!!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	

top (in	Corrected Slot Stream Advance (in integral Capacity Slots)	Corrected Station Wait Time (s)	Corrected Station Stop Time (with 1 extra slot)	Corrected Station Wait Time (s)	Corrected Station Stop Time (with 2 extra slots)	Corrected Station Wait Time (s)	Corrected Station Stop Time (with 3 extra slot3)	Corrected Station Wait Time (s)	...	Corrected Station Stop Time (with 7 extra slots)	Corrected Station Wait Time (s)
72	1	353.81	2	713.81	3	1073.81	4	1433.81		8	2873.81
727	1	320.46	2	647.73	3	975.00	4	1302.27		8	2611.37
82	1	292.55	2	592.55	3	892.55	4	1192.55		8	2392.55
147	1	268.84	2	545.77	3	822.69	4	1099.61		8	2207.30
431	1	248.42	2	505.57	3	762.71	4	1019.85		8	2048.42
39	1	230.64	2	470.64	3	710.64	4	950.64		8	1910.64
46	1	215.00	2	440.00	3	665.00	4	890.00		8	1790.00
494	1	201.11	2	412.88	3	624.64	4	836.41		8	1683.46
53	1	188.69	2	388.69	3	588.69	4	788.69		8	1588.69
762	1	177.51	2	366.98	3	556.46	4	745.93		8	1503.82
16	1	167.37	2	347.37	3	527.37	4	707.37		8	1427.37
131	1	158.13	2	329.56	3	500.99	4	672.42		8	1358.13
292	1	149.66	2	313.30	3	476.94	4	640.57		8	1295.12
117	1	141.87	2	298.39	3	454.91	4	611.43		8	1237.52
29	1	134.66	2	284.66	3	434.66	4	584.66		8	1184.66
39	1	127.96	2	271.96	3	415.96	4	559.96		8	1135.96
284	1	121.72	2	260.18	3	398.64	4	537.10		8	1090.95
834	1	115.88	2	249.21	3	382.54	4	515.88		8	1049.21
964	1	110.39	2	238.96	3	367.54	4	496.11		8	1010.39
913	1	105.23	2	229.37	3	353.50	4	477.64		8	974.19
76	1	100.35	2	220.35	3	340.35	4	460.35		8	940.35
975	1	95.73	2	211.85	3	327.98	4	444.11		8	908.63
16	1	91.33	2	203.83	3	316.33	4	428.83		8	878.83
527	1	87.15	2	196.24	3	305.33	4	414.42		8	850.78
118	1	83.15	2	189.03	3	294.91	4	400.80		8	824.32
691	1	79.32	2	182.18	3	285.03	4	387.89		8	799.32
58	1	75.64	2	175.64	3	275.64	4	375.64		8	775.64
513	1	72.10	2	169.40	3	266.70	4	363.99		8	753.18
023	1	68.69	2	163.42	3	258.16	4	352.90		8	731.85
852	1	65.39	2	157.69	3	250.00	4	342.31		8	711.54
07	1	62.18	2	152.18	3	242.18	4	332.18		8	692.18
215	1	59.07	2	146.88	3	234.68	4	322.49		8	673.71
631	1	56.04	2	141.75	3	227.47	4	313.18		8	656.04
271	1	53.08	2	136.80	3	220.52	4	304.24		8	639.12
142	1	50.18	2	132.00	3	213.81	4	295.63		8	622.91
37	1	47.33	2	127.33	3	207.33	4	287.33		8	607.33
151	1	44.53	2	122.79	3	201.05	4	279.31		8	592.35
543	1	41.76	2	118.35	3	194.95	4	271.54		8	577.93
81	1	39.01	2	114.01	3	189.01	4	264.01		8	564.01
803	1	36.29	2	109.76	3	183.23	4	256.69		8	550.57
81	1	33.57	2	105.57	3	177.57	4	249.57		8	537.57
611	1	30.84	2	101.43	3	172.02	4	242.60		8	524.96
432	1	28.09	2	97.33	3	166.56	4	235.79		8	512.71
267	1	25.32	2	93.24	3	161.17	4	229.09		8	500.79
786	1	22.49	2	89.15	3	155.82	4	222.49		8	489.15
936	1	19.59	2	85.04	3	150.50	4	215.95		8	477.77
558	1	16.58	2	80.87	3	145.15	4	209.44		8	466.58
265	1	13.43	2	76.59	3	139.75	4	202.91		8	455.54
121	1	10.08	2	72.15	3	134.22	4	196.29		8	444.56
342	1	6.43	2	67.45	3	128.46	4	189.48		8	433.55
48	1	2.31	2	62.31	3	122.31	4	182.31		8	422.31
821	2	56.36	3	115.38	4	174.40	5	233.41		9	469.48
222	2	48.28	3	106.34	4	164.41	5	222.47		9	454.73
326	2	38.41	3	96.03	4	153.65	5	211.27		9	441.75
I!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!		#NUM!	#NUM!

Metro routes typically consist of a number of sections, with a central core of pure metro, where all trains stop at all stations, then Low Speed semi-metro sections beyond the core in both directions, over which some trains travel non-stop, to become stopping trains again in the outer sections, which are Medium Speed semi-metros. The line speed actually changes at a station where all trains stop, the stopping trains of the inner, Low Speed section often terminate at this station, and the continued service is taken over by the former non-stop trains. By this means significantly shorter journey times between the outer reaches and the central core are made possible, avoiding such disagreeable present journeys as, for example, between West Ruislip and Central London.

One genuine difference, and a defining one, between Low Speed metros or semi metros and those in other speed ranges is that, in the Low Speed range, line capacity **increases** with line speed. This means that we adopt the highest appropriate line capacity and speed, and, if the capacity is more than actually needed, introduce a proportion of phantoms. (If reducing the actual trains by half seems excessive, we can be more subtle than that. But note that, by adopting a proportion or phantoms of less than 50%, the clock-face timetable is retained, but the regular time distance between trains is lost.)

Another genuine difference is that, for a semi-metro although not for a true metro, the line speed can (and usually will) change. But before explaining this, the actual performance needs to be analysed,

It can happen that the station stop distance, being the sum of deceleration and acceleration distance, (and equal to the station calling section length,) actually exceeds the distance between adjacent stations. This is quite usual, albeit not frequent, with High Speed Railways, where this value can be quite large (14.9km = 9.2miles for line speed 269kph = 167mph, 40tph, and 22km = 13.7miles for line speed 327kph = 203mph, 32kph). In such cases the values have to be calculated explicitly and input manually (in calculating journey times, for instance).

But the station stop distance has reached only 2.21km = 1.37miles by the time line capacity reaches its maximum at 103.71kph = 64.45mph, so, provided that there are no stations closer together than this distance, the calculation process for metros described above can be used with confidence. For the capacity value we're really interested in, 60tph, the minimum inter-station distance is only 1.25km = 0.78 miles. (If there are pairs of stations closer that the above distance, then the calculations are only valid for lower line speeds, which yield the lower inter-station distance.) I don't think this is likely to be a problem. Referring back to the earlier table, for the line speed corresponding to 64tph in the Medium Speed range, the minimum inter-station distance is 3.92km = 2.44 miles. This might be more of a problem.

For pure metros, every train is a stopping train. Alternatively, every capacity slot sub-stream is a stopping one. There are really only three capacity values worth considering, 48, 50 and 60tph. (As usual, my best choices are in red.)

Line Cap-acity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph) _	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
48	75	13.49	48.58	30.19	0.48 / 0.30	2 / 3 / 4	114 / 189 / 264	2½ / 3¾ / 5
50	72	14.41	51.89	32.24	0.55 / 0.34	2	106	2.4 = 2m24s
60	60	21.63	77.88	48.39	1.25 / 0.78	2 / 3 / 4 / 5 / 6	62 / 122 / 182 / 242 / 302	2 / 3 / 4/ 5 / 6

For capacity 50tph, the value chosen does actually give what is technically a clock-face timetable, of 2 sub-streams of 25tph. The others don't.

For a pure metro, it is possible to vary the station wait times, by varying the number of platforms. It may well be that the anticipated passenger volumes at one particular station make a longer wait time desirable at that station. An integer number of time slots can be added to the slot stream advance (and therefore to the number of sub-streams) and to the station wait time. But this number is not arbitrary: the slot stream advance must still be an integer sub-multiple of the line capacity, since this is one of the necessary conditions for a viable timetable. So this is not a wonderful facility, but may be of use in special cases. (60tph has possible values of 2,3,4,5 and 6!).

For metros and semi-metros, station wait times of around 1 minute are generally acceptable; anything over say 2½ minutes is almost certainly too long.

Readers may fairly wonder why anyone should care about clock-face, with frequencies of around one every minute. But the **line** frequency is spread over several platform faces, each of which enjoys only a fraction of the overall value, and a clock-face timetable applies to a particular platform face. So they remain valuable, even at metro level.

Note that the values (apart from the slot time, of course,) are all strikingly different from those encountered in the previous section for those capacities. Naturally so, for these line capacities are points on the ascending side of the capacity graph, whereas the previous ones are all from the other side of the hump.

The technique of scheduling more capacity than in fact is going to be used, and leaving half or more of it as phantom slots, was introduced in the previous section. This is more common in metros, which frequently need to split their services. Almost all metros show a wide variation in the capacities actually used, between the central core and the outer ends.

One point needs careful clarification. As has been explained previously, adjacent trains, from adjacent sub-streams, cannot both be diverging, except for the Low Speed range, for which the deceleration time is less than the slot time, so that a stopping train has already reached the station before the following train reaches the start of (physical) station loop. This is perfectly correct, but applies only to (High and Medium Speed) **station loops**, which consist of a **single track** diverging from the main line, allowing overtaking. This is the situation with all semi-metros, but it does not apply to **pure** metros, **in any speed range**. The way trains on a pure metro approach a station has been described in the third paragraph of the present section. The critical point is that alternate trains diverge **along alternate tracks**; a particular train has therefore always reached the station well before the next train due to use the same **approach track** (not the same **platform**) has even reached the dividing points.

Metro stuff is effectively impossible to visualise for reasons I did my best to explain at the end of the previous section. All I can do here is to state the maths. Sorry.

Line capacity = 60tph.   Slot time = 60sec. Slot length = 1.2980km.
Line speed = 21.63m/s = 77.88kph = 48.39mph.
Deceleration time = 43.27sec = 0.7211 slots   Deceleration distance = 0.4680km = 0.3606 slots
Acceleration time = 72.11sec = 1.2019 slots   Acceleration distance = 0.7800km = 0.6009 slots

Station Calling Section travelling time = 115.38sec = 1.9230 slots.
Station Calling Section distance = 1.2480km = 0.9615 slots

By the time the stopping train has travelled the length of the station calling section, its empty slot, which it gave up on entering the section, has travelled 1.9230 slots (time or distance). It is thus 0.9615 slots (time or distance) beyond the end of the section. In order to make this 1 slot exactly, it must travel a further 0.0385 slots (time or distance). This implies that the train must wait for 0.0385 **time** slots = 2.30sec at the station.

We now follow the progress of slot stream and stopping train.

Time Slot 1:
The empty slot advances 1 slot. It is now 1 slot beyond start of section and also 0.0385 slots beyond end of section. The slot containing the next stopping train has arrived at start of section.
The train decelerates for 0.7211 time slots and reaches the station, where it waits for 0.0385 time slots. Finally, it departs from the station and accelerates for 0.2404 time slots. It thus has 0.9615 time slots of acceleration still to do.

During Time slot 2:
The empty slot advances a further 0.9615 slots. It is now 1.9615 slots beyond start of section, and **1 slot exactly beyond end of section.** The slot stream advance is thus 1.
The second (empty) slot advances 0.9615 slots. **It therefore coincides precisely with end of section.**
The train accelerates for a further 0.9615 time slots, thus completing its acceleration. **It therefore coincides precisely with end of section and is travelling at line speed.**
(The second stopping train also decelerates, waits and accelerates for 0.7211, 0.0385 and 0.2019 time slots respectively, a total of 0.9615 time slots – but so what?)

This describes a pure metro. All the trains are in a single slot stream, using a single platform. There is no overtaking; there cannot be, as the trains are all on the same track. The capacity slot stream, being a purely virtual concept, is in no way obstructed by this, and advances by one slot (time or distance) as against the stopping train.

This is, of course, a totally unrealistic situation. While it would actually work, a station wait time of under 3sec is no use for anything. The wait time and the slot stream advance must therefore be increased by (at least) 1 slot time (or distance, for the slot stream advance). This means that the slot stream is now divided into 2 sub-streams, and both platform faces are now in use. One of the sub-streams is for stopping trains, all using the same platform, (this is not necessarily so, but it's easier to imagine that way,) with a station wait time of 62.31sec, and the other is either also stopping, with likewise a station wait time of 62.31sec, in which case we have a pure metro, or non-stop, overtaking, in which case we have a (Low Speed) semi-metro.

It is instructive to demonstrate this for a High Speed (Pure) Metro also. In fact, the values are identical with the overtaking case, pp.20-21. However, it is given again, here, but adding the extra 1 time slot to make the slot stream advance 4, from the beginning, rather than as a post-hoc correction.

Taking the capacity value of 32tph:

Line capacity = 32tph. Slot time = 112.5sec. Slot length = 10.2146km.
Line speed = 90.80m/s =326.87kph = 203.11mph.
Deceleration time = 181.59sec = 1.6142 slots Deceleration distance = 8.2441km = 0.8071 slots

Acceleration time = 302.66sec = 2.6902 slots   Acceleration distance = 13.7401km = 1.3451 slots
Station Calling Section travelling time = 484.25sec = 4.3044 slots
Station Calling Section distance = 21.9842km = 2.1522 slots

By the time the stopping train has travelled the length of the station loop, its empty slot, which it gave up on entering the loop, has travelled 4.3044 slots (time or distance) on the main line. It is thus 2.1522 slots (time or distance) beyond the end of the (virtual) station loop. In order to make this 3 slots exactly, it must travel a further 0.8478 slots (time or distance). This implies that the train must wait for 0.8478 **time** slots = 95.38sec at the station. But we know in advance that this would give a slot stream advance of 3, when the value required is actually 4. So take that value ab initio, thus station wait time = 1.8478 time slots,
= 207.88sec.

We now follow the progress of slot stream and stopping train.

Time Slot 1:
The empty slot advances 1 slot along the main line.
The train decelerates for 1 time slot. It thus has 0.6142 time slots of deceleration still to do.

Time slot 2:
The empty slot advances a further 1 slot along the main line. It is now 2 slots beyond start of section.
The train completes its deceleration in 0.6142 time slots, and reaches the station. It waits there for 0.3858 time slots. It thus has 1.4620 time slots still to wait.

Time Slot 3:
The empty slot advances a further 1 slot along the main line. It is now 3 slots beyond start of section. It has also passed the end of the (virtual) station section; it is 0.8478 slots beyond end of section.
The train waits at the station for 1 time slot. It thus has 0.4620 time slots still to wait

Time Slot 4:
The empty slot advances a further 1 slot along the main line. It is now 4 slots beyond start of section, and 1.8478 slots beyond end of section.
The slot containing the next stopping train **for that particular platform** has arrived at start of section.
The train waits at the station for a further 0.4620 time slots. It then departs, performing the first 0.5380 time slots of its acceleration. It thus has 2.1522 time slots of acceleration still to do.

Time Slot 5:
The empty slot advances a further 1 slot along the main line. It is now 5 slots beyond start of section, and 2.8478slots beyond end of section.
The second empty slot advances 1 slot.
The train performs 1 time slot of acceleration. It thus has 1.1522 slots of acceleration still to do.
The second train decelerates for 1 time slot. It thus has 0.6142 time slots of deceleration still to do.

Time Slot 6:
The empty slot advances a further 1 slot along the main line. It is now 6 slots beyond start of section, and 3.8478slots beyond end of section.
The second empty slot advances a further 1 slot. It is now 2 slots beyond start of section.
The train performs 1 time slot of acceleration. It thus has 0.1522 time slots of acceleration still to do.
The second train completes its deceleration in 0.6142 time slots, and reaches the station. It waits there for 0.3858 time slots. It thus has 1.4620 time slots still to wait.

During Time Slot 7:
The empty slot advances a further 0.1522 slots along the main line, to 6.1522 slots from start of section and the (corrected) slot stream advance is **4 slots exactly beyond end of section.**
The second empty slot advances a further 0.1522 slots along the main line. It is now 2.1522 slots beyond start of section. **It therefore coincides precisely with end of section.**
The train performs its final 0.1522 time slots of acceleration. **It therefore coincides precisely with end of section and is travelling at line speed.**
(The second train also waits at the station for 0.1522 time slots – but so what?)

Summarising: Each train takes 6.1522 time slots to call at the station and re-join the main line, thus:

deceleration	1.6142 time slots,
station wait	1.8478 time slots,
acceleration	2.6902 time slots,
`	6.1522 time slots in total.

Note that this applies to every train in each of the 4 sub-streams. Each train occupies the capacity slot given up by the next train in its own sub-stream.

This could, theoretically at least, be operated with just 2 platform faces. Imagine that trains in sub-streams 1 and 3 both used the same platform. So the second train would decelerate for 1 time slot in Time Slot 3, above, while the first train was waiting at the station, and reach the station, 0.6640 time slots into Time Slot 4, (0.6142 – 0.4620 =) 0.1522 time slots = 17.12 sec after the first train had departed. Tight scheduling is, after all, what Same Speed is all about. I merely point out the possibility, without (as yet) a recommendation!

Change of Line Speed, for a Semi-Metro

A metro service has the same line capacity throughout, (60tph – that's the most appropriate; there's no reason to use any other value,) but the line speed changes (from 48 to 86mph) when it becomes a Medium Speed semi-metro. It may switch directly to this from pure metro, but there will usually be an intervening section of Low Speed semi-metro, (if there are inter-station distances less than the minimum for the Medium Speed case; if there aren't, there's no reason not to switch directly).

The change of line speed will normally (certainly for a metro route) take place at a station. Trains decelerate from 48mph on the Low Speed side, to stop at the station, then accelerate to 86mph on continuing beyond the station. It is possible, though unlikely, that some trains will not stop at the station. For these, non-stop trains, they approach the station at 48mph, then, on passing the station, begin the acceleration to 86mph. The capacity slot stream itself accelerates, beginning at the station, from the lower to the higher speed; slot stream and non-stop train thus accelerate in lockstep, the change in speed makes no difference whatever to their relationship. In the reverse direction, slot stream and non-stop train decelerate in lockstep, reaching the lower speed as they together pass the station, and then continue together at the lower speed. This is obvious, for non-stop trains, but in fact the equivalent effect applies to trains which call at the station also. The acceleration from lower to higher line speed takes place as the final component of the train's acceleration to the higher speed. In the other direction, the deceleration to the lower speed takes place as the first component of the train's deceleration to zero at the station. Train and slot stream no longer proceed in lockstep, as with the non-stop case, (the acceleration of the slot stream takes place as the first component, immediately on passing the station, and in the other direction the deceleration takes place as the last component, immediately before,) but the effects are identical. What this means, is that this speed-change portion can be completely ignored in the calculations, since it makes no difference whatever to the relationship between train and slot stream. In effect the line speed, and hence the speed of the slot stream, changes instantaneously on passing the station, and is thus constant, although at different values, on both sides. This further means that the line capacity and thus the slot time is constant throughout the calculation, which, strictly speaking, is not the case for the section where the line speed changes. (The slot **length**, being the distance travelled at constant line speed in the slot time, is obviously different for different line speeds.) This makes a great simplification in what would otherwise be a rather intractable problem. But note that it is still necessary to convert the **distance** values on one side of the station, so that they are all expressed relative to the same speed throughout (this is to ensure that the fundamental relationship, that the time taken by the train to travel over the station loop is twice the time taken by the empty slot to travel the same distance, at the chosen speed, still applies). Note also that the equalisation time, applied to **integralise** the slot stream advance, is different for the two directions.

Note that it is perfectly okay to speak of station loops for a semi-metro, since overtaking **is** involved.

This may well sound like a computational three-card trick, so here are the actual calculations, for the usual 60tph case.

Line capacity = 60tph. Slot time = 60sec.

Line speed (Low Speed value, V_L) = 21.63m/s = 77.88kph = 48.39mph (Slot length)$_L$ = 1.2980km
(Medium Speed value, V_M) = 38.37m/s = 138.12kph = 85.83mph (Slot length)$_M$ = 2.3020km

Low to Medium Speed (V_L => V_M):

Deceleration time (V_L => 0) = 43.27sec = 0.7211 time slots
Deceleration distance = 0.4680km = 0.3606 (distance slots)$_L$ = 0.2033 (distance slots)$_M$
Acceleration time (0 => V_M) = 127.89sec = 2.1315 time slots
Acceleration distance = 2.4533km = 1.0657 (distance slots)$_M$
Acceleration time (V_L => V_M) = 55.78sec = 0.9297 time slots
Acceleration distance = 1.6733km = 0.7269 (distance slots)$_M$

Station Loop travelling time = 171.16sec = 2.8527 time slots
Station Loop distance = 2.9233km = 1.2699 (distance slots)$_M$

By the time the train has travelled the station loop distance, the empty slot has travelled 2.8527 slots (time or (distance)$_M$). Thus the empty slot is 1.5828 slots (time or (distance)$_M$) beyond end of loop. This implies that the train must wait for 0.4172 time slots (= 25.03sec) at the station to make the slot stream advance = 2, exactly. This was calculated ignoring the speed change portion, but in following the progress of train and slot stream, there is no reason why the speed change portion should not be included

Time Slot 1:
The empty slot advances 0.2033 time slots to reach the station, (it actually advances 0.4680km, which is 0.3606 time slots at V_L or, equivalently, 0.2033 time slots at V_M,) then accelerates (from V_L) for 0.7967 time slots. It thus has a further 0.9297 – 0.7967 = 0.1330 time slots still to accelerate (to reach V_M). It is now 1 time slot from start of loop.

The train decelerates for 0.7211 time slots to reach the station, where it waits for 0.2789 time slots. It thus has a further 0.4172 – 0.2789 = 0.1383 time slots still to wait.

Time Slot 2:
The empty slot accelerates for 0. 1330 time slots and reaches V_M. It then advances a further 0.8670 time slots at V_M. It is now 2 time slots from start of loop and 2 – 1.2699 = 0.7301 time slots beyond end of loop.
The train waits for 0. 1383 time slots, then accelerates for 0.8617 time slots. It thus has 2.1315 – 0.8617 = 1.2698 time slots still to accelerate.

Time Slot 3:
the empty slot advances a further 1 time slot at V_M. It is now 3 time slots from start of loop, and 1.7301 time slots beyond end of loop.
The train accelerates for 1 time slot. It thus has 0.2698 time slots still to accelerate.

During Time Slot 4:
The empty slot advances a further 0.2699 time slots at V_M. It is now 3.2699 time slots from start of loop and 2 time slots precisely beyond end of loop. The slot stream advance is thus 2.
The train accelerates for the remaining 0.2698 time slots, achieving line speed V_M precisely as it reaches end of loop. (We will brazenly ignore the rounding error of 1 part in 2700!!)

A further 2 time slots need to be added to the slot stream advance and to the station wait time, to make the slot stream advance and thus the number of sub-streams 4, as required by this line speed. The station wait time is thus 2.4172 time slots, = 145.03sec.

Medium to Low Speed (V_M => V_L):

Deceleration time (V_M => 0) = 76.73sec = 1.2789 time slots
Deceleration distance = 1.4720km = 0.6394 $(\text{distance slots})_M$ = 1.1341 $(\text{distance slots})_L$
Acceleration Time (0 => V_L) = 72.11sec = 1.2019 time slots
Acceleration distance = 0.7800km = 0.6009 $(\text{distance slots})_L$
Deceleration time (V_M => V_L) = 33.46sec = 0.5577 time slots
Deceleration distance = 1.0040km = 0.4361 $(\text{distance slots})_M$ = 0.7735 $(\text{distance slots})_L$

Station Loop travelling time = 148.84sec = 2.4807 time slots
Station Loop distance = 2.2520km = 2.0780 $(\text{distance slots})_L$

By the time the train has travelled the station loop distance, the empty slot has travelled 2.4807 slots (time or $(\text{distance})_M$). Thus the empty slot is 0.4027 slots (time or $(\text{distance})_M$) beyond end of loop. This implies that the train must wait for 0.5973 time slots (= 35.84sec) at the station to make the slot stream advance = 1, exactly. This was calculated ignoring the speed change portion, but in following the progress of train and slot stream, there is no reason why the speed change portion should not be included

Empty slot and train both decelerate from V_M to V_L taking, obviously, the same time – 33.46sec = 0.5572 time slots. The empty slot completes this deceleration at the point when it reaches speed V_L, simultaneously with reaching the station. It continues beyond the station at constant speed V_L. The train begins its deceleration immediately on departing from start of loop, It continues to decelerate beyond V_L

down to zero, which it reaches at the point when both it and the empty slot reach the station simultaneously. Therefore, the time advanced by the empty slot from leaving the start of loop to beginning its deceleration is precisely the same as that taken by the train to decelerate from V_L to zero.

Time Slot 1:
The empty slot advances 0.7211 time slots at speed V_M, then decelerates for 0.2789 time slots. It thus has 0.5577 – 0.2789 = 0.2788 time slots still to decelerate. It is now 1 time slot from start of of loop.
The train decelerates for 1 time slot. It thus also has 0.2788 time slots still to decelerate.

Time Slot 2:
The empty slot decelerates for 0.2788 time slots, and reaches tha station as its speed reaches V_L. It then advances a further 0.7212 time slots at speed V_L beyond the station. It is now 2 time slots from start of loop.
The train performs its final 0.2788 time slots of deceleration, reaching the station and stopping there. It waits at the station for 0.5973 time slots, then accelerates for 0.1239 time slots. It thus has 1.2019 – 0.1239 = 1.0780 time slots still to accelerate.

Time Slot 3:
The empty slot advances 1 time slot at speed V_L. It is now 3 time slots from start of loop and 0.9220 time slots beyond end of loop.
The train accelerates for 1 time slot. It thus has 0.0780 time slots still to accelerate.

During Time Slot 4:
The empty slot advances 0.0780 time slots at speed V_L It is now 3.0780 time slots from start of loop, and 1 time slot precisely beyond end of loop. The slot stream advance is thus 1.
The train accelerates for the remaining 0.0780 time slots, achieving line speed V_L precisely as it reaches end of loop.

An extra time slot is added to the station wait time and to the slot stream advance, since the slot stream advance and thus the number of sub-streams must be 2 for this line speed. The station wait time is thus 1.5973 time slots = 95.84sec.

In performing the above pair of calculations, it does seem to make a difference on which side the distance values are converted, and it should be the side after the station. I got this right by chance the first time, going from V_L to V_M, and left it like that for the other calculation. But this turned into a very fraught process. Switching it to go the other way completely obliterated the problem, and the above results came out pretty much by themselves.

Summarising all the values of interest for metros and semi-metros at the 60tph capacity:

Line Cap-acity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph) _	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
60	60	21.63	77.88	48.39	1.25 / 0.78	2	62	2
					Low to	Medium	145	
60	60	38.37	138.12	85.87	3.92 / 2.44	4	138	4
					Medium to	Low	96	

Change of Line Speed for a High (or Medium) Speed Railway

The previous section dealt with the important special case of switching line speed between the two possible values, in the Low and Medium Speed ranges, for the same value of line capacity, which is the (equal) maximum capacity at both of those speeds.

The line speed of all Same Speed railways must be constant over each section, but it is possible that this may vary between different sections. Generally this means **either** that the line capacity also varies, so that it is still at the maximum (but different) value for each speed individually, **or** that the capacity remains constant throughout at the value corresponding to the higher speed, but this is no longer the maximum capacity at the lower speed. Precisely how this is handled depends on several factors, of which the most important is whether or not the speed change is permanent, i.e. whether, once having switched to the lower speed, that line speed then persists for the rest of the journey, or whether it is transient, and later (usually, but not always, quite soon) reverts to the higher speed. Note that, in considering line speed changes, we are primarily concerned with decelerating to a lower speed; accelerating (back) to the higher speed involves no complications – simply accelerate as required.

There may seem to be a contradiction in saying, on the one hand, that the line speed is constant within each section, but that, on the other, the trains decelerate to a lower speed. **Line** speed, being the maximum speed that trains may travel, as well as the speed at which they normally do travel, does remain fixed for each section, but trains may, when required, travel at a lower speed, while actually **changing** speed. (The actual speed obviously varies when accelerating from / decelerating to a station stop.) Note that all changes of speed must take place in the higher speed section; the train must already be travelling at the lower speed when it enters the lower-speed section, or **still** travelling at the lower speed until it has **completely** left the lower-speed section and entered the higher-speed section – only then can acceleration begin. Each train, together with its containing capacity slot, decelerates or accelerates in lock-step. The actual slot size, time or distance or both, changes as the speed changes.

This topic is in fact surprisingly difficult; it has taken me an extraordinarily long time to get a grip on it and to write a satisfactory exposition of it. Its consequences are of immense importance.

The **overall** line capacity of **any** (Same Speed) route is defined by the **maximum** line speed over all the sections. **Temporary** changes of line speed are not usually encountered for High Speed railways, at least, not for the core High Speed section, which will be new-build, to a uniform line speed. But it **can** happen, particularly in the early stages of implementation, where sections of classic route may be incorporated as (hopefully!) a temporary measure, to get some services running as soon as possible. (The temptation to economise in construction costs by allowing for lower speeds in more challenging landscapes should be resisted, or at least needs very careful justification: such economy will incur extra costs for ever afterwards in operational complexities and inefficiencies.)

Medium Speed railways, particularly when these are conversions to Same Speed standards of classic routes, generally include line speed variability and permanent speed restrictions which it may not be practically possible, (more likely, not economically worthwhile,) to remove. Any sections of lower line speed, as here, would (theoretically) offer higher line capacities. But this can't, in general, actually be taken advantage of: where would the extra trains come from or go to?

Temporary sections of lower line speed are almost invariably handled by maintaining the line capacity of the High Speed sections, so that the line capacity remains constant over all sections, even if the line speed doesn't. This is achieved by maintaining the same **time slot** value throughout (since line capacity in tph =

3600 / slot time in seconds). Naturally, this changes the capacity slot **length**, since this is equal to the distance travelled at (local) line speed in the slot time.

It may not – indeed, it almost certainly will not – be immediately obvious, but what this means is that, in this particular case, the slot length decreases, in absolute terms, but **increases, relative to the Train Separation Distance for that line speed,** as the (instantaneous) line speed itself decreases, but slot time and line capacity remain constant. (This is true for High and Medium speed ranges, where line capacity decreases as line speed increases, but not for the Low Speed range, where the opposite applies.) The confusion arises because, in the standard case for Same Speed railways, where slot time and length are such that the line capacity is always at its maximum for the line speed, the slot length and time **both decrease** as the line speed decreases, (and the line capacity increases,) but the slot length remains equal to the TSD (basic or extended, as appropriate,) for that (instantaneous) speed.

The original, fundamental definition of the capacity slot length, which is the foundation of the entire Capacity Slot Model, is that it is equal to TSD(e) for the High Speed range and TSD(b) elsewhere. In other words it's all about maintaining the necessary separation between adjacent trains. This remains perfectly clear if the calculations start from the line speed as the independent variable, and thus from the capacity slot length, from which the slot time and thus the line capacity are then derived. This was indeed my original approach, as explained at the beginning of the section 'Timetabling Considerations and Sweet-Speeds'. But this is rather lost sight of in the (far more convenient) approach where the calculations start from the line capacity as the independent variable, and thus from the slot time, from which the line speed and thus the slot length are then derived. It has proved necessary to go back to the original definitions to achieve the essential mental clarity needed to understand **precisely** what is happening here.

Accordingly, when the line speed is temporarily reduced, but the line capacity and thus slot time held constant, the capacity slot length, and thus train separation, increases relative to the TSD for the reduced line speed. Adjacent trains are thus further apart than they strictly need to be for that line capacity and (non-corresponding) line speed. This means that exactly the same maths applies, even though that section is being operated at less than its (theoretically) maximum capacity. The great advantage of this is that the slot stream advance, and thus the capacity sub-streams, (but **not** the station wait times, if there are any, since these **are** affected by the slot length,) are unchanged. When the time comes to re-accelerate back up to the higher speed, the slot length and TSD automatically increase back to their original values, which are, of course, equal, and everything is as it was originally. A further advantage is that all this is achieved quite automatically, without stopping.

As was explained in the section 'Timetabling Considerations and Sweet-Speeds', there are only a very few line speeds – the Sweet-Speeds – appropriate for Same Speed railways. This refers to the **highest** value of the line speed, which determines the overall line capacity. In the previous three sections we have been concerned only with operation at maximum capacity. This is no longer true for temporary deceleration to lower line speeds, whose natural, Same Speed line capacity would be higher. This has the quirky consequence that the lower line speed can be **any** (lower) speed whatever – not restricted to Sweet-Speeds. So, therefore, existing permanent speed restrictions on converted classic lines can always be accommodated at their existing speeds. (Thus, having, over the previous three sections, stressed the sole availability of the sweet-speeds for operation of Same Speed railways – which is absolutely true as the **overall** condition of a Same Speed Railway – this present section explains how that restriction is relaxed for **temporary** reductions of line speed. This is inevitably confusing, with which I sympathise.)

This technique is always possible; actually, it is the natural situation. If a continuous stream of trains, running under Same Speed principles, begins to decelerate, at a particular location, to a lower, target speed and, having reached that speed, to continue at that as the new, constant line speed, then the line capacity is automatically preserved at the original value.

A Same Speed train cannot decelerate arbitrarily, or the following train would get too close to it, closer than the minimum separation distance. Strictly, this is the **basic** separation distance, TSD(b); for High Speed trains, the leeway in TSD(e) has to be used up first. But either way, the situation is reached (in the High Speed case) or already **is** (in the Medium Speed case) where the decelerating train is travelling at the Turnout Limit Speed or lower, while the following train is (very slightly more than) TSD(b) behind and still travelling at line speed. At this point therefore. the line must divide into two decelerating tracks; the train, still decelerating, takes one track and, as soon as it has completely diverged, i.e. the speed has reduced to the B**uffer-end Speed, (**at which point it is **exactly** TSD(b) ahead of the following train, which is still travelling at line speed,) **and** the switch has just been reset behind it to point to the **other** decelerating track, so that it is no longer in the path of the following train. At this instant, the (front end of the) decelerating train is precisely the **constant** portion of TSD(b), i.e. the buffer length, beyond the switch points, and the following train, still travelling at constant line speed, is precisely the **variable** portion of TSD(b) before the switch points, which are set so as to route it onto the other decelerating track. (The composition of the constant, buffer component of the TSD was explained in detail in the section ‘Consequences of the Results’.)

As was explained in the third paragraph of the previous but one section, on metros, exactly the same applies on the approach to a metro station.

What happens next is very tedious to describe, and even worse to follow; real brain-numbing stuff! There are three possible and quite separate cases to consider.

1. If the original section’s line speed is in the High Speed range **and** the destination section’s line speed, i.e. the Target Speed, is in the Medium Speed range, (or even the Low Speed range,) then the first train continues decelerating on the deceleration track, until it reaches the target speed, after which it continues at that constant speed until it reaches the end of the deceleration track. At that point, the deceleration tracks merge to reconstitute the main line and, simultaneously, the section boundary with the following, lower-speed section is reached, so the train passes immediately onto the lower-speed section.
   This obviously begs the question of what determines the length of the deceleration tracks. The answer is: when the actual distance between the two trains once again exceeds TSD(b), calculated on the instantaneous speed of the second, following train. That is precisely true, if un-illuminating. The following examples will illustrate and, I hope, clarify.
2. If the original section’s line speed is in the High Speed range **and** the destination section’s line speed is **also** in the High Speed range, (but lower than the original, obviously,) then the first train, behaves exactly as above until it has decelerated down to the Buffer-end Speed. That completes its actual deceleration, and it continues at that speed until it reaches the end of the deceleration track. At this point, now moving back onto the reconstituted main line, it begins its re-acceleration back up to the target speed. When that speed is reached, that is the completion of that train’s **net** deceleration, and it continues at that speed.
3. If the original section’s line speed is in the Medium Speed range, then it is by definition already less than or equal to the Buffer-end Speed. The following train is precisely TSD(b) behind. The train travels the buffer length, at line speed, across the junction. Note that it does not begin its

deceleration immediately; it is still precisely TSD(b) ahead of the following train. The points are then switched to connect to the other deceleration track, so the train is no longer in the path of the following train. **Now** it can begin its deceleration. It performs the deceleration until it reaches the target speed, and then continues at that constant speed until it reaches the end of the deceleration track, the decelerating tracks merge to reconstitute the main line and, simultaneously, the section boundary with the following, lower-speed section is reached, so the train passes immediately onto the lower-speed section – exactly as in case 1 above. The second train performs no deceleration on the main line, but travels at constant line speed until it reaches the junction, whereupon it takes the other deceleration track.

The above cases will be illustrated thus:

1. 32tph constant line capacity, line speed 93.60m/s decelerating to target line speed 38.37m/s.
2. 32tph constant line capacity, line speed 93.60m/s decelerating to target line speed 74.71m/s.
3. 60tph constant line capacity, line speed 36.37m/s decelerating to target line speed 25.00m/s.

The results are presented as an Excel line chart, with elapsed time as the independent variable. The quantities plotted are: speed and distance travelled for each of the two trains, the Basic Train Separation Distance for train 2 (since the critical issue is the distance of train 2 behind train 1), the actual, instantaneous separation distance, and the amount by which this exceeds TSD(b). The graph is produced twice, at different scales, to highlight speed and distance behaviour, with lots of elucidatory text added.

The results are divided into several different sections determined by what the trains are actually doing or the functional location that one or other has reached, and the exposition is structured by these sections.

Note particularly that, although only two trains are illustrated in the graphs, they must be understood as representing a constant stream of trains, each of which affects and is affected by both the preceding and the following train.

Note the lozenge shape of the speed graphs of the two trains (lines 2 and 4). Opposite pairs of sides are identical, illustrating that the same elapsed time applies for each train's deceleration between the same pair of speeds (obviously), but also that the same elapsed time applies to both trains at uniform but different speeds, and this is the capacity slot time, 112.50s for the High Speed cases. Once both trains have decelerated to the same speed (when line 4 merges with line 2) then the separation distance between them remains the same, so long as their speeds remain the same. This distance is the capacity slot length. Thus, for the normal deceleration case, as here, line capacity remains constant, or, rather, the same capacity applies at both original line speed and target speed. (The concept of capacity doesn't have precise meaning when the train speeds are in the process of changing.)

I believe this constitutes a formal proof of preservation of capacity, through normal deceleration.

Case 1 Deceleration between High and Medium Speed ranges.
Specimen Example 32tph throughout.
Decelerating from Overall line speed 90.80m/s to Target line speed 38.37m/s.

The speed graph is simple to follow. Train 1 decelerates from the overall line speed to the target line speed, and then continues at that constant speed. Train 2 travels at constant line speed for a distance equal to the capacity distance slot, throughout train 1's deceleration, and also for a short time beyond, getting closer to train 1 until the distance between them is at its minimum, **relative to TSD(b) for train 2**, at which time train 2 has reached the start of deceleration point, and begins its own deceleration. As train 2 decelerates, train 1 travels at constant target line speed. The distance between them continues to decrease – of course it does, train 2 is still travelling faster – but what changes is that the TSD(b) needed between them decreases faster still. Train 2's speed decreases linearly, but TSD(b) decreases quadratically, and quite quickly falls below the actual separation distance between the trains. At this point, the deceleration tracks merge to re-form the main line, and the same point is also the section boundary between the Overall and Target line speeds. This constitutes the completion of the deceleration process, for train 1. TSD(b) continues to decrease faster than does the speed until train 2 reaches the target speed, beyond which both trains continue at that constant speed. The trains travel on different tracks for the time during which their separation distance would be below the minimum TSD(b) for the instantaneous line speed of train 2 – if they were on the same track.

All this is on the speed graph, but much easier to see on the distance graph, where the speed lines are omitted, and the vertical scale is four times as big. Concentrate on lines 6 (TSD(b) for train 2) and 7 (actual separation distance). These lines cross twice: at elapsed time 67.54s, when train 1 reaches the Buffer-end point, and at 164.86s, where it reaches the end of the deceleration track. (The deceleration track actually begins at 53.82s, where train 1 begins its divergence at the turnout limit speed.)

For this case only, graphs are given for the acceleration case also, but only to confirm that the process is automatic, and separation distances remain adequate throughout.

0.00s Train 1 begins its deceleration.

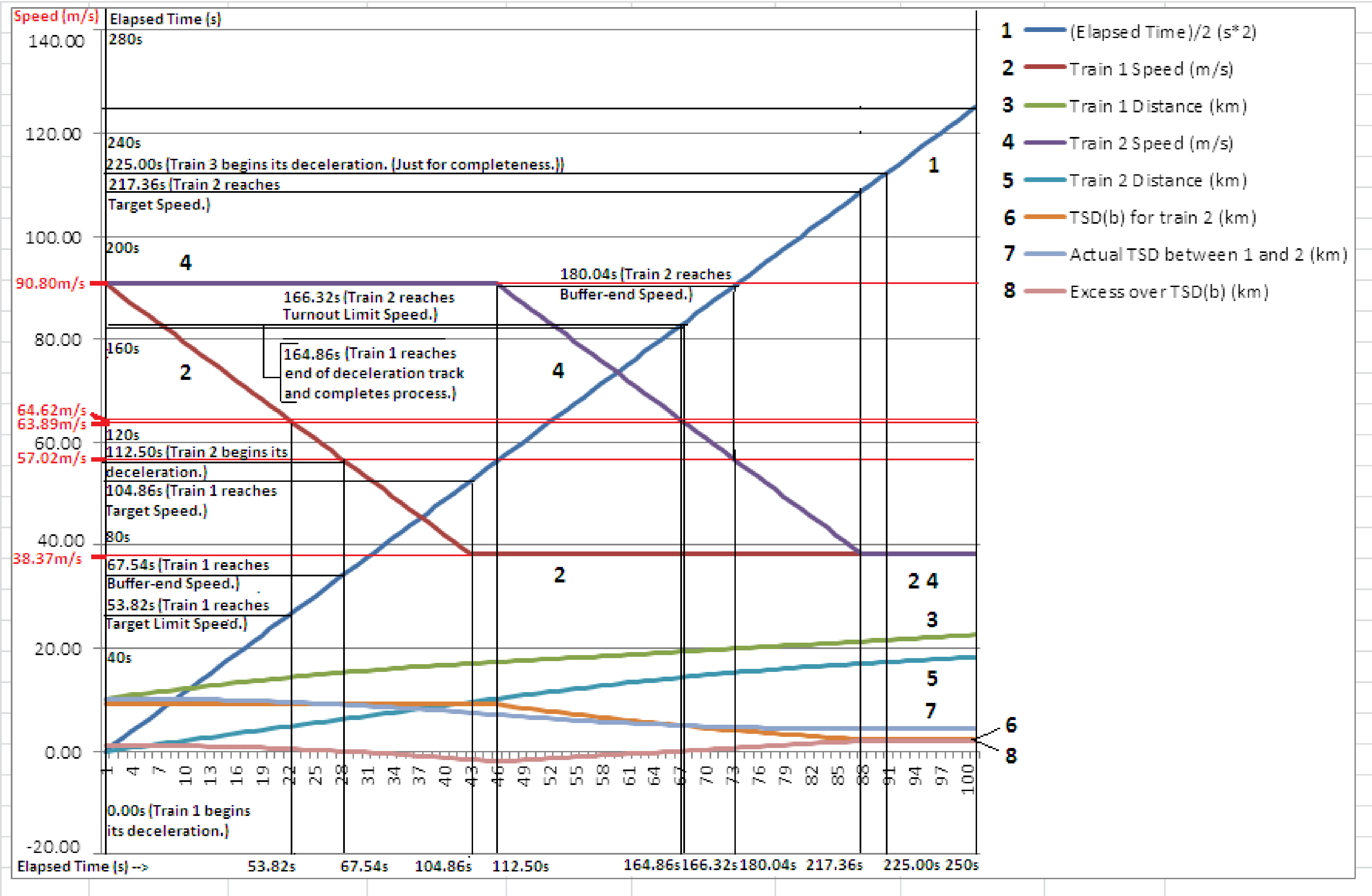
Train 1, at the start of its deceleration, is 10.2146km, i.e. one capacity slot length, = TSD(e), ahead of train 2. This is the normal separation distance between adjacent trains travelling at line speed. Train 2's distance travelled is taken as zero at this point in time, the origin of the deceleration process.

53.82s Train 1 reaches the bifurcation of the main line into two deceleration tracks.

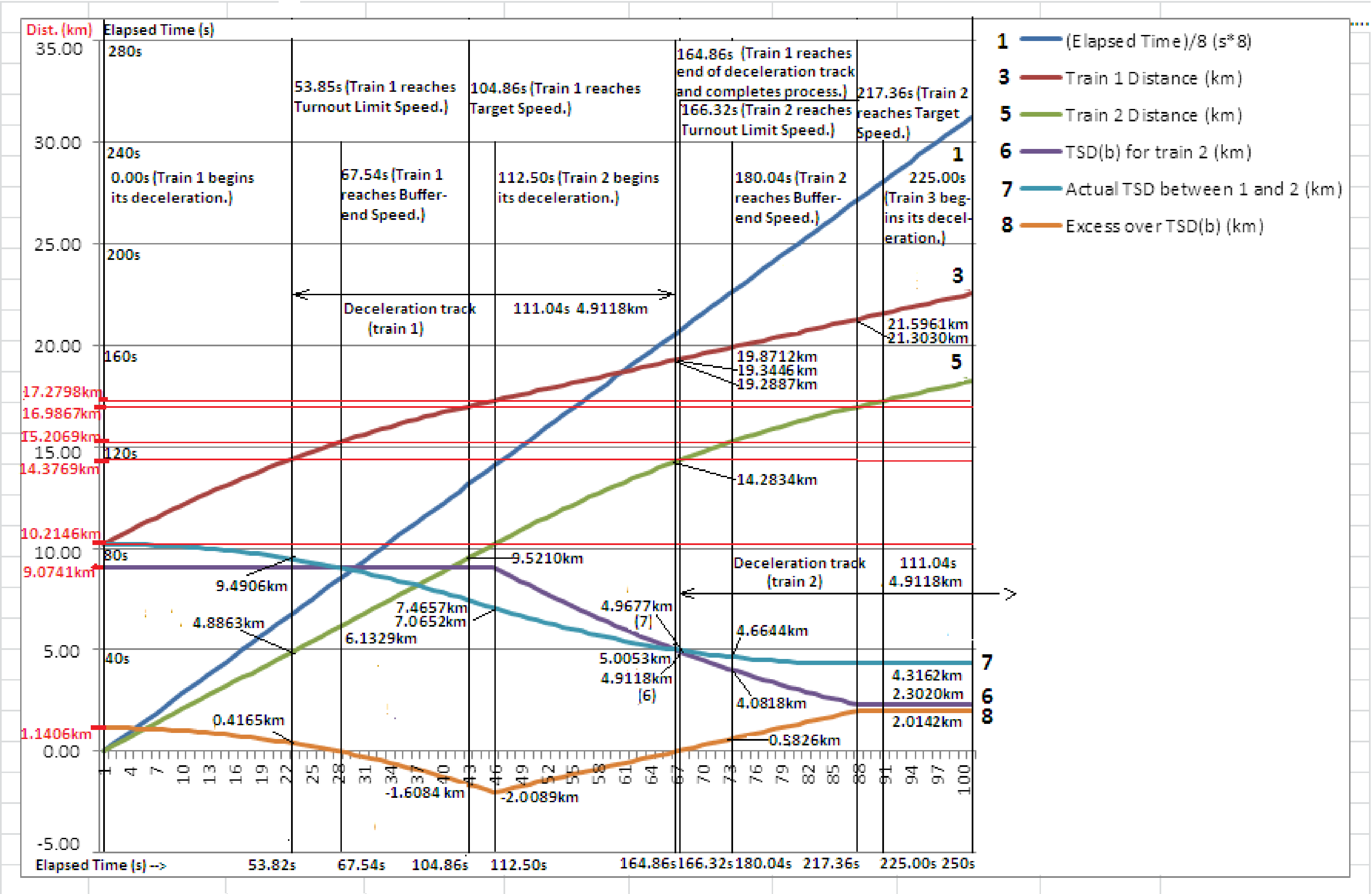
Train 1 has travelled 4.1623km and its speed has fallen to the Turnout Limit Speed of 63.89m/s, 230kph. In the same time, train 2 has travelled 4.8863km at overall line speed. The separation distance between rhe trains has reduced to 9.4906km. Train 1 is directed by the switch onto deceleration track 1.

67.54s Train 1 decelerates to the Buffer-end Speed on deceleration track 1.

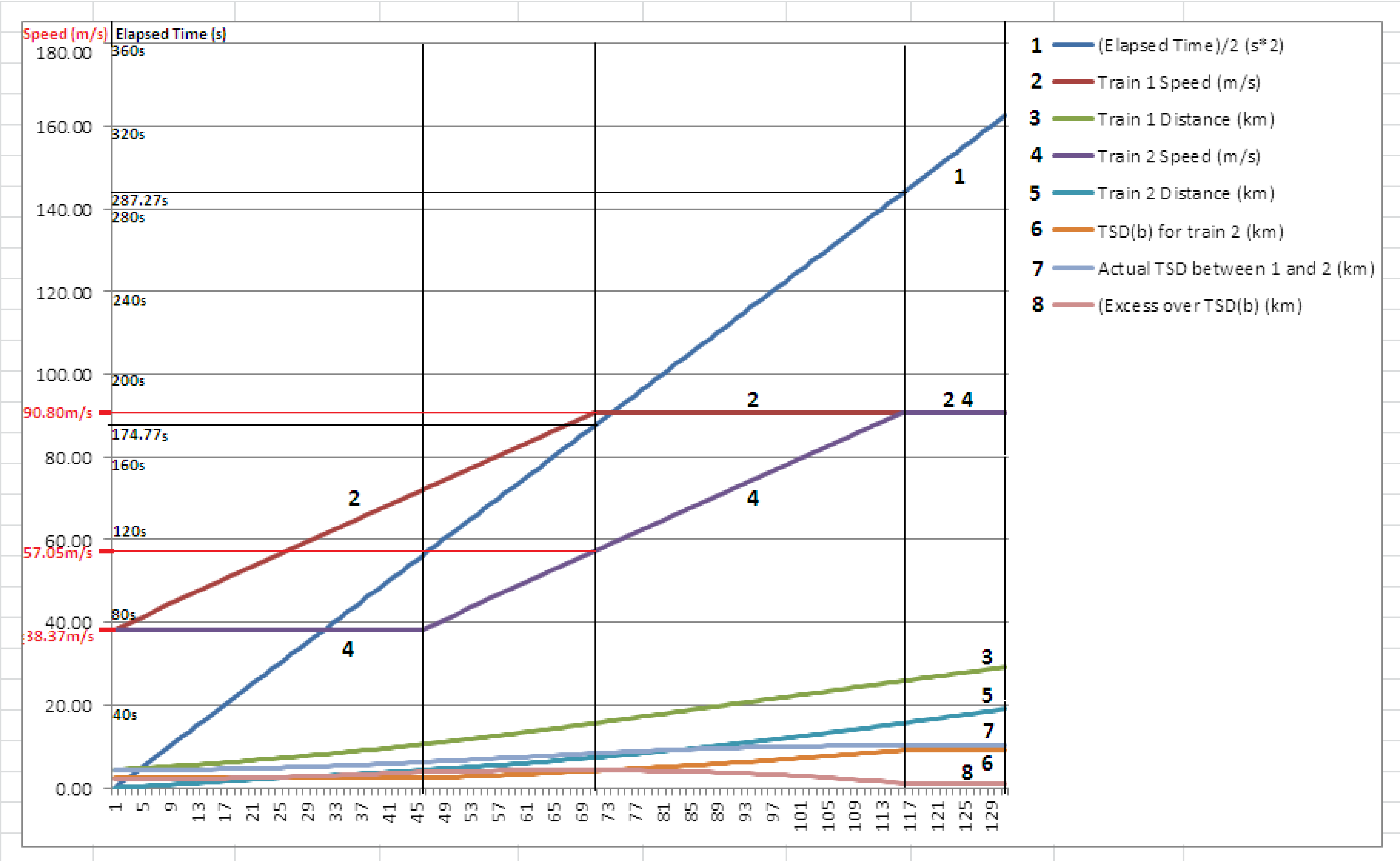
Train 1 travels the buffer distance, 830m, onto deceleration track 1, at which point it has decelerated to the Buffer-end Speed, 57.02m/s. The switch has moved, behind train 1. to point to deceleration track 2, onto which train 2 will be routed when it reaches the switch. In the same time train 2 has travelled 1.2466km at constant overall line speed. The separation distance between the trains is now 9.0741km, TSD(b) precisely, the absolute minimum for the overall line speed, at which train 2 is still travelling, and will for some time yet. At this instant, train 1 is 830m, the buffer length, constant portion of TSD(b), on



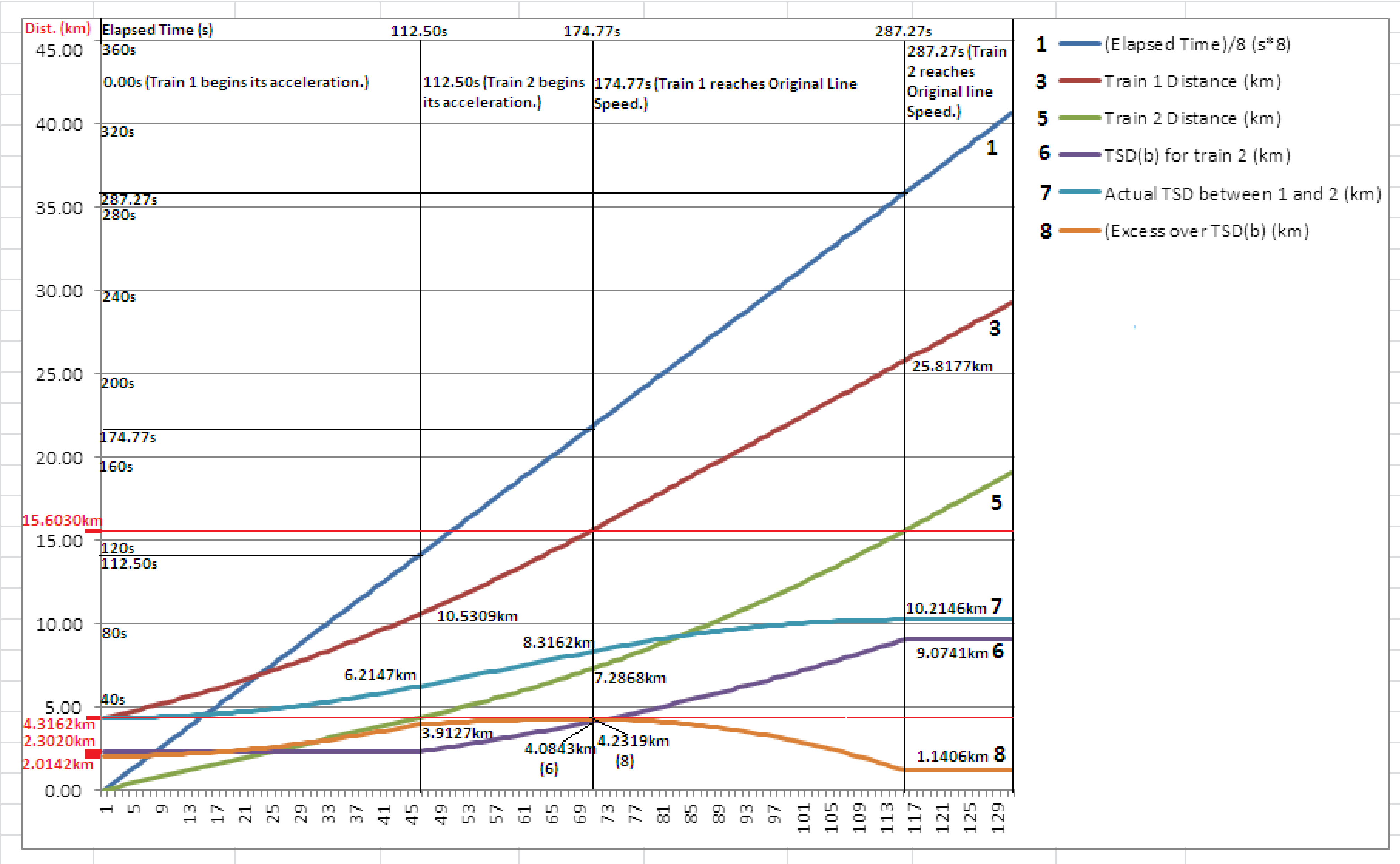
Deceleration Graphs at constant capacity 32tph, to a time scale unit of 2sec, to spread out on the page. Annotated, focussed on speed.



Deceleration Distance Graphs at constant capacity 32tph, to a time scale unit of 8sec, enlarging the distance graphs by 8*scale. Annotated.



Re-acceleration Graphs at constant capacity 32tph, to a time scale unit of 2sec, to spread out on the page. Annotated.



Re-acceleration Distance Graphs at constant capacity 32tph, to a time scale unit of 8sec, enlarging the distance graphs by 8*scale. Annotated.

-track 1 beyond the switch, and train 2 is 8.2441km, the dynamic portion of TSD(b), before the switch, still on the main line but effectively on deceleration track 2, since that is where it will be routed. Train 1 is thus no longer in the path of train 2.

104.86s Train 1 reaches the target line speed.

Train 1 travels a further 1.7798km and reaches the target line speed of 38.37m/s. It will maintain that as a constant speed from this point on, for the time being on deceleration track 1. In the same time, train 2 has travelled 3.3881km at constant overall line speed.

112.50s Train 2 begins its deceleration.

Train 1 travels a further 0.2931km at constant target line speed. In the same time, train 2 travels 0.6936km at constant overall line speed. Train 1 has thus travelled a distance 7.0652km from the point at which it began its deceleration, at which point it was 10.2146km ahead of train 2, and, in the same time, train 2 has travelled 10.2146km, so the separation distance between the trains is now 7.0652km, 2.0089km **less than** the minimum, TSD(b), of 9.0741km.

This is where it gets challenging. The preceding graphs illustrate the interactions between just two trains. But the actuality is a continuous sequence of trains, and each train interacts simultaneously with the train in front, **and the train behind**.

Train 2 now performs **precisely,** relative to the following, **train 3**, as train 1 in the preceding paragraphs did, relative to train 2. Specifically, the elapsed times, distances travelled, and train separation distances are identical.

Train 2, at the start of its deceleration, is 10.2146km ahead of train 3. It is also its minimum distance, 7.0652km, relative to the required TSD(b), behind train 1.

164.86s Train 1 reaches end of deceleration track (TSD = TSD(b) for train 2, precisely).

Train 1 has travelled 2.0089km at constant target line speed since train 2's start of deceleration, and has thus cleared the deficit in separation distance at that location, and reached the point at which it is precisely TSD(b) (calculated for train 2's instantaneous speed of 64.62m/s) ahead of train 2, which has, in the same time decelerated from 90.80m/s to 64.62m/s, and travelled a distance of 4.0688km.

Although the TSD is still decreasing, and will continue to decrease until train 2 reaches the target speed, its excess over the instantaneous TSD(b) will continue to increase until train 2 reaches the target speed. This (i.e. train 1's location) is precisely the point at which the deceleration tracks are no longer required, and therefore should merge to re-constitute the main line.

Train 1 has thus completed the deceleration process.

The length of each deceleration track (i.e. since train 1 reached the TLS) is thus (19.2887 – 14.3769) = 4.9118km. In fact, and most surprisingly, this is a universal constant; the deceleration track length always has this value (for **High Speed** lines), (for the standard values of deceleration, turnout limit speed and buffer length,) irrespective of the original and target speeds. It is easily proved that this is so, but I have yet to come up with a rigorous explanation of **why** it is so, though it must be for essentially the same reason as the fact that the (physical) station loop length is constant for the High Speed range, since it likewise depends on v_t. (The actual value is that of the basic train separation distance for the turnout limit speed, $TSD(b)_{vt}$.)

The elapsed time taken to travel the length of the deceleration track (i.e. the front of the train travelling from the diverging switch points to the converging switch points) does depend on the target speed, which is not surprising, but on nothing else, which perhaps is. The formula is

$$\mathbf{t_{dt} = v_t/a_d - (v_g^2/2a_d - b)/v_g}$$

The elapsed time taken to travel the deceleration track, for the present case, is 111.04s. The elapsed time periods when the trains are on the deceleration tracks are indicated on the distance graph. In the present example, the elapsed time at which train 1 reaches the end of deceleration track and train 2 reaches the turnout limit speed are almost identical. This is purely fortuitous and has no particular significance.

166.32s Train 2 reaches the bifurcation of the main line into two deceleration tracks.

Train 2 has travelled 4.1623km from the start of deceleration, and its speed has fallen to the Turnout Limit Speed of 63.89m/s, 230kph. In the same time, train 3 has travelled 4.8863km at overall line speed. The separation distance between the trains has reduced to 9.4906km. Train 2 is directed by the switch onto deceleration track 2.

180.04s Train 2 decelerates to the Buffer-end Speed on deceleration track 2.

Train 2 travels the buffer distance, 830m, onto deceleration track 2, at which point it has decelerated to the Buffer-end Speed, 57.02m/s. The switch has moved, behind train 2, to point to deceleration track 1, onto which train 3 will be routed when it reaches the switch. In the same time train 3 has travelled 1.2466km at constant overall line speed. The separation distance between the trains is now 9.0741km, TSD(b) precisely, the absolute minimum for the overall line speed, at which train 3 is still travelling, and will for some time yet. At this instant, train 2 is 830m, the buffer length, constant portion of TSD(b), on track 2 beyond the switch, and train 3 is 8.2441km, the dynamic portion of TSD(b), before the switch, still on the main line but effectively on deceleration track 1, since that is where it will be routed. Train 2 is thus no longer in the path of train 3.

Also in the same time, train 1, (which has of course now completed its deceleration,) travels 0.5266km at constant target line speed. The separation between trains 1 and 2 is now 4.6644km, as compared with TSD(b) for train 2 of 4.0818km.

217.36s Train 2 reaches the target line speed.

Train 2 travels a further 1.7798km and reaches the target line speed of 38.37m/s. It will maintain that as a constant speed from this point on, for the time being on deceleration track 2. In the same time, train 3 has travelled 3.3881km at constant overall line speed.

Also in the same time, train 1 travels 1.4318km at constant target line speed. The distance between trains 1 and 2 is now 4.3162km, and will remain constant at that value as long as they are both travelling at the target line speed.

225.00s Train 3 begins its deceleration.

Train 2 travels a further 0.2931km at constant target line speed. In the same time, train 3 travels 0.6936km at constant overall line speed. Train 2 has thus travelled a distance 7.0652km from the point at which it began its deceleration, at which point it was 10.2146km ahead of train 3, and, in the same time, train 32 has travelled 10.2146km, so the separation distance between the trains is now 7.0652km, Also in the same time, train 1 also travels 0.2931km at constant target line speed (of course it does!). The distance between trains 1 and 2 is now 4.3163km, (still!).

277.36s Train 2 reaches end of deceleration track (TSD = TSD(b) for train 3, precisely)

Train 2 has travelled 2.0089km at constant target line speed since train 3's start of deceleration, and has thus cleared the deficit is separation distance at that location, and reached the point at which it is precisely TSD(b) (calculated for train 3's instantaneous speed of 64.62m/s) ahead of train 3, which has, in the same time decelerated from 90.80m/s to 64.62m/s, and travelled a distance of 4.0688km.

Although the TSD is still decreasing, and will continue to decrease until train 3 reaches the target speed, its excess over the instantaneous TSD(b) will continue to increase until train 2 reaches the target speed. This (i.e. train 2's location) is precisely the point at which the deceleration tracks are no longer required, and therefore now merge to re-constitute the main line.

Train 2 has thus completed the deceleration process.

Case 2 Deceleration within High Speed range.
Specimen Example 32tph throughout.
Decelerating from Overall line speed 93.60m/s to Target line speed 74.71m/s.

The speed graph is rather more complicated than for the first example, but still simple to follow. The main point to note is that the elapsed time range is 40% greater – 350s vs 250s. Accordingly, the scales adopted use units of 4s and 16s, as compared with 2s and 8s, to get the best available spread over the page.

Train 1 decelerates from the overall line speed to an intermediate target line speed of 57.02m/s, the Buffer-end Speed, and then continues at that constant speed until it reaches the end of the deceleration track, where the two tracks merge to reconstitute the main line, and train 1 begins its re-acceleration up to the final Target speed. When it reaches that speed, then its net deceleration process is complete, and it maintains that constant speed for as long as required.

Train 2 travels at constant line speed for a distance equal to the capacity distance slot, exactly the same as for the first example, throughout train 1's deceleration, (which is smaller than for the first example, since the intermediate target speed is higher than the (final) target speed of that example,) and thus extends for a longer time beyond, getting closer to train 1 until the distance between them is at its minimum, **relative to TSD(b) for train 2**, at which time train 2 has reached the start of deceleration point, and begins its own deceleration.

As train 2 decelerates, train 1 travels at constant target line speed. The distance between them continues to decrease in absolute terms, but increases relative to the (instantaneous) value of TSD(b) for train 2. When the separation distance reaches TSD(b) for train 2, then train 1 has reached the end of the deceleration track, as mentioned above. Train 1 begins its re-acceleration, while train 2 continues its deceleration until it reaches the Turnout Limit Speed, at the bifurcation of the main line into deceleration tracks, and is directed onto track 2, on which it continues its deceleration down to the Buffer-end Speed, at which point the switch ghas been reset to direct the next, train 3, onto track 1.

Having reached the Buffer-end Speed, the intermediate target speed, train 2 continues at that speed until it reaches the end of the deceleration track, and re-accelerated up to the (final) Target Speed, on reaching which it has completed its net deceleration.

0.00s Train 1 begins its deceleration.

Train 1, at the start of its deceleration, is 10.2146km, i.e. one capacity slot length, = TSD(e), ahead of train 2. This is the normal separation distance between adjacent trains travelling at line speed. Train 2's distance travelled is taken as zero at this point in time, the origin of the deceleration process.

53.82s Train 1 reaches the bifurcation of the main line into two deceleration tracks.

Train 1 has travelled 4.1623km and its speed has fallen to the Turnout Limit Speed of 63.89m/s, 230kph. In the same time, train 2 has travelled 4.8863km at overall line speed. The separation distance between rhe trains has reduced to 9.4906km. Train 1 is directed by the switch onto deceleration track 1.

67.54s Train 1 decelerates to the Buffer-end Speed on deceleration track 1.

Train 1 travels the buffer distance, 830m, onto deceleration track 1, at which point it has decelerated to the Buffer-end Speed, 57.02m/s. The switch has moved, behind train 1. to point to deceleration track 2, onto which train 2 will be routed when it reaches the switch. In the same time train 2 has travelled 1.2466km at constant overall line speed. The separation distance between the trains is now 9.0741km, TSD(b) precisely, the absolute minimum for the overall line speed, at which train 2 is still travelling, and will for some time yet. At this instant, train 1 is 830m, the buffer length, constant portion of TSD(b), on track 1 beyond the switch, and train 2 is 8.2441km, the dynamic portion of TSD(b), before the switch, still on the main line but effectively on deceleration track 2, since that is where it will be routed. Train 1 is thus no longer in the path of train 2.

Until this point, the behavious of the two trains has been identical to that for the first example. But now, the buffer-end speed forms tha intermediate target speed, so there is no need for train 1 to decelerate further, so it simply continues at that speed.

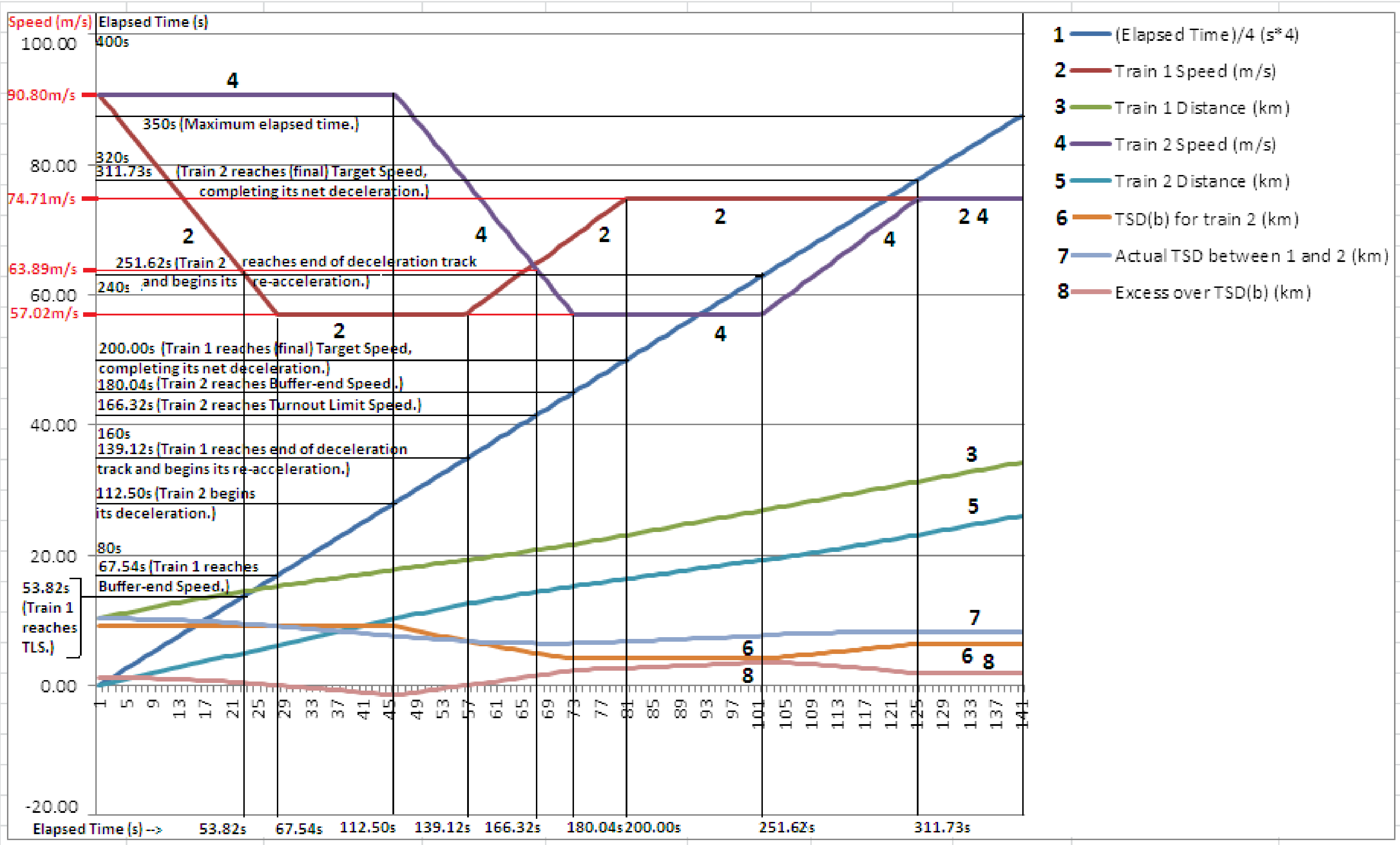
112.50s Train 2 begins its deceleration.

Train 1 travels a further 2.5620km at constant intermediate target speed. In the same time, train 2 travels 4.0817km at constant overall line speed. These values are much higher for the first example, (c.f. 0.2931km and 0.6936km, respectively,) because the deceleration has been less. Train 1 has now travelled a distance of 7.5543km from the point at which it began its deceleration, at which point it was 10.2146km ahead of train 2, and, in the same time, train 2 has travelled 10.2146km, so the separation distance between the trains is now 7.5543km, 1.5198km **less than** the minimum, TSD(b), of 9.0741km (c.f. 2.0089km in the first example) There is thus less deficit to recover..

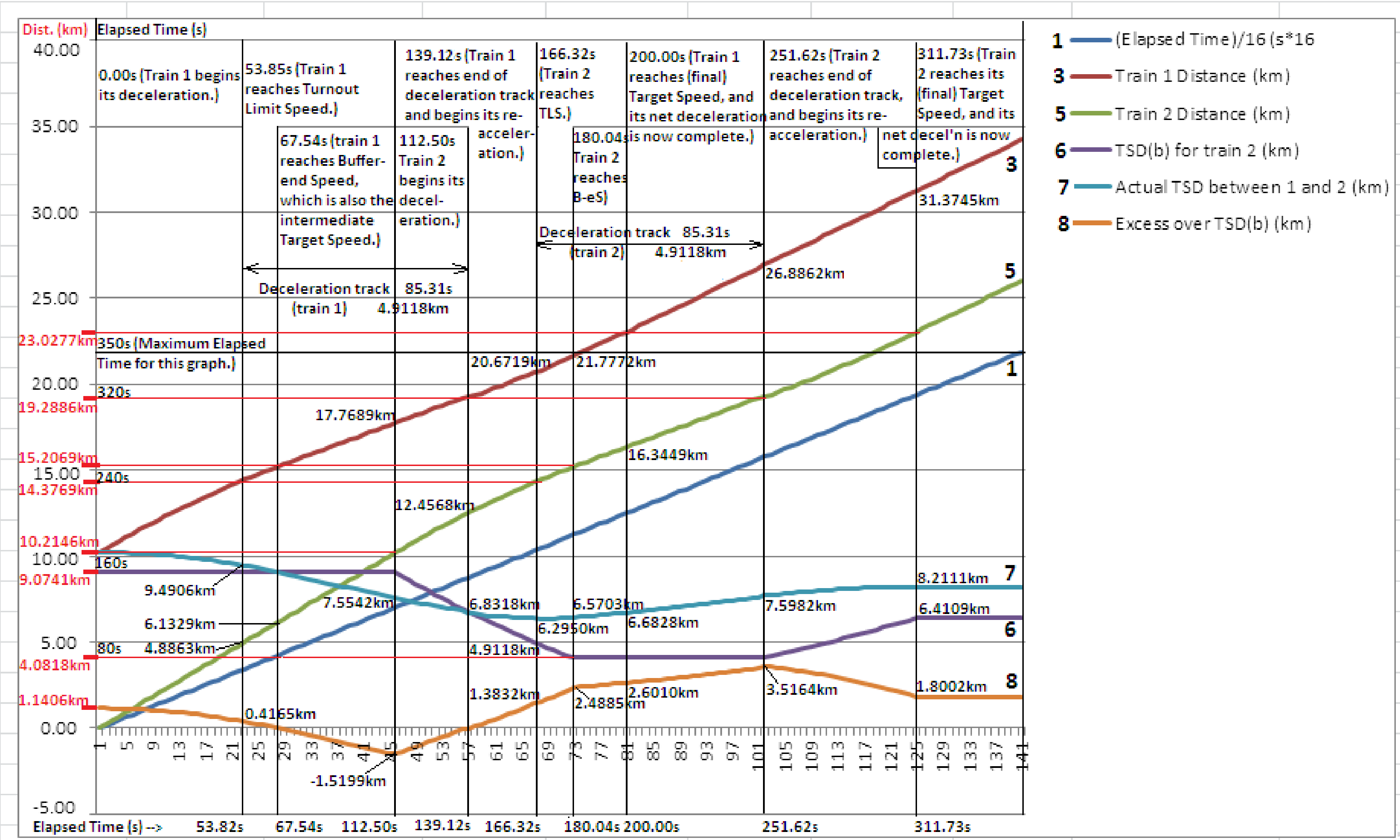
Train 2, at the start of its deceleration, is 10.2146km ahead of train 3. It is also its minimum distance, 7.5543km, relative to the required TSD(b), behind train 1.

139.15s Train 1 reaches end of deceleration track (TSD = TSD(b) for train 2, precisely)

This, unsurprisingly, happens a lot earlier – over 25s – than in the first example. But although the elapsed times are very different, the times actually travelled by train 1 when it reaches the end of the deceleration track are identical, at 19.2886km, (from a starting value of 10.2146km when it began its deceleration,) to a rounding error of less than 1 part in a million. The decelerating track distance (from the point at which



Deceleration Graphs at constant capacity 32tph, to a time scale unit of 4sec, to spread out on the page. Annotated, focussed on speed.



Deceleration Distance Graphs at constant capacity 32tph, to a time scale unit of 16sec, enlarging the distance graphs by 16*scale. Annotated.

train 1 reached the TLS) is thus 4.9118km (to rounding error) for both examples.

At this point, (that train 1 has just reached,) the deceleration tracks merge to re-form the main line, and the switch had been pre-set to pass train 1, from deceleration track 1, back onto the main line. Note that, since this is a **converging** junction, the only timing consideration is that the back end of the train must cross the junction before the switch can be reset. Train 1 is travelling at the Buffer-end Speed as it reaches the switch points, and can begin its re-acceleration at that point, as it crosses the switch. Given that the train length is only 400m, and the time taken to reset the switch is 4s, then the time taken for train 1 completely to cross the switch, at constant target or, as here, intermediate target speed, is 630m or 11.05s. Note that this has no effect on the location of the end of deceleration track, but it does mean that an extra 630m or 11.05s is required for train 1 completely to cross the switch, and the switch to be reset to receive the next train off the other deceleration track, train 2 on track 2 in this instance.

Note a further strange and surprising effect: for a very short period of time, the main line will be continuous. At elapsed time 150.20s, train 1 has completely re-joined the main line from deceleration track 1, and the converging switch has been reset to accept the next train, train 2, from deceleration track2. But train 2 has not actually moved on to the deceleration track yet – it is about to do so at elapsed time 166.32s, but until then is still on the main line before the switch, as also is the following train 3. But for the next 16.12s, train 1 is on the main line beyond the deceleration track 2 and trains 2 and 3 are on the main line before the deceleration track, and deceleration track 2 is connected to the main line at both ends. Track 1 on the other hand is completely isolated, not connected at either end.

This is, I believe, perfectly easy to understand, nonetheless it came as quite a surprise to me.

Train 1 now begins its re-acceleration up to the (final) target speed.

166.32s Train 2 reaches the bifurcation of the main line into two deceleration tracks.

Train 2 has travelled 4.1623km from the start of deceleration, and its speed has fallen to the Turnout Limit Speed of 63.89m/s, 230kph. In the same time, train 3 has travelled 4.8863km at overall line speed. The separation distance between the trains has reduced to 9.4906km. Train 2 is directed by the switch onto deceleration track 2, while behind it, the switch is reset to direct the next train, train 3, onto deceleration track 1. There is thus no longer a continuous, through route – train 3 will be sent onto track 1, while train 2 will continue along track 2 until it passes back onto the main line (whereupon the converging switch will be reset to accept the next train, train 3, from track 1; track 1 will then form part of the through route for next 16s.

180.04s Train 2 decelerates to the Buffer-end Speed on deceleration track 2.

Train 2 travels the buffer distance, 830m, onto deceleration track 2, at which point it has decelerated to the Buffer-end Speed, 57.02m/s. The switch has moved, behind train 2, to point to deceleration track 1, onto which train 3 will be routed when it reaches the switch. In the same time train 3 has travelled 1.2466km at constant overall line speed. The separation distance between the trains is now 9.0741km, TSD(b) precisely, the absolute minimum for the overall line speed, at which train 3 is still travelling, and will for some time yet. At this instant, train 2 is 830m, the buffer length, constant portion of TSD(b), on track 2 beyond the switch, and train 3 is 8.2441km, the dynamic portion of TSD(b), before the switch, still on the main line but effectively on deceleration track 1, since that is where it will be routed. Train 2 is thus no longer in the path of train 3.

At the same time, train 1, since reaching the end of the deceleration track, has performed part of its re-acceleration, on the re-merged main line, and has travelled 2.4886km .The separation between trains 1 and 2, back on the same track, is now 6.5703km, as compared with TSD(b) for train 2 of 4.0818km.

200.00s Train 1 reaches the (final) target line speed.

Train 1 accelerates a further 1.2505km and reaches the (final) target line speed of 74.71m/s, and has now completed the net deceleration process. It will maintain that as a constant speed from this point on, indefinitely.

Also in the same time, train 2 travels 6.6828km at constant intermediate target line speed. The distance between trains 1 and 2 is now 4.3163km.

251.62s Train 2 reaches end of deceleration track (TSD = TSD(b) for train 3, precisely)

Train 2 has travelled 4.0817km at constant intermediate target line speed since reaching this speed, reaching the end of deceleration track 2, and beginning its re-acceleration to (final) target speed.

311.73s Train 2 reaches the (final) target line speed.

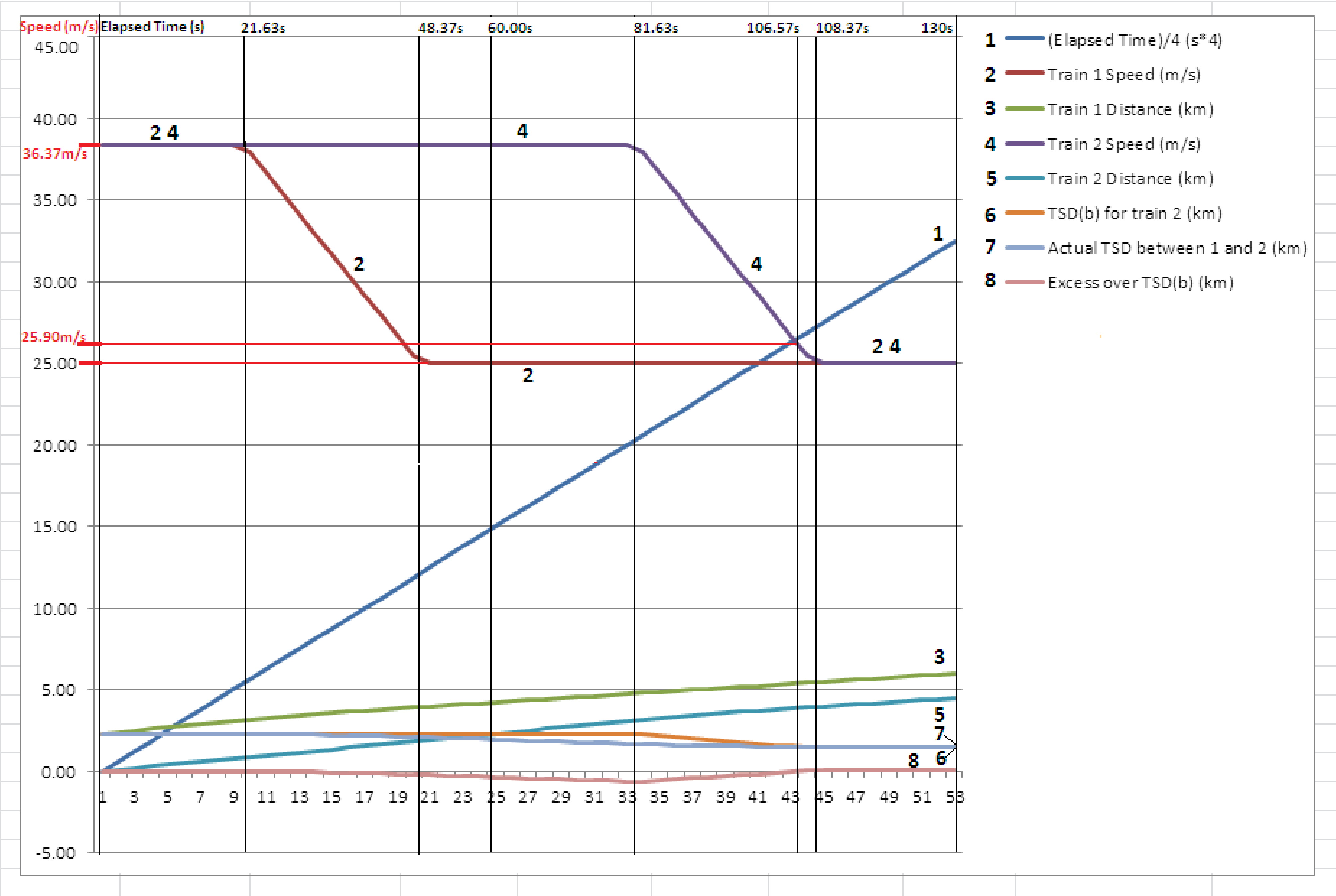
Train 2 accelerates for 3.7391km and reaches the (final) target speed of 74.71m/s., and has now completed the net deceleration process. It will maintain that as a constant speed from this point on, indefinitely. It is now 8.2111km behind train 1, which will likewise remain constant while both trains maintain this speed.

Case 3 Deceleration Medium and Low Speed ranges.
Specimen Example 60tph throughout.
Decelerating from Overall line speed 37.37m/s to Target line speed 25m/s.

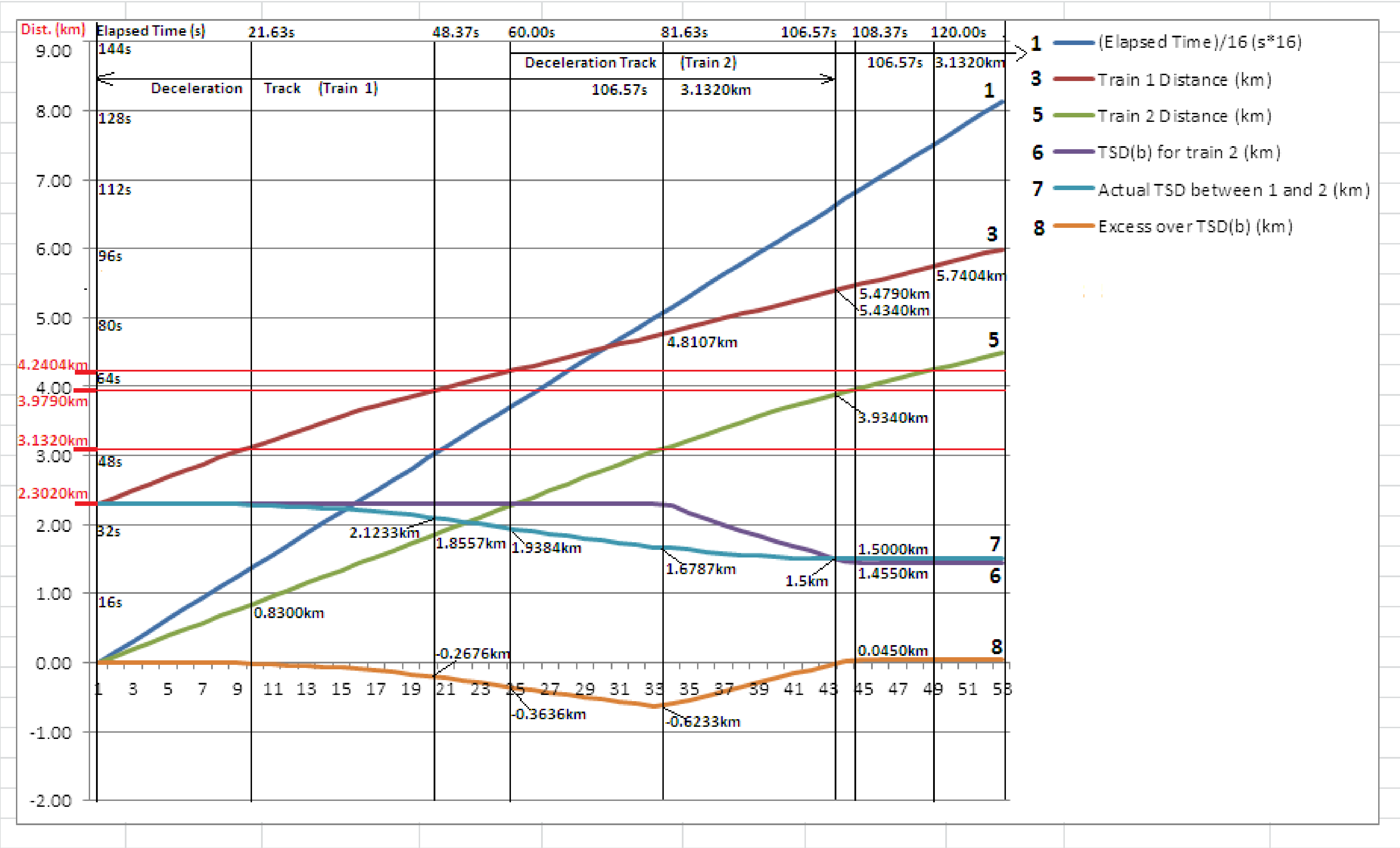
The speed graph for this case is almost trivially simple: train 1 decelerated from original line speed to target line speed and then continues at that speed indefinitely. Train 2 travels 1 capacity slot distance (= TSD(b) at original line speed) then decelerates similarly. It may be claimed, correctly, that case 1, above, does no more. But for the High Speed case there is the necessity to perform part of the deceleration on the main line (or, at least, while still in the path of the following train,) so there are several other features of interest, times, speeds and distances. None of this applies to the medium speed range – by definition, indeed. The trains are already travelling the minimum distance apart, at a line speed of less than or equal to the Buffer-end Speed.

What **is**, at first sight, quirky, is that the initial section of the train’s deceleration is performed at that constant speed. Train 1 travels a distance equal to the buffer length onto deceleration track 1 at constant line speed, a fixed distance, the capacity slot distance, ahead of train 2. This takes the first 21.63s. At this point it is no longer in the path of train 2, the switch having reset behind it to direct train 2 onto deceleration track 2 in due course, and its actual deceleration then begins. Note that train 2’s deceleration begins at elapsed time 60s, the first 21.63s of it being still at line speed.

The train travels at constant target speed until it reaches the end of the deceleration loop, at which point it re-joins the main line, and the deceleration process is complete.



Deceleration Graphs at constant capacity 60tph, to a time scale unit of 4sec, to spread out on the page. Annotated, focussed on speed.



Deceleration Distance Graphs at constant capacity 60tph, to a time scale unit of 16sec, enlarging the distance graphs by 16*scale. Annotated.

0.00s Train 1 begins its virtual deceleration, at the birurcation of the main line into two ` deceleration tracks.

Train 1, at the start of its deceleration, is i.e. one capacity slot length, = TSD(b), ahead of train 2. This is the normal separation distance between adjacent trains travelling at line speed. Train 2’s distance travelled is taken as zero at this point in time, the origin of the deceleration process.

For Medium Speed lines, TSD(b) is the separation distance between adjacent trains, i.e. they are already as close to each other as is permissible. No deceleration takes place on the main line. The bifurcation of the main line takes place at this point, at the very beginning of the deceleration, which is actually a virtual deceleration, since train 1 runs onto deceleration track 1 at constant line speed, and continues at that speed until it has travelled the buffer length onto that track.

21.63s Train 1 begins its actual deceleration.

Train 1 travels the buffer length, 830m, onto deceleration track 1, at full line speed. At this point, the switch has moved, behind train 1, to point to track 2, onto which train 2 will be routed, when it reaches the switch. In the same time, train 2 has unsurprisingly also travelled 830m, on the main line. The separation between the trains is still 2.3020km. train 1 is the buffer length, 830m, beyond the switch and train 2 is 1.4720km before it, still on the main line, but effectively on deceleration track 2, since that is where it will be routed.

Train 1 is thus no longer in the path of train 2, and may, finally, begin its actual deceleration.

48.37s Train 1 reaches the target line speed.

Train 1 ttravels the distance 0.847km, and reaches the target speed of 25m/s. It will maintain that as a constant speed from this point on, for the time being on deceleration track 1. In the same time, train 2 has travelled 1.0257km at constant overall line speed.

60.00s Train 2 begins its virtual deceleration.

Train 1 travels a further 0.2614km at constant target line speed. In the same time, train 2 travels 0.5463km at constant, overall line speed. And reaches the bifurcation of the main line.

81.63s Train 2 begins its actual deceleration.

Train 2 travels the buffer length, 830m, onto deceleration track 2, at overall line speed. At this point, the switch has moved, behind train 2, to point to track 1, onto which train 3 will be routed, when it reaches the switch. In the same time, train 3 has unsurprisingly also travelled 830m, on the main line. The separation between the trains is still 2.3020km; train 2 is the buffer length, 830m, beyond the switch and train 3 is 1.4720km before it, still on the main line, but effectively on deceleration track 1, since that is where it will be routed.

Train 2 is thus no longer in the path of train 3, and may, finally, begin its actual deceleration. At this point it is also at its minimum distance, 1.6787km, relative to the required TSD(b), behind train 1

106.57s Train 1 reaches end of deceleration track (TSD = TSD(b) for train 2, precisely).

Train 1 has travelled 0.6237km at constant target line speed, since train 2 began its (actual) deceleration, and has thus cleared the deficit in separation distance at that location, and reached the point at which it is precisely (TSD(b) (calculated for train 2's instantaneous speed of 25.90m/s) ahead of train 2, which has in the same time decelerated from 36.37m/s to 25.90m/s, and travelled a distance of 0.8020km.

Although the TSD is still decreasing, and will continue to decrease until train 2 reaches the target speed, its excess over the instantaneous TSD(b)until train 2 reaches the target speed. This (i.e. train 1's location) is precisely the point the deceleration tracks are no longer required, and therefore should merge to re-constitute the main line.

Train 1 has thus completed the deceleration process.

The length of each deceleration track (i.e. since the beginning of the entire process, when train 1 began its virtual deceleration) is thus (5.4340 – 2.3020) = 3.1320km. This is no longer, for Medium Speed cases, a constant value, as it is for High Speed, but it is, very clearly, the same thing, as will be seen in the next section, when the formulae are derived. It depends on the original line speed, but not the target speed.

The actual formula for the deceleration track length is

$$\mathbf{s_{dt} = (v_l^2/2a_d + b) + b = TSD(b)_{vl} + b}$$

so the value in the present case is 3.1320km. The formula for the overall time to traverse the deceleration track is $\mathbf{t_{dt} = b/v_l + v_l/a_d - (v_g^2/2a_d - b)/v_g}$

so the value in the present case is 106.57s.

108.37s Train 2 reaches the target line speed.

Train 2 travels a further 0.0450km to reach the target speed. In the same time train 1 travels the same distance – it isn't **exactly** the same, but for a time of only 1.8s, the difference doesn't show.

The distance between trains 1 and 2 as now 1.5km precisely, and will remain at tha value while their speeds remain the same.

166.57s Train 2 reaches end of deceleration track (TSD = TSD(b) for train 3, precisely).

Train 2 has now completed the deceleration process.

Deceleration Track Length for Overall Line Speed in the High Speed Range

v_l = (original) line speed in m/s (a Sweet-Speed)
v_t = Turnout Limit Speed = 230kph = 63.89m/s
a_d = deceleration rate = 0.5m/s^2
v_g = Target Speed in m/s
c_l = line capacityin tph
c_t = capacity slot time in sec, = 3600/c_l
c_s = capacity slot length, = vc_t where v is the current, instantaneous speed
b = buffer component of Train Separation Distance, = 830m.

Elapsed time for train to decelerate v_l => v_t, $t_{lt} = (v_l – v_t)/a_d$
Distance travelled during deceleration v_l => v_t $s_{lt} = (v_l^2 – v_t^2)/2a_d$

Elapsed time for train to decelerate v_l => v_g $t_{lg} = (v_l – v_g)/a_d$
Distance travelled during deceleration v_l => v_g $s_{lg} = (v_l^2 – v_g^2)/2a_d$

Having decelerated to the target speed, train 1 travels, at constant speed v_g, for time t_1 sec, thus travelling a distance $s_1 = v_gt_1$ m, at which point it reaches the end of the deceleration track.

Train 2, on the other hand, travels:

1. a distance c_{sl}, at constant speed v_l, in time c_t, thus $c_{sl} = v_lc_t$
2. followed by a deceleration for time t_2 sec at constant rate a_d, reaching a speed v_2 m/s. Thus $v_l – v_2 = a_dt_2$.

The total elapsed times for both trains are equal, i.e. $t_{lg} + t_1 = c_t + t_2$
or $(v_l – v_g)/a_d + t_1 = c_t + (v_l – v_2)/a_d$
(v_l,v_g, c_t and a_d are known, t_1, v_2 are unknown).

When train 1 reaches the end of the deceleration track, then the distance between the trains is equal to the basic TSD, TSD(b), for the instantaneous speed of train 2 at that point. Initially, train 1 (at the start of its deceleration) is c_{sl}, = v_lc_t, ahead of train 2. When train 1 reaches the end of the deceleration track, the TSD is: $(c_{sl} + s_{lg} + v_gt_1) – [c_{sl} + (v_l^2 – v_2^2)/2a_d]$
i.e. $(v_l^2 – v_g^2)/2a_d – (v_l^2 – v_2^2)/2a_d + v_gt_1 = (v_2^2 – v_g^2)/2a_d + v_gt_1$

And this must be equal to TSD(b) for speed v_2, i.e. $v_2^2/2a_d + b$

Therefore $v_gt_1 = v_g^2/2a_d + b$ or $t_1 = v_g/2a_d + b/v_g$

Thus t_1 varies only with v_g.

The length of the deceleration track is $s_{dt} = s_{lg} – s_{lt} + s_1$
$= (v_l^2 – v_g^2)/2a_d – (v_l^2 – v_t^2)/2a_d + v_gt_1$
$= (v_t^2 – v_g^2)/2a_d + v_g^2/2a_d + b$

$$\mathbf{s_{dt} = v_t^2/2a_d + b = TSD(b)_{vt} = 4.9118km}$$

This applies universally, since v_t, b and a_d are invariant. The length of the deceleration track, for any line capacity and thus (Sweet-) line speed to any target speed, is the same, and equal to the Basic Train Separation Distance for the Turnout Limit Speed.

Note that this concerns only the length of the deceleration track; it does **not** imply that the second train is (instantaneously) travelling at the TLS. Having said that, the special case where that **is** actually true is of special interest, as will be explained shortly.

The elapsed time to travel the length of the deceleration track, (i.e. for the front end of the train to travel from the points of the diverging switch to the points of the converging switch,) is

$$t_{dt} = t_{tg} + t_1$$
$$= (v_t - v_g)/a_d + v_g/2a_d + b/v_g$$
$$\mathbf{= v_t/a_d - (v_g^2/2a_d - b)/v_g = 127.78 - (v_g^2/2a_d - b)/v_g}$$

(Note that that last quantity is not a basic TSD – the buffer constant is subtracted. Note also that 127.78 is 127.777….)

This **does** vary with the target speed, which is not surprising, but with nothing else, which perhaps is.

Deceleration Track Length for Overall Line Speed in the Medium Speed Range

v_l = (original) line speed in m/s (a Sweet-Speed)
v_g = target Speed in m/s
a_d = deceleration rate = 0.5m/s^2
b = buffer component of Train Separation Distance, = 830m.
c_l = line capacityin tph
c_t = capacity slot time in sec, = 3600/c_l
c_s = capacity slot length, = vc_t where v is the current, instantaneous speed
c_{sl} = capacity slot length at speed v_l, =v_lc_t
c_{sg} = capacity slot length at speed v_g, =v_gc_t

Train 1:

0. Train 1 is initially the slot distance c_{sl} ahead of train 2.
1. Train 1 travels the distance b at constant speed v_l onto deceleration track 1
   $t_{b1} = b/v_l$
   $s_{b1} = b$
2. Train 1 decelerates $v_l \Rightarrow v_g$
   $t_{lg} = (v_l - v_g)/a_d$
   $s_{lg} = (v_l^2 - v_g^2)/2a_d$
3. Train 1 travels at constant speed v_g to end of deceleration track 1
   $t = t_1$
   $s = s_1$ such that $s_1 = v_gt_1$

Train 2:

1. Train 2 travels the distance slot length plus buffer length at constant speed v_l
   $t_{b2} = (c_{sl} + b)/v_l$
   $s_{b2} = c_{sl} + b$
2. Train 2 decelerates $v_l \Rightarrow v_2$ at the instant that train 1 reaches the end of deceleration track 1
   $t_2 = (v_l - v_2)/a_d$
   $s_2 = (v_l^2 - v_2^2)/2a_d$

Travelling to the point in time where train 1 reaches the end of deceleration track 1:

$$t_{b1} + t_{lq} + t_1 \quad = \quad t_{b2} + t_2$$

i.e. $\quad b/v_l + (v_l – v_g)/a_d + t_1 \; = \quad (c_{sl} + b)/v_l + (v_l – v_2)/a_d$

So $\quad \mathbf{v_2 = v_g + a_d(c_t – t_1)}$

The distance between the trains at this point is:

$$\begin{aligned}
&(c_{sl} + s_{b1}+s_{lg} + s_1) – (c_{sl} + b + s_2)\\
= \quad &(v_l^2 – v_g^2)/2a_d + v_gt_1 – (v_l^2 – v_2^2)/2a_d\\
= \quad &v_gt_1 + (v_2^2 – v_g^2)/2a_d
\end{aligned}$$

And this is equal to the instantaneous TSD(b) for train 2

i.e. $\quad v_2^2/2a_d + b$

So $\quad \mathbf{t_1 = (v_g^2/2a_d + b)/v_g \quad = TSD(b)_{vg}/v_g}$

(This is the time travelled at constant speed v_g, and is identical to the High Speed case. All the values are known)

The length of the deceleration track is thus:

$$\begin{aligned}
s_{dt} = \quad &s_{b1} + s_{lg} + s_1\\
= \quad &b + (v_l^2 – v_g^2)2a_d + (v_g^2/2a_d + b)\\
= \quad &v_l^2/2a_d + 2b
\end{aligned}$$

or $\quad \mathbf{s_{dt} = (v_l^2/2a_d + b) + b = TSD(b)_{vl} + b}$

This is directly equivalent to the High Speed case, since v_l is the speed at which the train is travelling when it enters the deceleration track (but for the High Speed case, that is the turnout limit speed, which is constant). The extra b for the Medium Speed case is of course the initial virtual acceleration component.

Remember that the (physical) station loop length is constant for all line speeds in the High Speed range, since it depends only on the turnout limit speed, this being the speed at which a train enters the physical loop, whereas for the Medium and Low Speed ranges, it varies with the line speed. Deceleration to a lower speed thus, unsurprisingly, behaves in the same manner as deceleration all the way down to zero.

The overall elapsed time for a train to travel the length of the deceleration track (i.e. the front of thetrain travelling from the diverging switch points to the converging switch points) is likewise directly equivalent to the High Speed case. It it:

$$\begin{aligned}
t_{dt} = \quad &t_{bl} + t_{lg} + t1\\
= \quad &b/v_l + (v_l – v_g)/a_d + (v_g^2/2a_d + b)/v_g\\
= \quad &b/vl + (vlvg – vg2)/advg + (v_g^2/2a_d + b)/v_g
\end{aligned}$$

or $\quad \mathbf{t_{dt} = b/v_/ + v_l/a_d – (v_{g2}/2a_d – b)/}v_g$

Terminal Stations

Terminal stations are the real capacity (and other) bugbear, at least, large terminal stations in London are, like, for example, Euston. Each platform of a terminal station can handle only 2tph – 20 minutes to unload, service and reload the train, plus 10 minutes contingency. Attempting to satisfy the entire load of a HS line in a single terminal station, as HS2 Ltd., with their lunatic plans for Euston, vaingloriously assert they can do, is a catastrophe in the making. An acceptable level of capacity can be provided, in a terminal station, only by a completely unacceptable metastasis of platforms, and of station area.

But all is not lost. The correct way to design a Same Speed Railway of **any** category, but High Speed in particular, is roots – trunk – branches. Multiple services from different origins – the roots – progressively merge into a single trunk and travel the bulk of their journeys at high speed on the trunk. They then progressively diverge from the trunk – the branches – to reach their destinations. Each origin and destination has only one or two services, so, even at only 2tph per terminal platform, doesn't need many platforms to accommodate them. In any case, trains don't **have** to be serviced at the station platform itself. After unloading, they could be moved to a servicing area and processed at leisure, returning to the platform in good time for their next assignment, with plenty of time to reload in comfort. Such luxuries are absent from congested metropolitan termini. The roots and branches can often, at least towards the ends, be existing classic routes. HS2 Ltd. is evidently of the considered opinion that the best place to terminate a HS line is on the trunk!!!

The solution to this farrago is to do away with terminal stations, at least, big ones in London. A new, underground, through station should be built at Euston Cross (i.e. linking Euston, St. Pancras and King's Cross). With station wait times of up to 10 minutes allowed, 3 or 4 platform faces in each direction should be sufficient, with a single pair of approach tunnels. Services pass underneath London and out to the other side, fanning out to serve several terminal destinations, such as Maidstone, Gillingham/Rainham, Dover, Margate and Eastbourne, each of which, being served by only a fraction of the total, would need little if any new infrastructure.

The above statement of station capacity was my original estimate, pulled out of the air, more or less. But I can do a lot better now. The section between Old Oak Common and Stratford HL (South) simply needs to be operated as a 32tph, constant-capacity line, running at the reduced line speed of 20.11m/s, i.e. 45mph, which can be accommodated by perfectly ordinary switches – no special trackwork needed at all. The results are now given. They may look (and, indeed, are) very similar to those in the previous section, but note that that describes an overtaking situation, whereas this is a pure metro, similar to that in the section on metros, starting on p.25. The terminology is thus slightly different.

Line capacity = 32ph.   Slot time = 112.5sec.   Slot length = 2.2627km.
Line speed = 20.11m/s = 72.41kph = 45mph.
Deceleration time = 40.23sec = 0.3576 slots   Deceleration distance = 0.4045km = 0.1788 slots
Acceleration time = 67.04sec = 0.5959 slots   Acceleration distance = 0.6742km = 0.2980 slots
Station Calling Section travelling time = 107.27sec = 0.9535 slots.
Station Calling Section distance = 1.0787km = 0.4767 slots

By the time the stopping train has travelled the length of the station calling section, its empty slot, which it gave up on entering the section, has travelled 0.9535 slots (time or distance) on the main line. It is thus 0.4767 slots (time or distance) beyond the end of the section. In order to make this 1 slot exactly, it must travel a further 0.5233 slots (time or distance). This implies that the train must wait for 0.5233 **time** slots = 58.87sec at the station.

We now follow the progress of slot stream and stopping train.

Time Slot 1:
The empty slot advances 1 slot along the main line. It is now 1 slot beyond start of section, but has also (already!) passed the end of section. It is 0.5233 slots beyond end of section.
The slot containing the next stopping train arrives at start of section.
The train decelerates for 0.3576 time slots, and reaches the station, where it waits for 0.5233 time slots. It then accelerates for 0.1192 time slots. It thus has 0.4767 time slots of acceleration still to do.

During Time slot 2:
The empty slot advances a further 0.4767 slots. It is now 1.4767 slots beyond start of section, and **1 slot exactly beyond end of section.** The slot stream advance is thus 1.
The second (empty) slot advances 0.4767 slots. **It therefore coincides precisely with end of section.**
The train accelerates for a further 0.4767 time slots, thus completing its acceleration. **It therefore coincides precisely with end of section and is travelling at line speed.**
(The second stopping train also decelerates for 0.3576 time slots, and then waits at the station for 0.1191 time slots, a total of 0.4767 time slots – but so what?)

The slot stream advance is only 1; it needs to be 4 to be consistent with the original Same Speed value. (Like the slot time, the slot stream advance and thus the number of capacity sub-streams, must be constant throughput for constant capacity over different line speeds.) The station wait time must therefore be 3.5233 time slots, = 396.37sec, = 6min36.37sec.

A station wait time of 396sec would be far too long for a normal (albeit Same Speed) metro, but perfect for long distance High Speed Trains, **running under pure metro conditions** to cross London. Between Old Oak Common and Stratford, a line speed of 45mph would be perfectly adequate.

In principle, (and in actuality, for some of the routes I have considered,) we could reach a situation where all trains pass through (i.e. underneath) London, and none actually starts or terminates there. We thus face the possibility that most or all of the existing terminal stations in London, and possibly in other metropolitan areas, could become redundant, while actual rail travel went on increasing, facilitated by greatly increased available capacities. I merely flag this up as a possibility; it isn't going to happen next year or even next decade. But we should begin now to consider worthy alternative uses for some of the finest architecture in the country. (I think Liverpool Street is likely to be the first to become available.)

In practice, however, the rush hour would presumably not have gone away. There would still be a need for extra capacity at these times, and the existing terminal stations would provide this, leaving the inter-regional, cross-London connections carrying an essentially even base load throughout the day. The terminal stations would not be required for railway purposes outside the periods 7:30 – 9:30 and 16:30 – 18:30, say. So the above remark on finding worthy alternative uses for them still applies. My own initial ideas are for staging artistic and cultural events and small exhibitions in the passenger circulating areas. Most of the infrastructure required – cafes, toilets and retail units – is already there. The opening times for such events would be 10:00 – 16:00 and 19:00 onwards, Monday – Friday, (and all day at weekends,) to give time for set-up and dismantling after and prior to rail use, since the passenger circulating areas should certainly not be obstructed during rush hours.

Resilience

The focus of this article has been all about determining maximum values of line capacity, and about how such capacity levels are actually achieved. The results have all been of every last capacity slot in use. It would be very inadvisable trying to run a railway (**any** railway!) under such conditions, though, on reflection, that is the way conventional railways always have been run. **Safety** standards have been at a high levels for a long time now, but **resilience** is unknown and unprovided for. For Same Speed Railways, where everything and in particular all operating conditions are at the outer limits of tolerance, no service could reasonably be attempted under such conditions. The slightest divergence from the timetable would cause the whole system to come crashing down. There is, in other words, no built-in resilience.

As pointed out, traditional, mixed traffic railways always have operated with minimal (no!) resilience, and, every so often, suffer serious interruptions of service. The huge capacities offered by Same Speed Railways would magnify hugely the effect of a service interruption. So we don't have the luxury of ignoring resilience.

Realistically, there is only one sort of problem from which we can recover by instituting resilience – when a stopping train for some reason misses its scheduled restart time from a station. This can happen for all sorts of no-fault reasons, such as a passenger being taken suddenly ill, and the train having to be held waiting for an ambulance. This is actually very straightforward to defend against, and defended against it must be, since the train is immediately obstructing the following train and all subsequent trains in its own capacity slot sub-stream and in all other sub-streams (since the sub-streams are purely a logical abstraction – there is only a single physical track in each direction, occupied by all of them).

We consider first the overtaking categories. Assume that we have just the one stopping sub-stream, which is the most likely case.

The most direct approach is to assign a second sub-stream as a stopping sub-stream, but with no trains actually assigned to it; it serves purely as a resilience sub-stream for the actual stopping service. As soon as it is clear that there is a problem, and that a train is going to miss its restart time, the sub-stream is switched over to the other platform face, and the following trains are then no longer impeded. The capacity slot given up by the following stopping train, which ordinarily would have been taken over by the train with the problem, simply continues, empty thereafter. This is hardly a problem. The train with the difficulty simply stays in the platform until the difficulty has been resolved, and then departs, taking over the next slot (all of which are normally empty) in the resilience sub-stream, and travels in that sub-stream for the rest of its journey. There is of course no scope for it to regain any of the lost time; that is inevitable in the tightly-scheduled world of Same Speed railways, where no resources are wasted (as opposed to deliberately assigned for resilience). This is, I believe, a gratifyingly robust and efficient solution, but very expensive in line capacity, taking up, typically, 25% of the total.

A less extravagant alternative, for High Speed and Medium Speed railways, which rarely operate at full capacity in the stopping sub-stream, is to use the free capacity in that sub-stream for resilience. The stations are likely to be very adequately served by a train every 15 minutes. For my favourite line capacity of 32tph, with 4 sub-streams, that means 4 out of the 8 slots per hour in the stopping sub-stream are empty. Suppose the trains use both platforms at the stations alternately. If a train misses its restart time, the next three slots **at that platform** are empty, so it has another 2 chances of taking up a slot (the middle slot of the 3 corresponds to the train in the other platform). Then the next train arrives, uses the other platform, (because our train is blocking the one it would have used,) and departs at the correct time. Our train now has a further 2 opportunities to get moving. All the other trains continue to use the other platform. Even if only 50% of the slots are empty, that still means that the train has a free slot available every second slot in its sub-stream. Even if the train has broken down, then, provided it did actually reach a station, and provided also that a second train doesn't fail at the same station, the service continues running indefinitely. The failed train can be removed by emergency services overnight.

The slot window has been mentioned earlier. This gives a certain resilience in that it allow a train to depart any time during the ~1 minute before its scheduled departure time, and thus have more leeway in joining its new slot.

The above remarks apply to High and Medium Speed railways with overtaking; in fact they apply to semi-metros in all speed ranges. For pure metros, the situation is very different. For these, we don't bother at all with resilience, in the sense of trying to **recover** from an incident. Instead, we simply cancel the affected service. This may sound staggeringly cavalier, but the reasons are decisive.

Pure metros have very few capacity sub-streams, typically just two. Low Speed metros **invariably** have just two (in scheduled use). So sacrificing capacity for resilience is not an option.

At least one, and preferably two extra platform faces in each direction are added at each station. They are not normally in use. It a train hits a problem at a station, the slot sub-stream of which it is a member simply switches over to one of these resilience platforms, which is then uses indefinitely, for the rest of the day if need be. The train which missed its slot is cancelled, and left where it is, until the end of the day's service, if need be. The passengers on that train simply move to another platform and catch another train, an inconvenience certainly, but at the service frequencies provided, not a huge problem. The cancelled train has to sit it out because there are no scheduled free slots in a (pure) metro service.

Better still, (and the main reason for having **two** extra platform faces,) **both** slot sub-streams switch across to the reserve pair of platform faces, on opposite sides of the same island platform. Otherwise the service would be split between non-contiguous platform faces, requiring stair access to get between them. (Of course, in the disastrous but surely rare event of **two** failed trains at the same station, non-contiguous platforms would have to be used, since the second failure would be on one of the second pair of platforms, but the full service would still operate unless and until a **third** train failed there.)

In practice, it is unlikely that a metro will be operating at maximum capacity all the time, throughout the whole day, so there usually will be opportunity to recover the failed train before the end of the day's service.

Resilience is a very important subject. This section acknowledges its importance and suggests possible approaches.

Appendix C – Calculating Journey Times

Journey Time Estimates

Appendix B contains the basic information used in calculating journey times, though this is no longer its prime intention. The method of calculation is very straightforward, though perhaps not immediately obvious, and very readily automated in a spreadsheet.

The basic method of estimating journey times is to separate the journey into discrete sections, generally between station stops. It is assumed that the train accelerates from zero at the initial station up to line speed, and decelerates from line speed to zero at the second station. (The acceleration and deceleration times and distances are the same for every line section which exceeds in length the sum of these two distances, and are listed in the table of Fundamental Dynamic Values on p.10.) The section in between is travelled at constant line speed. These times are accumulated for the various sections of the journey. A standard wait time of 3 minutes is added at each intermediate station (thus excluding, obviously, the originating and destination stations). And that, in essence, is all that is involved. A number of refinements are available to deal with particular exceptional cases, and these are explained in the following sections.

This, of course, is a very simple approach, with obvious objections. It does, however, in my opinion, give the best available **estimate** of journey times, given just distance information. To get better accuracy requires detailed knowledge of the route including a complete speed profile. Such an approach is no longer an **estimate**, but a factual and exact prediction.

The above two paragraphs are how the topic was originally introduced in Appendix B. They are completely correct, for the standard case. But subsequent experience in generating journey time estimates has revealed very many so-called 'particular exceptional cases', so many indeed that they can no longer be regarded as either particular or exceptional.

Actually, the above claim is ridiculously modest. The results so derived are precise and exact, not estimates at all, for a Same Speed Railway, High Speed or not, which is performing exactly according to the theory.

The Mk1A changes, incorporating inter alia sections of classic route within the main line, require changes of line speed without stopping. This is not a problem; the situation is covered by the formulae derived in the calculus crib on p11.

That is also a former appendix B paragraph, (but much later, v4.0, December 2016,) and marks the point at which the (no-longer-) exceptional conditions started to multiply. Changes of line speed within a section oblige that section to be split, so sections were no longer exclusively between stations.

Attempting to assemble actual timetables (starting with the Scottish Network, HS13/HS14 Route and Service Plans v.3.0 in October 2017) requires the estimation of *passing times* (so represented in the spreadsheets) at a large number of locations of interest for some reason or other. Indeed, all journey-time-estimate spreadsheets are now produced in two versions, a summary version, giving times only at stations and at line-speed-change locations between stations, and the full version showing passing times wherever of interest (and typically up to three times as long as the summaries).

Even with all this increasing complexity, the method of calculation is unchanged in its essentials, but there are now a lot of special cases to keep track of. Accordingly, this new Appendix C was produced, to expound journey time calculations in all their variety, and elucidate some fascinating arcana which have been discovered in the process, of which my personal favourite is the concept of Propinquant Junctions (you can tell I'm the victim of a classical education!).

Basics

For constant acceleration / deceleration, a, the time taken, to accelerate from zero to speed v / decelerate from speed v to zero, is t = v/a. The distance, s, over which this takes place is $s = at^2/2$, $= v^2/2a$. The time taken to travel a distance s at constant speed v is t = s/v. These are all the basic formulae required.

Denoting the constant rates of acceleration and deceleration as a_a and a_d, the times to accelerate from zero to speed v and to decelerate from v to zero as t_a and t_d, and the corresponding distances as s_a and s_d, then: $s_a = v^2/2a_a$ $\quad s_d = v^2/2a_d$ $\quad t_a = v/a_a$ $\quad t_d = v/a_d$.
Given that $a_a = 0.3m/s^2$ and $a_d = 0.5m/s^2$, standard values of s_a s_d t_a and t_d are calculated for a series of line speeds v which are of interest, specifically:

		Constants			
		a (m/s**2)	v (m/s)	t=v/a (s)	s=v**2/2a (m)
100mph / 160kph		0.5	44.4	88.9	1975.3
125mph / 200kph		0.5	55.6	111.1	3086.4
140mph / 225kph		0.5	62.5	125.0	3906.3
143mph / 230kph		0.5	63.9	127.8	4081.8
186.5mph / 300kph		0.5	83.3	166.7	6944.4
223.7mph / 360kph		0.5	100.0	200.0	10000.0
250mph / 400kph		0.5	111.1	222.2	12345.7
100mph / 160kph		0.3	44.4	148.1	3292.2
125mph / 200kph		0.3	55.6	185.2	5144.0
140mph / 225kph		0.3	62.5	208.3	6510.4
143mph / 230kph		0.3	63.9	213.0	6803.0
186.5mph / 300kph		0.3	83.3	277.8	11574.1
223.7mph / 360kph		0.3	100.0	333.3	16666.7
250mph / 400kph		0.3	111.1	370.4	20576.1
Train Length Effect		0.0	63.9	6.3	400.0

This rectangle of cells is included in every spreadsheet, in an area below the active rows. The values are actually calculated within the spreadsheet, according to the above formulae. The values of interest for v, t and s are accessed by **absolute reference**, (such as A7,) and do not therefore need to be calculated within each individual active row. (The spreadsheet is clever enough to adjust these references, in the formula contained in the journey-time cell of each individual active row, if rows are deleted or new rows inserted. The administrative housekeeping is thus completely automatic.)

The bottom row, different from the others, gives the values for a train, of length 400m, traversing a junction at constant speed 230kph.

Each individual active row of the spreadsheet represents a particular section of a journey. The single data column contains the section lengths. This value is fed into one of the standard formulae, below, held in another column, to calculate the journey time over that section. Other columns accumulate distances and times for the overall journey, a second ‘times’ column adding in the effects of station wait times, to give total, elapsed journey times.

In the following standard formulae, s_l is section length (m) and v_l is line speed (m/s).

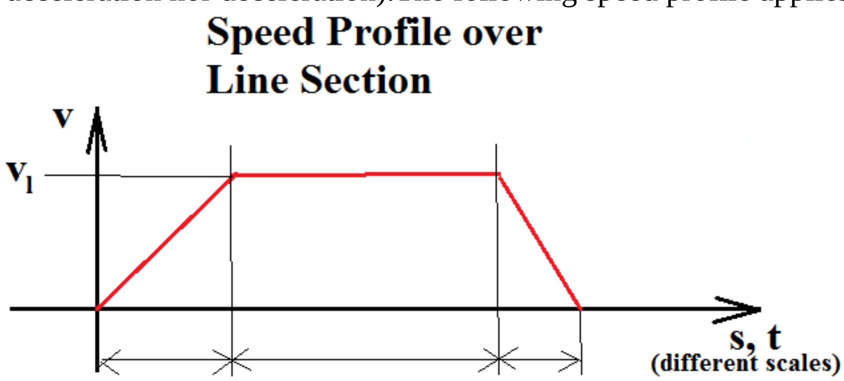
The Standard Formulae

1. Section travelled at constant speed throughout. The train is already travelling at line speed on entering the section, and continues at line speed on leaving it, typically between stations at which it doesn’t stop, though other services do, which is (one possible reason) why the end points are locations of interest.

   $t = s_l/v_l$
2. Section between stations, at both of which the train stops.

   $t = (s_l – s_a – s_d)/v_l + t_a + t_d$

   This may look a bit intimidating, but its meaning is very straightforward. $s_l – s_a – s_d$ is the distance travelled at line speed, v_l, i.e. the section length less the distances taken up by acceleration to and deceleration from line speed. Dividing this by the line speed gives the time taken, travelling at line speed, to which are simply added the acceleration and deceleration times, to get the total time, start to stop. Once having understood this, the following formulae should be readily intelligible, being just special cases of this formula (as, indeed, is #1, which is simply the limiting case when there is neither acceleration nor deceleration).The following speed profile applies:

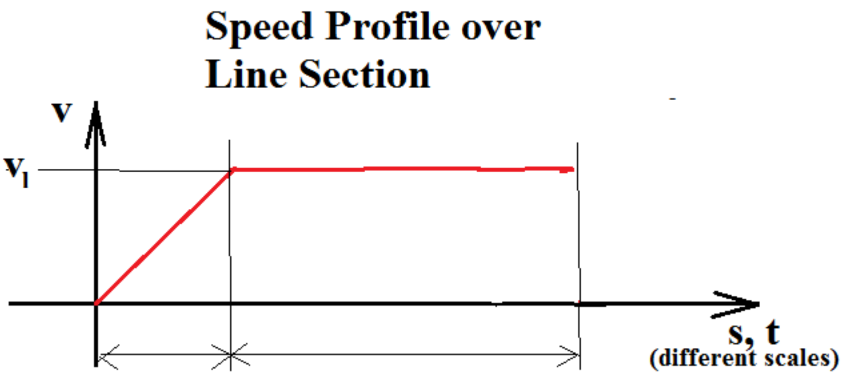
   

distances	s_a	$(s_l\text{-}s_a\text{-}s_d)$	s_d
times	t_a	$(s_l\text{-}s_a\text{-}s_d)/v_l$	t_d

   The graph depicted is of speed against time, and is, on the assumption of constant acceleration / deceleration, of straight-line segments. Distances are also indicated, but the graph of speed against distance for the varying parts would be parabolic. The ratios of acceleration to deceleration times and distances are the same, the inverse ratio of the acceleration rates, 5:3 for the values actually used, and here carefully depicted as such. But the ratios of acceleration + deceleration to constant speed portions are **not** the same for time and distance.
3. Section immediately following, or immediately before, a station where the train stops; continuing at line speed at the other end. Thus:

   (#3.1) the train accelerates up to line speed and then holds that speed to the end of the section

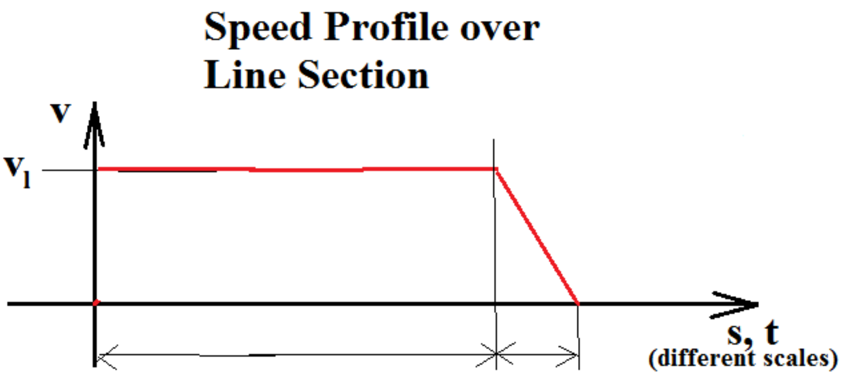
   $t = (s_l – s_a)/v_l + t_a$



distances	s_a	$(s_l\text{-}s_a)$
times	t_a	$(s_l\text{-}s_a)/v_l$

(#3.2) **or** the train enters the section at linespeed, but decelerates to zero at the end:

$t = (s_l - s_d)/v_l + t_d$



distances	$(s_l\text{-}s_d)$	s_d
times	$(s_l\text{-}s_d)/v_l$	t_d

Note that, for these open-ended profiles, the line speed of the next / previous section may be different from that of the present section. It may be **greater**, in which case the train accelerates to that new value once it has entered the next section / decelerates from that value to the present section's line speed before entering the present section, but cannot be **less**, since in that case the deceleration to that value must take place before entering the next section, i.e. before leaving the present section / the acceleration to the line speed of the present section must take place after the train has entered the present section.

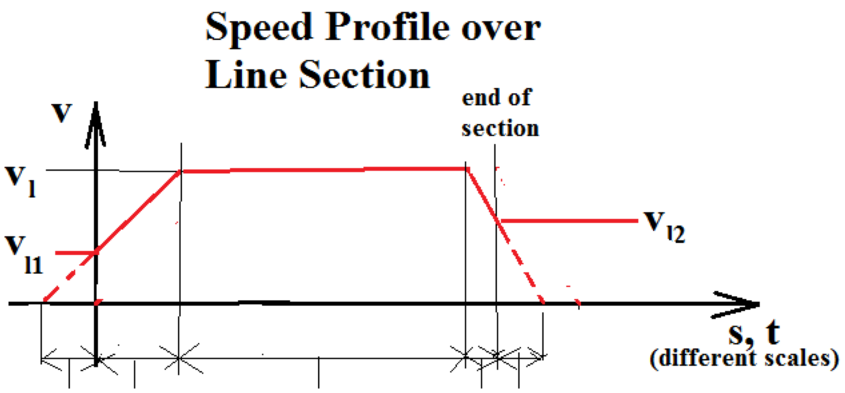
Strictly speaking, the train begins to accelerate to a higher line speed in the next section once it has **entirely** entered the next section, i.e. only when the back end of the train has actually left the present section, likewise acceleration to the higher line speed of the present section from the lower speed of the previous one can take place only when it has **entirely** entered the present section, i.e. only when the back end of the train has actually entered the present section. The precise calculations would thus have to take account of the train length. This may seem an unwanted complication (since it surely has only a small effect?), but in fact it is trivially easy to handle, so there is no reason not to include it. In fact, it occurs at every point where the line speed changes, applying to one of the adjacent sections or the other (in fact, always to the section of higher line speed). The train-length effects will be dealt with shortly, after the following case. These considerations also apply to case 1, above, open-ended at both ends.

4. The last, and most complicated of the standard cases, deals with the situation described immediately above, where the train is moving on entering and on leaving the present section, the

line speeds of the adjacent sections being lower than that of the present section, so that acceleration from the entering speed (the line speed of the previous section) to that of the present section, and deceleration to the line speed of the following section, must take place within the present section. The trick here is to treat it as if it were case 2, over a longer section length, so that it has accelerated from zero up to the line speed of the previous section by the time it enters the present section, and has decelerated to the line speed of the next section by the time it leaves the present section, and continues its steady deceleration to zero. The notation gets rather messy.
Let v_{l1}, s_{a1}, t_{a1} be the line speed, acceleration distance and time to that line speed for section 1, the preceding section, and v_{l2}, s_{d2}, t_{d2} the line speed, deceleration distance and time from that line speed to zero for section 2, the following section (and the values without 1 or 2 in the suffix those for the present section, as thus far). The result is:

$$t = \{(s_l - (s_a - s_{a1}) - (s_d - s_{d2})\}/v_l + (t_a - t_{a1}) + (t_d - t_{d2})$$
$$= (s_l - s_a - s_d + s_{a1} + s_{d2})/v_l + t_a + t_d - t_{a1} - t_{d1}$$

This is, admittedly, rather dreadful. I shan't even attempt to explain it in words, but the speed profile should provide the necessary elucidation.



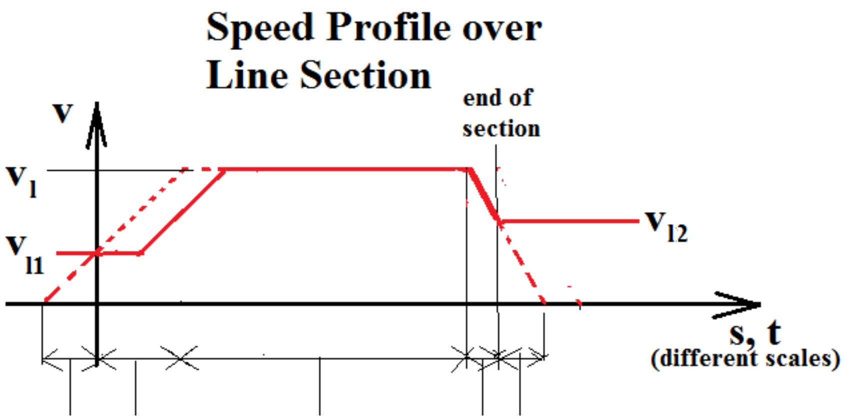
distances s_{a1} s_a-s_{a1} $\{s_l$-$(s_a$-$s_{a1})$-$(s_d$-$s_{d2})\}$ s_d-s_{d2} s_{d2}
times t_{a1} t_a-t_{a1} t_d-t_{d2} t_{d2}
$\{s_l$-$(s_a$-$s_{a1})$-$(s_d$-$s_{d2})\}/v_l$

It is readily seen from the diagram that this is essentially the same as case 2, but with modified values of $s_a - s_{a1}$, $t_a - t_{a1}$ for the acceleration distance and time, and $s_d - s_{d2}$, $t_d - t_{d2}$ for the deceleration.
The above treatment gives the full generality. This is not encountered in practice, since the long sections over which it would apply are always broken up into smaller sections by intermediate locations of interest. So this last case is only encountered, in practice, in the single-ended situations, described in case 3, above. So just leave out the acceleration or deceleration component, as appropriate, from the formula. (Actually, it does occur, occasionally, with a station at one end, Edinburgh Airport – Forth Bridge South and F.B. North – Kirkcaldy being such cases – see below.)

5. The effect of train length is small but significant, and so easy to deal with that there is absolutely no reason not to include it. At section boundaries where the line speed changes between sections, the time taken by the train to cross the boundary must be taken into account. (The notional train length is taken as 400m, or 16 carriages of 25m each. In practice this will be made up of two separate trains of 8 carriages, to allow for the train to be split / joined during the journey. If there is no such requirement, then a single, 8-car train may well suffice, but the calculations always assume the 16-car case.)
When the train passes a boundary between a lower and a higher line speed, it must continue at the

lower line speed until it has completely left the first section and is entirely in the second section, before in can begin to accelerate. When it passes from a higher to a lower speed, then it must decelerate within the current section, reaching the lower speed of the next section at the point where the head of the train reaches the boundary. The train actually crosses the boundary at a constant, (the lower,) speed. If the length of the train is s_t (400m in practice, but why not be completely general?) and the lower speed is v_t (almost always the turnout limit speed for a junction – 230kph – but not invariably so, and, again, why not be general?) then the time taken by the train to traverse the boundary, $t_t = s_t/v_t$. These train-length effects are applied to the higher speed section is each speed-change-boundary case.

The standard formulae above apply to a notional, zero-length train. To take the train length s_t into account, it is subtracted from the distance travelled at line speed, thus reducing the time traveling at line speed, when that distance is divided by the line speed, and then the actual time taken, t_t, traveling at speed v_t, added in, to get the actual time taken to get through that section. In the most general case:



This is essentially the same profile as case 4 above, so the various distances are not reproduced, being the same as before. The effect of train length is of course grossly exaggerated, but the point to note is that the same distance, train length, subtracted from the distance travelled at line speed, has a bigger effect on time the lower the speed at which it is travelled – obviously, since t=s/v for constant speed. The graph depicted is velocity against time, and clearly shows that the train length effect applies only when the line speed of the section is higher than the previous one. The method of calculation is clearly laid out above; the graph seeks to elucidate it and make its meaning clear.

6. The above treatment of train length is definitive for the case where there is a change of line speed between sections. However, there is another case, at least equally important, where the train crosses a diverging or converging junction from or joining the main line. (Trains traveling straight ahead on the main line, not diverging, continue across the junction at full line speed.) The diverging / converging route has the same line speed. It is therefore necessary for diverging / converging trains to decelerate from line speed down to the turnout limit speed (230kph), the entire train crossing the junction at this speed, and then accelerating back up to line speed. This situation was recognised early, and features in appendix B; the ‘Junction Effects’ table on p.15 contains a ‘Route Junction Time Penalty’ row, which is the extra time taken in decelerating, crossing the junction and re-accelerating as compared with the time taken to travel the same distance at line speed. (The values are 14 and 37 sec for 300 and 360kph – and 56 sec for 400kph, for what that’s worth.)
A special case of the above is for station loops. On a HS line with stations where some services stop and others pass through without stopping, there must be provision for overtaking. Thus the platforms are on (very long) station loops. This is a fundamental characteristic of HS railways. It is described on p.3, and featured in version 1.0 of the present article! The locations on the main

line of the junctions to and from the station loops are exactly prescribed by distances required for acceleration and deceleration from and to the station. The point being made here is that the measurement of these distances on **the main line** must allow for the train length **in the accelerating case (only)**, so that the train, accelerating at the constant rate from the station, reaches the turnout limit speed **only at the point where its back end has just crossed the junction** onto the main line. It thus continues its acceleration without a break right up to line speed. (This brings the actual junction 400m closer to the station than would be the case if the train had accelerated up to 230kph and then crossed the junction at that constant speed. It is also conceptually simpler, but a little more involved to describe.) For the deceleration case, the junction is at the full deceleration distance from the station. The train decelerates down to 230kph at the junction, and continues its steady deceleration as it crosses the junction.
I admit that this is an awfully pedantic point, but high speed railways require absolute precision in their operation. They also require absolute precision in thinking about them and designing them. Having made the point, it plays no further part in these calculations, but was actually brought to my attention by the treatment of Propinquant Junctions, which follows shortly.

The above formulae are used in the journey time spreadsheets, and cover nearly all cases completely automatically. The only things which need to be watched for are adjacent stations, diverging / converging junctions and adjacent junction pairs. Adjacent stations are explained beginning at p.55, with a convenient table on p.58 which determines whether or not two stations are truly ‘adjacent’ (depends on distance apart and line speed), and, if they are, gives the inter-station journey time, start to stop. This value is then entered explicitly in the Journey Time cell for the section, replacing the formula.

Diverging / converging junctions were mentioned in point 6 above, which quotes the relevant values for the junction time penalty. Adjacent Junctions are dealt with starting at p.60, again yielding a junction time penalty. These junction time penalties are simply added as an explicit value into the Journey Time cell formula.

One final detailed point in calculating overall elapsed journey times. The times given in the spreadsheet are all arrival times, so the wait time at stations needs to be added, and this must be added to the section beginning at the station – obviously so, since, when you think about it, that’s where the time penalty is incurred, before the train even leaves the station.

The following example exhibits **all** the above formulae. It deals with the three services between Edinburgh and Aberdeen, fast, stopping (both of which via Stirling and Perth,) and via Dundee. It has **lots** of *passing times*. (It also has extra standard values to cover the speed restrictions over the Forth and Tay bridges!)

Section	Distance (km)	Cumulative Distance (km)	Start - Stop Time (minutes)	Cumulative Journey Time (minutes)	Elapsed Time from Edinburgh, inc. Station Wait Times
Edinburgh Waverley HS - Haymarket HS	2.1	2.1	2.5	2.5	2.5
Haymarket HS - Edinburgh Airport	9.0	11.1	5.2	7.7	10.7
Edinburgh Airport - Stirling	45.0	56.1	12.7	20.4	26.4
Stirling - *Gleneagles (pass)*	28.0	84.1	9.2	*29.6*	*38.6*
Gleneagles (pass) - Perth	22.0	106.1	6.9	36.5	45.5
Perth - *Stanley Junction (pass)*	11.5	117.6	4.6	*41.1*	*53.1*
Stanley Junction (pass) - Coupar Angus)pass)	14.0	131.6	2.8	*43.9*	*55.9*
Coupar Angus (pass) - Forfar (pass)	26.0	157.6	5.2	*49.1*	*61.1*
Forfar (pass) - Bridge of Dun (pass)	26.0	183.6	5.2	*54.3*	*66.3*
Bridge of Dun (pass) - Craigo Junction (pass)	8.0	191.6	1.7	*56.0*	*68.0*
Craigo Junstion (pass) - Laurencekirk (pass)	8.6	200.2	2.3	*58.3*	*70.3*
Laurencekirk (pass)- Drumlithie Junction (pass)	12.0	212.2	3.2	*61.5*	*73.5*
Drumlithie Junction (pass) - Cowie Junction (pass)	11.0	223.2	2.9	*64.4*	*76.4*
Cowie Junction (pass) - Aberdeen	21.0	244.2	5.6	70.0	82.0
Stirling - Gleneagles	28.0	84.1	10.2	30.6	39.6
Gleneagles - Perth	22.0	106.1	8.6	39.3	51.3
Perth - *Stanley Junction (pass)*	11.5	117.6	4.6	*43.9*	*60.9*
Stanley Junction (pass) - Coupar Angus	14.0	131.6	4.2	48.1	65.1
Coupar Angus - Forfar	26.0	157.6	8.9	57.0	77.0
Forfar - Bridge of Dun	26.0	183.6	8.9	65.9	88.9
Bridge of Dun - *Craigo Junction (pass)*	8.0	191.6	4.0	*69.9*	*95.9*
Craigo Junstion (pass) - Laurencekirk	8.6	200.2	3.3	73.2	99.2

Laurencekirk - *Drumlithie Junction (pass)*	12.0	212.2	4.9	78.1	107.1
Drumlithie Junction (pass) - Stonehaven (classic route)	11.2	223.4	4.0	82.2	111.2
Stonehaven - *Cowie Junction (pass)*	2.0	225.4	1.9	84.1	116.1
Cowie Junction (pass) - Aberdeen	21.0	246.4	7.0	91.1	123.1
Edinburgh Airport - *Forth Bridge South (pass)*	10.0	21.1	4.8	*12.5*	*18.5*
Forth Bridge South (pass) - Forth Bridge North (pass)	5.3	26.4	4.0	*16.5*	*22.5*
Forth Bridge North (pass) - Kirkcaldy	7.9	34.3	4.2	20.7	26.7
Kirkcaldy - Leuchars Junction	55.9	90.2	19.2	40.0	49.0
Leuchars Junction - *Tay Bridge South (pass)*	9.0	99.2	4.7	*44.7*	*56.7*
Tay Bridge South (pass) - Dundee	4.4	103.6	5.0	49.7	61.7
Dundee - Arbroath	27.5	131.1	10.1	59.8	74.8
Arbroath - Montrose	22.7	153.8	8.8	68.6	86.6
Montrose - Laurencekirk	16.2	170.0	7.1	75.7	96.7
Laurencekirk - Stonehaven	23.2	193.2	9.0	84.7	108.7
Stonehaven - Aberdeen	23.0	216.2	5.4	90.1	117.1
Kirkcaldy - Ladybank	36.9	71.2	13.5	34.3	43.3
Ladybank - Newburgh	12.0	83.2	6.5	40.7	52.7
Newburgh - Perth	17.0	100.2	8.4	49.1	64.1

Propinquant Junctions

A (nearly) final, very esoteric point, concerns what might be called (and what I **shall** call) a Propinquant Junction, i.e. close by but not so close as to be termed adjacent, and needs to be expounded, so we know how to deal with it.

The method of dealing with a diverging or converging junction, by adding a standard Junction Time Penalty, is referred to in point 6 on p.35, and the actual calculation is still in Appendix B, on p.15, and included in the Route Junction Time Penalty row in the Junction Effects table on p.16. What has perhaps not been adequately emphasised is that this very simple case and its treatment assumes that the line speed is the same on the diverging / converging route as on the main line, **and** that trains are travelling at full line speed as they approach the junction, the diverging / converging train decelerating down from line speed to the turnout limit speed, at which it crosses the junction, and then accelerating back up to line speed. This is not always the case. In fact, it is surprisingly rarely the case!

It may, and surprisingly often **does**, happen that a diverging junction is encountered when a train has just departed from a station stop, but has not yet accelerated up to full line speed. It may well be (in fact, under the new regime of passing times, it definitely **will** be,) that the passing time for the junction is of interest. (It is worth pointing out that all these latest, off-beat effects have only come to light since I started calculating passing times.) Two cases are encountered:

1. the train has not yet reached even the turnout limit speed, by the time it reaches the junction; or
2. the train has accelerated past the turnout limit speed, but has not yet reached the line speed.

The first case is trivially simple;.the second case isn't. Note that the concept of a propinquant junction as just described applies to **diverging** junctions and **accelerating** trains. The concept applies also to **converging** junctions and **decelerating** trains, but this will be expounded separately, later, so as not to confuse the issue. (It's confusing enough already!)

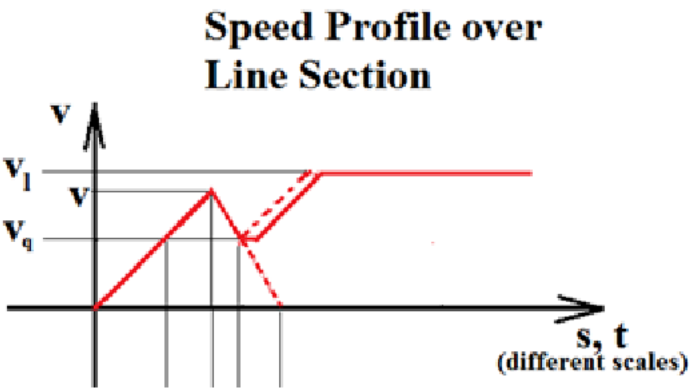
Either way, we rely on the standard formulae

$v = at \quad s = at^2/2 = v^2/2a \qquad$ (and $t = s/v$, for constant speed).

(Remember that the first three of these simple formulae only apply when starting from / slowing down to zero speed – the general case is expounded in appendix B, starting at p.11, but these simple cases are so very convenient that it's well worth framing the problem so as to be able to use them, and this is, in fact, always possible.)

For the first case, we know the distance s of the junction, so $t = \sqrt{(2s/a)}$. For the following section, perform the calculations as if it included the section to the junction, and then subtract the t value just calculated for the junction. Trivially simple, indeed.

The second case is illustrated by the following profile:



t_{q1} t t_{q2} t'
s_{q1} s s_{q2} s'

Given v_q (invariably the turnout limit speed, 230kph, 63.9m/s,) and s_{q2}, the junction distance, find t_{q2}.

Purely for consistency, the effect of train length in traversing the junction is indicated, (grossly exaggerated, of course,) even though the effect is confined within the following section, having no effect on the present calculations.

The train accelerates (with acceleration rate a_a m/s^2) up to some intermediate maximum speed, v, at distance s and time t, whereupon it must immediately begin its deceleration (at deceleration rate a_d m/s^2) to speed v_q at distance s_{q2} and time t_{q2}. (Thereafter, after crossing the junction, it resumes its acceleration up to the line speed v_l, but we're not interested in that.) Supposing that it continued its deceleration down to zero, this it would reach at distance s' and time t'.

It initially reaches (and accelerates beyond) speed v_q at distance s_{q1} and time t_{q1}, where:

$t_{q1} = v_q/a_a$ $\quad$ $s_{q1} = v_q^2/2a_a$ Note that these are fixed values, so can be treated as constants in the calculations following. For the virtual deceleration portion:

$t' - t_{q2} = v_q/a_d$ $\quad$ $s' - s_{q2} = v_q^2/2a_d$

thus $t_{q2} = t' - v_q/a_d$ $\quad$ $s' = s_{q2} + v_q^2/2a_d$

We know that, for a given speed, acceleration / deceleration times and distances to / from that speed are inversely proportional to the acceleration / deceleration rates. Thus:

$t/(t' - t) = s/(s' - s) = a_d/a_a$

so $a_a t = a_d(t' - t)$ and $a_a s = a_d(s' - s)$

so $(a_a + a_d)t = a_d t'$ and $(a_a + a_d)s = a_d s'$ so, finally

$t = a_d t'/(a_a + a_d)$ $\quad$ $s = a_d s'/(a_a + a_d) = (a_d/(a_a + a_d))*(s_{q2} + v_q^2/2a_d)$

The maximum intermediate speed v is:

$v = a_a t = a_d(t' - t)$

also $s = a_a t^2/2 = v^2/2a_a$

and $(s' - s) = a_d(t' - t)^2/2 = v^2/2a_d$

Thus $s' = v^2/2a_a + v^2/2a_d = v^2(a_a + a_d)/2a_a a_d$

But we already have:

$s' = s_{q2} + v_q^2/2a_d$ so, finally

$v^2 = 2a_a a_d(s_{q2} + v_q^2/2a_d)/(a_a + a_d)$ $\quad$ $v = \sqrt{}(2a_a a_d(s_{q2} + v_q^2/2a_d)/(a_a + a_d))$

Now that we have solved for v, other results, derived earlier, can be expressed without it, so:

$t = v/a_a = [\sqrt{}(2a_a a_d(s_{q2} + v_q^2/2a_d)/(a_a + a_d))]/a_a$

$(t' - t) = v/a_d = [\sqrt{}(2a_a a_d(s_{q2} + v_q^2/2a_d)/(a_a + a_d))]/a_d$

so $t' = t + v/a_d = v(1/a_a + 1/a_d) = [\sqrt{}(2a_a a_d(s_{q2} + v_q^2/2a_d)/(a_a + a_d))]*(a_a + a_d)/a_a a_d$

Finally! The result we've all been waiting for:

$$t_{q2} = t' - v_q/a_d$$
$$= [\sqrt{}(2a_a a_d(s_{q2} + v_q^2/2a_d)/(a_a + a_d))]*(a_a + a_d)/a_a a_d - v_q/a_d$$

This may look frightening, but it's all in a day's work for a spreadsheet.

In addition, suppose that, instead of accelerating as far as possible and then decelerating, as soon as speed reached v_q it was held at that value until distance s_{q2} had been reached, when acceleration would be resumed. This would clearly be slower, but exactly what sort of time penalty would be involved? We shall now see.

The above exposition is ordered in what is, in my opinion, the most logical order. In programming the spreadsheet, I don't actually code such horrible expressions as have been encountered, hut, instead, calculate the individual values in the order in which they appear, above. Thus, initially, the values for t_{q1} and s_{q1}, and other fixed value stuff, which then act as constants in the following calculations. Then, for each row, the data value for s_{q2} is given, from which are derived, in order, s', s, v^2, v, t, t', t_{q2}, and the fixed-speed elapsed time and time penalty. The values are calculated over the entire range of interest, thus from v = 230kph, 63.9m/s, to 360kph, 100m/s, as the line speed. Here they are:

Vq	Aa	Ad	Vq**2	Vq/Ad	Aa+Ad	Aa*Ad	See below		Tq1	Sq1
63.9	0.3	0.5	4081.8	127.8	0.8	0.2	5.3	0.6	213.0	6803.0

Junction Distance Sq2 (m)	S ' (m)	S (m)	V**2	V (m/s)	T (s)	T ' (s)	Tq2 (s)	Speed Vq between Sq1 and Sq2	Time penalty (s)
6803.0	10884.8	6803.0	4081.8	63.9	213.0	340.7	213.0	213.0	0.0
7003.0	11084.8	6928.0	4156.8	64.5	214.9	343.9	216.1	216.1	0.0
7203.0	11284.8	7053.0	4231.8	65.1	216.8	346.9	219.2	219.2	0.1
7403.0	11484.8	7178.0	4306.8	65.6	218.8	350.0	222.2	222.4	0.1
7603.0	11684.8	7303.0	4381.8	66.2	220.7	353.0	225.3	225.5	0.2
7803.0	11884.8	7428.0	4456.8	66.8	222.5	356.0	228.3	228.6	0.3
8003.0	12084.8	7553.0	4531.8	67.3	224.4	359.0	231.3	231.7	0.5
8203.0	12284.8	7678.0	4606.8	67.9	226.2	362.0	234.2	234.9	0.7
8403.0	12484.8	7803.0	4681.8	68.4	228.1	364.9	237.1	238.0	0.9
8603.0	12684.8	7928.0	4756.8	69.0	229.9	367.8	240.1	241.1	1.1
8803.0	12884.8	8053.0	4831.8	69.5	231.7	370.7	242.9	244.3	1.3
9003.0	13084.8	8178.0	4906.8	70.0	233.5	373.6	245.8	247.4	1.6
9203.0	13284.8	8303.0	4981.8	70.6	235.3	376.4	248.7	250.5	1.9
9403.0	13484.8	8428.0	5056.8	71.1	237.0	379.3	251.5	253.7	2.2
9603.0	13684.8	8553.0	5131.8	71.6	238.8	382.1	254.3	256.8	2.5
9803.0	13884.8	8678.0	5206.8	72.2	240.5	384.8	257.1	259.9	2.9
10003.0	14084.8	8803.0	5281.8	72.7	242.3	387.6	259.8	263.0	3.2
10203.0	14284.8	8928.0	5356.8	73.2	244.0	390.3	262.6	266.2	3.6
10403.0	14484.8	9053.0	5431.8	73.7	245.7	393.1	265.3	269.3	4.0
10603.0	14684.8	9178.0	5506.8	74.2	247.4	395.8	268.0	272.4	4.4
10803.0	14884.8	9303.0	5581.8	74.7	249.0	398.5	270.7	275.6	4.9
11003.0	15084.8	9428.0	5656.8	75.2	250.7	401.1	273.4	278.7	5.4
11203.0	15284.8	9553.0	5731.8	75.7	252.4	403.8	276.0	281.8	5.8
11403.0	15484.8	9678.0	5806.8	76.2	254.0	406.4	278.6	285.0	6.3
11603.0	15684.8	9803.0	5881.8	76.7	255.6	409.0	281.3	288.1	6.8
11803.0	15884.8	9928.0	5956.8	77.2	257.3	411.6	283.9	291.2	7.4
12003.0	16084.8	10053.0	6031.8	77.7	258.9	414.2	286.4	294.4	7.9
12203.0	16284.8	10178.0	6106.8	78.1	260.5	416.8	289.0	297.5	8.5
12403.0	16484.8	10303.0	6181.8	78.6	262.1	419.3	291.6	300.6	9.1
12603.0	16684.8	10428.0	6256.8	79.1	263.7	421.9	294.1	303.7	9.7

12803.0	16884.8	10553.0	6331.8	79.6	265.2	424.4	296.6		306.9	10.3
13003.0	17084.8	10678.0	6406.8	80.0	266.8	426.9	299.1		310.0	10.9
13203.0	17284.8	10803.0	6481.8	80.5	268.4	429.4	301.6		313.1	11.5
13403.0	17484.8	10928.0	6556.8	81.0	269.9	431.9	304.1		316.3	12.2
13603.0	17684.8	11053.0	6631.8	81.4	271.5	434.3	306.5		319.4	12.9
13803.0	17884.8	11178.0	6706.8	81.9	273.0	436.8	309.0		322.5	13.5
14003.0	18084.8	11303.0	6781.8	82.4	274.5	439.2	311.4		325.7	14.2
14203.0	18284.8	11428.0	6856.8	82.8	276.0	441.6	313.9		328.8	14.9
14403.0	18484.8	11553.0	6931.8	83.3	277.5	444.0	316.3		331.9	15.7
14436.7	18518.5	11574.1	6944.4	83.3	277.8	444.4	316.7		332.4	15.8
14603.0	18684.8	11678.0	7006.8	83.7	279.0	446.4	318.7		335.0	16.4
14803.0	18884.8	11803.0	7081.8	84.2	280.5	448.8	321.0		338.2	17.1
15003.0	19084.8	11928.0	7156.8	84.6	282.0	451.2	323.4		341.3	17.9
15203.0	19284.8	12053.0	7231.8	85.0	283.5	453.5	325.8		344.4	18.7
15403.0	19484.8	12178.0	7306.8	85.5	284.9	455.9	328.1		347.6	19.5
15603.0	19684.8	12303.0	7381.8	85.9	286.4	458.2	330.4		350.7	20.3
15803.0	19884.8	12428.0	7456.8	86.4	287.8	460.5	332.8		353.8	21.1
16003.0	20084.8	12553.0	7531.8	86.8	289.3	462.9	335.1		357.0	21.9
16203.0	20284.8	12678.0	7606.8	87.2	290.7	465.2	337.4		360.1	22.7
16403.0	20484.8	12803.0	7681.8	87.6	292.2	467.4	339.7		363.2	23.6
16603.0	20684.8	12928.0	7756.8	88.1	293.6	469.7	341.9		366.4	24.4
16803.0	20884.8	13053.0	7831.8	88.5	295.0	472.0	344.2		369.5	25.3
17003.0	21084.8	13178.0	7906.8	88.9	296.4	474.2	346.5		372.6	26.2
17203.0	21284.8	13303.0	7981.8	89.3	297.8	476.5	348.7		375.7	27.0
17403.0	21484.8	13428.0	8056.8	89.8	299.2	478.7	350.9		378.9	27.9
17603.0	21684.8	13553.0	8131.8	90.2	300.6	480.9	353.2		382.0	28.8
17803.0	21884.8	13678.0	8206.8	90.6	302.0	483.2	355.4		385.1	29.8
18003.0	22084.8	13803.0	8281.8	91.0	303.3	485.4	357.6		388.3	30.7
18203.0	22284.8	13928.0	8356.8	91.4	304.7	487.5	359.8		391.4	31.6
18403.0	22484.8	14053.0	8431.8	91.8	306.1	489.7	362.0		394.5	32.6
18603.0	22684.8	14178.0	8506.8	92.2	307.4	491.9	364.1		397.7	33.5
18803.0	22884.8	14303.0	8581.8	92.6	308.8	494.1	366.3		400.8	34.5
19003.0	23084.8	14428.0	8656.8	93.0	310.1	496.2	368.4		403.9	35.5
19203.0	23284.8	14553.0	8731.8	93.4	311.5	498.4	370.6		407.0	36.5
19403.0	23484.8	14678.0	8806.8	93.8	312.8	500.5	372.7		410.2	37.5
19603.0	23684.8	14803.0	8881.8	94.2	314.1	502.6	374.9		413.3	38.5
19803.0	23884.8	14928.0	8956.8	94.6	315.5	504.7	377.0		416.4	39.5
20003.0	24084.8	15053.0	9031.8	95.0	316.8	506.9	379.1		419.6	40.5
20203.0	24284.8	15178.0	9106.8	95.4	318.1	509.0	381.2		422.7	41.5
20403.0	24484.8	15303.0	9181.8	95.8	319.4	511.0	383.3		425.8	42.6
20603.0	24684.8	15428.0	9256.8	96.2	320.7	513.1	385.4		429.0	43.6
20803.0	24884.8	15553.0	9331.8	96.6	322.0	515.2	387.4		432.1	44.7
21003.0	25084.8	15678.0	9406.8	97.0	323.3	517.3	389.5		435.2	45.7
21203.0	25284.8	15803.0	9481.8	97.4	324.6	519.3	391.6		438.4	46.8
21403.0	25484.8	15928.0	9556.8	97.8	325.9	521.4	393.6		441.5	47.9
21603.0	25684.8	16053.0	9631.8	98.1	327.1	523.4	395.6		444.6	49.0

21803.0	25884.8	16178.0	9706.8	98.5	328.4	525.5	397.7	447.7	50.1
22003.0	26084.8	16303.0	9781.8	98.9	329.7	527.5	399.7	450.9	51.2
22203.0	26284.8	16428.0	9856.8	99.3	330.9	529.5	401.7	454.0	52.3
22403.0	26484.8	16553.0	9931.8	99.7	332.2	531.5	403.7	457.1	53.4
22584.9	26666.7	16666.7	10000.0	100.0	333.3	533.3	405.6	460.0	54.4

The two cells missing in the first line ('see below') are:

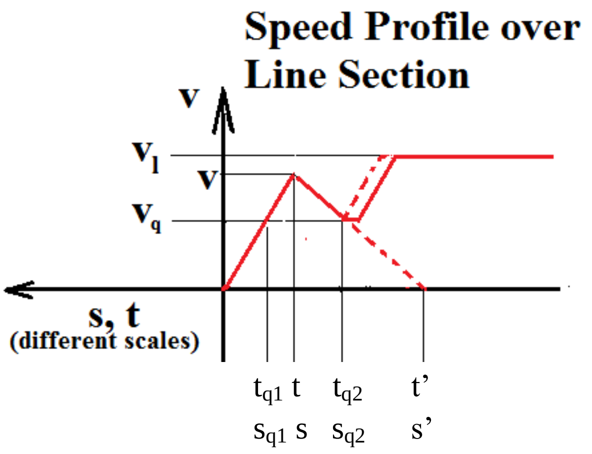
(Aa+Ad)/(Aa*Ad)	Ad/(Aa+Ad)

Note that, in generating the above spreadsheet, the value of s_{q2} for the first row, the distance to accelerate to 230kph, was input to a high precision, so is **exactly** the correct value, and the value for all the succeeding rows generated simply by adding 200, and the values in all the other columns derived as explained above. There are two exceptions to this: the values for both line speeds of interest, 300 and 360kph, were calculated explicitly. The starting point here was the value for v, 83.3333… and 100m/s exactly, working backwards along the columns to derive, finally, sq2. Having got that, the remaining columns after v were calculated by the existing formulae. These two rows have bold borders to emphasise the extra degree of precision, as indeed has the first. (Although the numeric results throughout are presented to a single decimal place, the values in the spreadsheet can be displayed to any desired degree of precision, up to the spreadsheet limit of around 15 significant digits, which really ought to be enough for anybody – it's only engineering, after all, not particle physics)

The main problem with printing out stuff from Excel is getting it to fit within a page width. Unless it is a very small spreadsheet, in its number of columns, all manner of tricks are necessary. Eventually, if everything else fails, it simply has to be sectioned, and different ranges of columns copied to different pages, which makes it pretty much unreadable online, but, in the printed version one can at least stick the pages together side by side, so the full rows appear, when unfolded.

The takeaway from all this is that improvement in accuracy made available by considering the true speed profile is small, compared with taking a uniform speed once v_q is reached – less that 55sec in the worst case. The temptation is to say that it doesn't matter, since these are, after all, only estimates. In fact, however, a similar situation has already been encountered, in the case of adjacent stations (see later, starting at p.51), and there the problem cannot be ignored, as there is no other way of calculating the times. The results for adjacent stations can be read off directly from the convenient table on p.54. The table just derived, above, can perform exactly the same function for propinquant junctions, the junction distance is given in the first column, and the time to that junction, for a **diverging** train, simply read off from the t_{q2} column. **Non-diverging** trains simply continue accelerating, through the junction if necessary, until they reach line speed, and their time to the junction simply calculated – if they're still accelerating there, (otherwise use the standard section formula #3 on p.32, above, for the accelerating case,) – from $t_{q2} = \sqrt{(2s_{q2}/a_a)}$. So although, indeed, the effects are small, they might as well be included as the exact results are now readily available. Another reason for not ignoring them is that there are in fact quite a few of them (once having identified the situation, they seem to be everywhere!). The main practical difficulty is simply in recognising these cases (as it is for adjacent stations).

As noted earlier, propinquant junctions can be **converging**, for **decelerating** trains. It is more difficult to describe than the diverging, accelerating case, and best introduced by the speed profile:



(As before, and purely for consistency, the effect of train length in traversing the junction is indicated, even though the effect is confined within the preceding section, having no effect on the present calculations. **This is wrong, but stay with the argument, as the calculation is easily corrected later.**)

The profile may appear rather confusing, in that it has the mirror image of the usual coordinate axes; ascending time (and distance) is now to the left. It is depicted like this to make absolutely clear its essential identity with the previous case, also so that all the notation of the previous case still applies, as do the calculations, save only that acceleration and deceleration switch places, so a_a becomes a_d, and a_d becomes a_a. (This is effected with extreme efficiency by simply switching the contents of the relevant two cells; no other change is required to the spreadsheet apart from the manual adjustment to the two rows corresponding exactly to v = 300 and 360kph). What was the origin is now (so to speak,) the destination. Actually, that is, physically, quite correct. The train is approaching the destination, (which is the reference datum for everything,) where its arrival time is taken as zero. It has to decelerate from line speed, v_l, to the turnout limit speed, v_q, in order to traverse a converging junction, at a distance s_{q2} and time t_{q2} before the station, (thus actually negative values,) where it joins the main line. However, if it continued to decelerate through and beyond the junction, it would come to a standstill before it had reached the station. It must therefore accelerate back up to some instantaneous, intermediate maximum speed, v, at distance s and time t before the station, before resuming its deceleration, to come to a stop precisely on arrival at the station (distance and time both zero). During that second deceleration, it again passes through speed v_q, at distance s_{q1} and time t_{q1} before the station. It will now be appreciated why I didn't attempt to describe the situation purely verbally. I hope readers find the exposition clear enough, given the diagram to illustrate it.

As before, the problem is: given v_q and s_{q2}, find t_{q2}. The previous calculations apply, switching a_a and a_d. In the following, bold type indicates that the value is now known, i.e. solved for. The results are:

$\mathbf{t_{q1} = v_q/a_d}$ $\qquad$ $\mathbf{s_{q1} = v_q^2/2a_d}$

$t_{q2} = t' - v_q/a_a$ $\qquad$ $\mathbf{s' = s_{q2} + v_q^2/2a_a}$

$t = a_a t'/(a_a + a_d)$ $\qquad$ $\mathbf{s = a_a s'/(a_a + a_d) = (a_a/(a_a + a_d))*(s_{q2} + v_q^2/2a_a)}$

$\mathbf{v^2 = 2a_a a_d(s_{q2} + v_q^2/2a_a)/(a_a + a_d)}$

$\mathbf{t = v/a_d = [\sqrt{(2a_a a_d(s_{q2} + v_q^2/2a_a)/(a_a + a_d))}]/a_a}$

$\mathbf{t' = v(a_a + a_d)/a_a a_d = [\sqrt{(2a_a a_d(s_{q2} + v_q^2/2a_a)/(a_a + a_d))}]*(a_a + a_d)/a_a a_d}$

$$\mathbf{t_{q2} = t' - v_q/a_a = [\sqrt{(2a_a a_d(s_{q2} + v_q^2/2a_a)/(a_a + a_d))}]*(a_a + a_d)/a_a a_d - v_q/a_a}$$

The programming of the spreadsheet is exactly as before. The values are calculated over the entire range of interest, from 230kph to 360kph, as before, and the values are calculated for a distance increment of 200m, as before. The results are:

Vq	Ad	Aa	Vq**2	Vq/Aa	Aa+Ad	Aa*Ad	See below		Tq1	Sq1
63.9	0.5	0.3	4081.8	213.0	0.8	0.2	5.3	0.4	127.8	4081.8

Junction Distance Sq2 (m)	S ' (m)	S (m)	V**2	V (m/s)	T (s)	T ' (s)	Tq2 (s)	Speed Vq between Sq2 and Sq1	Time penalty (s)
4081.8	10884.8	4081.8	4081.8	63.9	127.8	340.7	127.8	127.8	0.0
4281.8	11084.8	4156.8	4156.8	64.5	128.9	343.9	130.9	130.9	0.0
4481.8	11284.8	4231.8	4231.8	65.1	130.1	346.9	134.0	134.0	0.1
4681.8	11484.8	4306.8	4306.8	65.6	131.3	350.0	137.0	137.2	0.1
4881.8	11684.8	4381.8	4381.8	66.2	132.4	353.0	140.1	140.3	0.2
5081.8	11884.8	4456.8	4456.8	66.8	133.5	356.0	143.1	143.4	0.3
5281.8	12084.8	4531.8	4531.8	67.3	134.6	359.0	146.1	146.6	0.5
5481.8	12284.8	4606.8	4606.8	67.9	135.7	362.0	149.0	149.7	0.7
5681.8	12484.8	4681.8	4681.8	68.4	136.8	364.9	152.0	152.8	0.9
5881.8	12684.8	4756.8	4756.8	69.0	137.9	367.8	154.9	156.0	1.1
6081.8	12884.8	4831.8	4831.8	69.5	139.0	370.7	157.8	159.1	1.3
6281.8	13084.8	4906.8	4906.8	70.0	140.1	373.6	160.6	162.2	1.6
6481.8	13284.8	4981.8	4981.8	70.6	141.2	376.4	163.5	165.3	1.9
6681.8	13484.8	5056.8	5056.8	71.1	142.2	379.3	166.3	168.5	2.2
6881.8	13684.8	5131.8	5131.8	71.6	143.3	382.1	169.1	171.6	2.5
7081.8	13884.8	5206.8	5206.8	72.2	144.3	384.8	171.9	174.7	2.9
7281.8	14084.8	5281.8	5281.8	72.7	145.4	387.6	174.6	177.9	3.2
7481.8	14284.8	5356.8	5356.8	73.2	146.4	390.3	177.4	181.0	3.6
7681.8	14484.8	5431.8	5431.8	73.7	147.4	393.1	180.1	184.1	4.0
7881.8	14684.8	5506.8	5506.8	74.2	148.4	395.8	182.8	187.3	4.4
8081.8	14884.8	5581.8	5581.8	74.7	149.4	398.5	185.5	190.4	4.9
8281.8	15084.8	5656.8	5656.8	75.2	150.4	401.1	188.2	193.5	5.4
8481.8	15284.8	5731.8	5731.8	75.7	151.4	403.8	190.8	196.6	5.8
8681.8	15484.8	5806.8	5806.8	76.2	152.4	406.4	193.4	199.8	6.3
8881.8	15684.8	5881.8	5881.8	76.7	153.4	409.0	196.1	202.9	6.8
9081.8	15884.8	5956.8	5956.8	77.2	154.4	411.6	198.7	206.0	7.4
9281.8	16084.8	6031.8	6031.8	77.7	155.3	414.2	201.2	209.2	7.9
9481.8	16284.8	6106.8	6106.8	78.1	156.3	416.8	203.8	212.3	8.5
9681.8	16484.8	6181.8	6181.8	78.6	157.2	419.3	206.4	215.4	9.1
9881.8	16684.8	6256.8	6256.8	79.1	158.2	421.9	208.9	218.6	9.7
10081.8	16884.8	6331.8	6331.8	79.6	159.1	424.4	211.4	221.7	10.3
10281.8	17084.8	6406.8	6406.8	80.0	160.1	426.9	213.9	224.8	10.9
10481.8	17284.8	6481.8	6481.8	80.5	161.0	429.4	216.4	228.0	11.5
10681.8	17484.8	6556.8	6556.8	81.0	161.9	431.9	218.9	231.1	12.2
10881.8	17684.8	6631.8	6631.8	81.4	162.9	434.3	221.4	234.2	12.9
11081.8	17884.8	6706.8	6706.8	81.9	163.8	436.8	223.8	237.3	13.5

11281.8	18084.8	6781.8	6781.8	82.4	164.7	439.2	226.2		240.5	14.2
11481.8	18284.8	6856.8	6856.8	82.8	165.6	441.6	228.7		243.6	14.9
11681.8	18484.8	6931.8	6931.8	83.3	166.5	444.0	231.1		246.7	15.7
11715.5	18518.5	6944.4	6944.4	83.3	166.7	444.4	231.5		247.3	15.8
11881.8	18684.8	7006.8	7006.8	83.7	167.4	446.4	233.5		249.9	16.4
12081.8	18884.8	7081.8	7081.8	84.2	168.3	448.8	235.9		253.0	17.1
12281.8	19084.8	7156.8	7156.8	84.6	169.2	451.2	238.2		256.1	17.9
12481.8	19284.8	7231.8	7231.8	85.0	170.1	453.5	240.6		259.3	18.7
12681.8	19484.8	7306.8	7306.8	85.5	171.0	455.9	242.9		262.4	19.5
12881.8	19684.8	7381.8	7381.8	85.9	171.8	458.2	245.3		265.5	20.3
13081.8	19884.8	7456.8	7456.8	86.4	172.7	460.5	247.6		268.6	21.1
13281.8	20084.8	7531.8	7531.8	86.8	173.6	462.9	249.9		271.8	21.9
13481.8	20284.8	7606.8	7606.8	87.2	174.4	465.2	252.2		274.9	22.7
13681.8	20484.8	7681.8	7681.8	87.6	175.3	467.4	254.5		278.0	23.6
13881.8	20684.8	7756.8	7756.8	88.1	176.1	469.7	256.8		281.2	24.4
14081.8	20884.8	7831.8	7831.8	88.5	177.0	472.0	259.0		284.3	25.3
14281.8	21084.8	7906.8	7906.8	88.9	177.8	474.2	261.3		287.4	26.2
14481.8	21284.8	7981.8	7981.8	89.3	178.7	476.5	263.5		290.6	27.0
14681.8	21484.8	8056.8	8056.8	89.8	179.5	478.7	265.8		293.7	27.9
14881.8	21684.8	8131.8	8131.8	90.2	180.4	480.9	268.0		296.8	28.8
15081.8	21884.8	8206.8	8206.8	90.6	181.2	483.2	270.2		300.0	29.8
15281.8	22084.8	8281.8	8281.8	91.0	182.0	485.4	272.4		303.1	30.7
15481.8	22284.8	8356.8	8356.8	91.4	182.8	487.5	274.6		306.2	31.6
15681.8	22484.8	8431.8	8431.8	91.8	183.6	489.7	276.8		309.3	32.6
15881.8	22684.8	8506.8	8506.8	92.2	184.5	491.9	278.9		312.5	33.5
16081.8	22884.8	8581.8	8581.8	92.6	185.3	494.1	281.1		315.6	34.5
16281.8	23084.8	8656.8	8656.8	93.0	186.1	496.2	283.3		318.7	35.5
16481.8	23284.8	8731.8	8731.8	93.4	186.9	498.4	285.4		321.9	36.5
16681.8	23484.8	8806.8	8806.8	93.8	187.7	500.5	287.5		325.0	37.5
16881.8	23684.8	8881.8	8881.8	94.2	188.5	502.6	289.7		328.1	38.5
17081.8	23884.8	8956.8	8956.8	94.6	189.3	504.7	291.8		331.3	39.5
17281.8	24084.8	9031.8	9031.8	95.0	190.1	506.9	293.9		334.4	40.5
17481.8	24284.8	9106.8	9106.8	95.4	190.9	509.0	296.0		337.5	41.5
17681.8	24484.8	9181.8	9181.8	95.8	191.6	511.0	298.1		340.6	42.6
17881.8	24684.8	9256.8	9256.8	96.2	192.4	513.1	300.2		343.8	43.6
18081.8	24884.8	9331.8	9331.8	96.6	193.2	515.2	302.2		346.9	44.7
18281.8	25084.8	9406.8	9406.8	97.0	194.0	517.3	304.3		350.0	45.7
18481.8	25284.8	9481.8	9481.8	97.4	194.7	519.3	306.4		353.2	46.8
18681.8	25484.8	9556.8	9556.8	97.8	195.5	521.4	308.4		356.3	47.9
18881.8	25684.8	9631.8	9631.8	98.1	196.3	523.4	310.5		359.4	49.0
19081.8	25884.8	9706.8	9706.8	98.5	197.0	525.5	312.5		362.6	50.1
19281.8	26084.8	9781.8	9781.8	98.9	197.8	527.5	314.5		365.7	51.2
19481.8	26284.8	9856.8	9856.8	99.3	198.6	529.5	316.5		368.8	52.3
19681.8	26484.8	9931.8	9931.8	99.7	199.3	531.5	318.5		372.0	53.4
19863.7	26666.7	10000.0	10000.0	100.0	200.0	533.3	320.4		374.8	54.4

As before, the missing cells in the first line are:

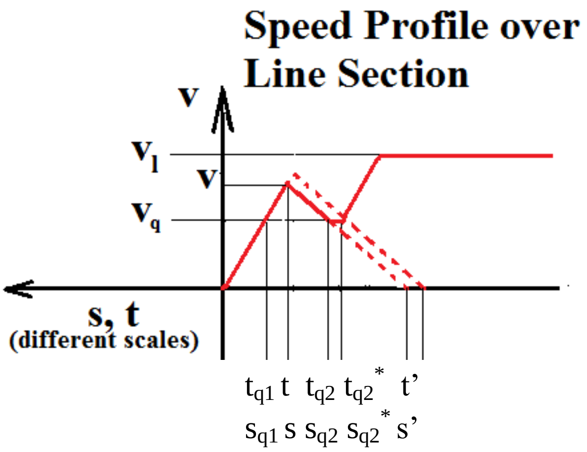
(Aa+Ad)/(Aa*Ad)	Aa/(Aa+Ad)

It will be noticed that the columns for s', v, t' and the time penalty for running at constant v_q between s_{q2} and s_{q1} (in the second case, as opposed to between s_{q1} and s_{q2} in the first,) are identical. (The generation of the s_{q2} values by incrementing from the first row, without any reset, is deliberately retained, purely to illustrate this, even though it results in a set of rather odd values.) I expected this would be the case for the first three, but the fourth did come as a surprise,

Corresponding values of both s and t for both cases are in the constant ratio 5:3 (first case : second), unsurprising given that acceleration : deceleration rates are 3:5. But there are no relationships between corresponding values of s_{q2} or t_{q2} between the two cases, that I have yet been able to detect; it would be surprising if there were. The corresponding distances and times are of course all shorter in the second case, unsurprisingly as deceleration is the predominant feature here, rather than acceleration, but not in any regular way..

All of this is immensely reassuring.

The above argument is not quite correct, but valuable nonetheless. The true situation is as illustrated next:



The results previously derived are exactly correct, for the quantities as indicated. But the variables t_{q2} and s_{q2} no longer apply to the junction itself, but to a location slightly after the junction, the distance being 400m, the standard train length, and the time c.6.3s, the time required for the train to cross over the junction at a steady speed of 230kph. So all that is required is to add 400m to the previous s_{q2}, to get the true junction distance, s_{q2}^*, and 6.3s to t_{q2}, to get the true junction time, t_{q2}^*. Here are the corrected values (retaining only the quantities of interest – everything else is as in the previous spreadsheet):

Sq2 (m)	True Junction Distance Sq2* (m)	Tq2 (s)	True Junction Time tq2* (s)	Speed Vq between Sq2* and Sq1	Time penalty (s)
4081.8	4481.8	127.8	134.0	134.0	0.0
4281.8	4681.8	130.9	137.2	137.2	0.0
4481.8	4881.8	134.0	140.2	140.3	0.1
4681.8	5081.8	137.0	143.3	143.4	0.1
4881.8	5281.8	140.1	146.3	146.6	0.2
5081.8	5481.8	143.1	149.3	149.7	0.3
5281.8	5681.8	146.1	152.3	152.8	0.5
5481.8	5881.8	149.0	155.3	156.0	0.7
5681.8	6081.8	152.0	158.2	159.1	0.9
5881.8	6281.8	154.9	161.1	162.2	1.1
6081.8	6481.8	157.8	164.0	165.3	1.3
6281.8	6681.8	160.6	166.9	168.5	1.6
6481.8	6881.8	163.5	169.7	171.6	1.9
6681.8	7081.8	166.3	172.6	174.7	2.2
6881.8	7281.8	169.1	175.4	177.9	2.5
7081.8	7481.8	171.9	178.1	181.0	2.9
7281.8	7681.8	174.6	180.9	184.1	3.2
7481.8	7881.8	177.4	183.6	187.3	3.6
7681.8	8081.8	180.1	186.4	190.4	4.0
7881.8	8281.8	182.8	189.1	193.5	4.4
8081.8	8481.8	185.5	191.8	196.6	4.9
8281.8	8681.8	188.2	194.4	199.8	5.4
8481.8	8881.8	190.8	197.1	202.9	5.8
8681.8	9081.8	193.4	199.7	206.0	6.3
8881.8	9281.8	196.1	202.3	209.2	6.8
9081.8	9481.8	198.7	204.9	212.3	7.4
9281.8	9681.8	201.2	207.5	215.4	7.9
9481.8	9881.8	203.8	210.1	218.6	8.5
9681.8	10081.8	206.4	212.6	221.7	9.1
9881.8	10281.8	208.9	215.2	224.8	9.7
10081.8	10481.8	211.4	217.7	228.0	10.3
10281.8	10681.8	213.9	220.2	231.1	10.9
10481.8	10881.8	216.4	222.7	234.2	11.5
10681.8	11081.8	218.9	225.2	237.3	12.2
10881.8	11281.8	221.4	227.6	240.5	12.9
11081.8	11481.8	223.8	230.1	243.6	13.5
11281.8	11681.8	226.2	232.5	246.7	14.2
11481.8	11881.8	228.7	234.9	249.9	14.9
11681.8	12081.8	231.1	237.3	253.0	15.7
11715.5	12115.5	231.5	237.7	253.5	15.8
11881.8	12281.8	233.5	239.7	256.1	16.4

12081.8	12481.8	235.9	242.1		259.3	17.1
12281.8	12681.8	238.2	244.5		262.4	17.9
12481.8	12881.8	240.6	246.8		265.5	18.7
12681.8	13081.8	242.9	249.2		268.6	19.5
12881.8	13281.8	245.3	251.5		271.8	20.3
13081.8	13481.8	247.6	253.8		274.9	21.1
13281.8	13681.8	249.9	256.2		278.0	21.9
13481.8	13881.8	252.2	258.5		281.2	22.7
13681.8	14081.8	254.5	260.7		284.3	23.6
13881.8	14281.8	256.8	263.0		287.4	24.4
14081.8	14481.8	259.0	265.3		290.6	25.3
14281.8	14681.8	261.3	267.5		293.7	26.2
14481.8	14881.8	263.5	269.8		296.8	27.0
14681.8	15081.8	265.8	272.0		300.0	27.9
14881.8	15281.8	268.0	274.2		303.1	28.8
15081.8	15481.8	270.2	276.5		306.2	29.8
15281.8	15681.8	272.4	278.7		309.3	30.7
15481.8	15881.8	274.6	280.8		312.5	31.6
15681.8	16081.8	276.8	283.0		315.6	32.6
15881.8	16281.8	278.9	285.2		318.7	33.5
16081.8	16481.8	281.1	287.4		321.9	34.5
16281.8	16681.8	283.3	289.5		325.0	35.5
16481.8	16881.8	285.4	291.7		328.1	36.5
16681.8	17081.8	287.5	293.8		331.3	37.5
16881.8	17281.8	289.7	295.9		334.4	38.5
17081.8	17481.8	291.8	298.0		337.5	39.5
17281.8	17681.8	293.9	300.2		340.6	40.5
17481.8	17881.8	296.0	302.3		343.8	41.5
17681.8	18081.8	298.1	304.3		346.9	42.6
17881.8	18281.8	300.2	306.4		350.0	43.6
18081.8	18481.8	302.2	308.5		353.2	44.7
18281.8	18681.8	304.3	310.6		356.3	45.7
18481.8	18881.8	306.4	312.6		359.4	46.8
18681.8	19081.8	308.4	314.7		362.6	47.9
18881.8	19281.8	310.5	316.7		365.7	49.0
19081.8	19481.8	312.5	318.8		368.8	50.1
19281.8	19681.8	314.5	320.8		372.0	51.2
19481.8	19881.8	316.5	322.8		375.1	52.3
19681.8	20081.8	318.5	324.8		378.2	53.4
19863.7	20263.7	320.4	326.6		381.1	54.4

Westerleigh (Propinquant) Junction

Although the tables derived for propinquant junctions in the previous section deal with the standard HS value of 230kph for the turnout limit speed, (v_q in the formulae,) the (formulaic) results are completely general. There are probably very few examples outside HS lines, but they do actually exist on classic routes, or at least one does, the only example so far discovered – Westerleigh Junction. This is where the SW/NE route diverges from the South Wales – London route. At Mk1A, HS4 and HS7 incorporate the classic routes, upgraded to a line speed of 125mph (200kph). Here the SW/NE route has a speed limit of 30mph (48kph) and, given the lousy alignment, it is not regarded as worthwhile to try to increase this. (There is no restriction on the GWML.)

The various quantities are:

$a_a = 0.3\text{m/s}^2$ $a_d = 0.5\text{m/s}^2$ $v_q = 48\text{kph} = 13.3333\text{m/s}$ $s_{q2} = 7400\text{m}$

(Line speed = 200kph = 55.6m/s, but, in the nature of a propinquant junction, this is never reached – not quite.)

The formula for a divergent propinquant junction is given on p.44. With the above values, the result is:

$$\mathbf{t_{q2} = 258s}$$

The formula for a convergent propinquant junction is given on p.48, but in this case the effect of the train traversing the junction at a steady v_q must be included. So take $s_{q2} = 7000\text{m}$, and add the time to travel 400m at 48kph (13.3333m/s) = 30s. The result is:

$$\mathbf{t_{q2} = 265s}$$

The following sections come from the old appendix B, and deal with the 'particular exceptional cases' which had been recognised at that time. Note that ***they are not wrong****, but, with the exception of Adjacent Stations, they are much rarer than had been thought. Indeed most of the examples are more correctly dealt with as propinquant junctions.*

Adjacent Stations

(Refer to the Junction Effects table on p14 for the values quoted below.)

Consider two stations, one after the other, and a train that stops at both, followed by one that is non-stop. For line speed 360kph, the total deceleration / acceleration distance for the first station stop is, as usual, 10.0 + 16.7 = 26.7km, in a time (excluding station wait time) of 200 + 333 = 533s. After this, the stopping train is travelling at full line speed. Providing the two stations are at least 26.7km apart then that is the full story; the behaviour around the second station is identical to that around the first.

But 26.7km, (16.7miles,) is a significant distance, and it could well be the case that two stations exist closer together than that. This needs further consideration. In this case, the train stopping at both would accelerate away from the first station to some intermediate speed, less than full line speed, then immediately switch to deceleration for the second station. In this situation, I strongly recommend that the train do **not** re-join the main line, even though it would have accelerated beyond the turnout speed limit, (unless the stations were even closer together than 4.1 + 6.8 = 10.9km, 6.8miles, which does seem improbably close,) since there is no benefit in its doing so, and it might obstruct a non-stop train on the main line. Instead, the station loops should continue between the stations, so we have a 4-track section, maximally 37.6km, (23.5miles,) in length. (The calculation is **twice** 26.7 less the deceleration distance on the main line before the first station – 5.9km – and the acceleration distance on the main line after the second – 9.9km.)

If s_a is the distance (<26.7km) between the stations, and v_a the maximum speed reached between them, and if s_1, s_2 are the acceleration / decelerating distances and t_1, t_2 the corresponding times, then:

$v_a = 0.3t_1 = 0.5t_2$, thus $t_2 = 0.6t_1$

$s_a = s_1 + s_2 = 0.3t_1^2/2 + 0.5t_2^2/2$

$t_2 = 0.6t_1$ so $s_1 = 0.3t_1^2/2$, $s_2=0.5(0.6t_1)^2/2 = 0.18t_1^2/2$

So $s_a = s_1 + s_1 = t_1^2(0.3 + 0.18)/2 = 0.24t_1^2$

Thus $t_1^2 = 4.167s_a$ so $t_1 = 2.04\sqrt{s_a}$ and $t2 = 0.6t1 = 1.225\sqrt{s_a}$

So $v_a = 0.3t_1 = 0.6124\sqrt{s_a}$

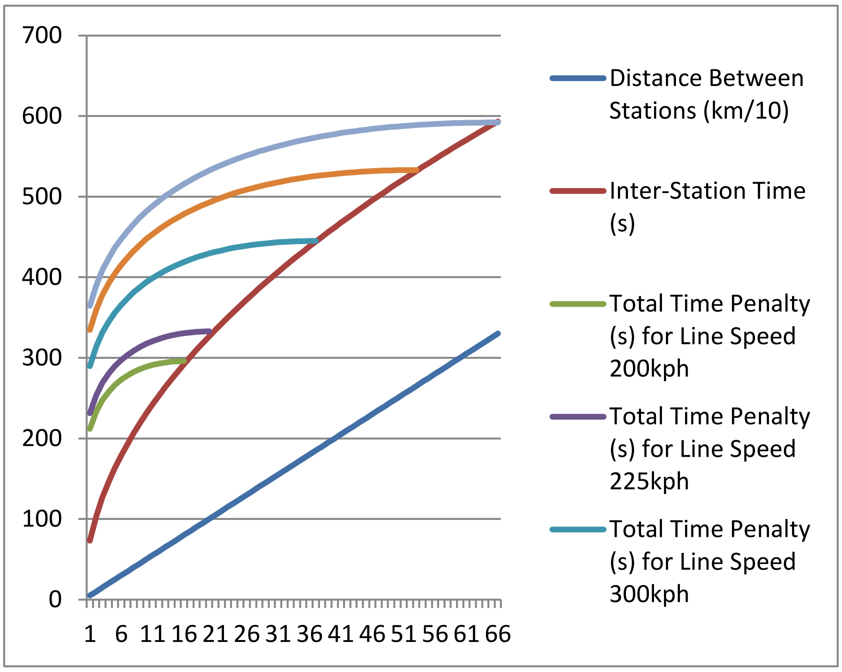
Results have been generated by spreadsheet (for line speeds 200, 225, 300, 360 and 400kph – only the final column is different) over the whole relevant range. The column headings are the symbols in the above equations. TP is Time Penalty, for the appropriate line speed. Getting the above equations right was extraordinarily troublesome and error-prone, and the penultimate column in the spreadsheet before the TP columns is there primarily to satisfy the reader (and me!!) that the results **are** correct. (The quirky unit chosen – tenths of a km, deci-km? – is dictated by the requirements of the line chart, to ensure that the values can actually be determined; the natural choice of km as the unit would have had the line horizontal at the bottom of the chart.)

The spreadsheet and chart illustrate two things. The column headed (t1+t2) I-S Time, and with the contents displayed in bold type, is the Inter-Station Time (s), i.e. the time actually taken from starting from the first station to stopping at the second, and is the value to take, in this particular situation, when deriving journey time estimates.

The overall time penalty for the double station stop includes the deceleration before the first station and the acceleration after the second. Note that this figure excludes the station wait times. It is the total time penalty imposed on the train by the fact of its stopping at the two stations, as compared with the time it would take to travel the same distance at line speed, without stopping. As the distance between stations approaches the distance at which line speed is reached between them, the overall time penalty converges on the total for two separate stops.

Sa (m)	Va (m/s)	t1 (s)	s1 (m)	t2 (s)	s2 (m)	**(t1+t2) I-S Time**	(s1+s2) (km/10)	TP (s) 200 kph	TP (s) 225 kph	TP (s) 300 kph	TP (s) 360 kph	TP (s) 400 kph
500	13.7	46	313	27	188	**73**	5	212	231	290	334	364
1000	19.4	65	625	39	375	**103**	10	233	254	314	360	390
1500	23.7	79	938	47	563	**126**	15	247	269	331	378	409
2000	27.4	91	1250	55	750	**146**	20	258	280	345	392	424
2500	30.6	102	1563	61	938	**163**	25	266	290	356	405	437
3000	33.5	112	1875	67	1125	**179**	30	273	297	366	415	448
3500	36.2	121	2188	72	1313	**193**	35	278	304	374	425	457
4000	38.7	129	2500	77	1500	**207**	40	282	309	381	433	466
4500	41.1	137	2813	82	1688	**219**	45	286	313	388	440	474
5000	43.3	144	3125	87	1875	**231**	50	289	317	394	447	482
5500	45.4	151	3438	91	2063	**242**	55	291	321	399	454	488
6000	47.4	158	3750	95	2250	**253**	60	293	323	404	459	495
6500	49.4	165	4063	99	2438	**263**	65	294	326	408	465	501
7000	51.2	171	4375	102	2625	**273**	70	295	328	412	470	506
7500	53.0	177	4688	106	2813	**283**	75	296	329	416	474	511
8000	54.8	183	5000	110	3000	**292**	80	296	330	419	478	516
8500	56.5	188	5313	113	3188	**301**	85		331	422	482	520
9000	58.1	194	5625	116	3375	**310**	90		332	425	486	525
9500	59.7	199	5938	119	3563	**318**	95		333	427	490	529
10000	61.2	204	6251	122	3750	**327**	100		333	429	493	532
10500	62.8	209	6563	125	3938	**335**	105			431	496	536
11000	64.2	214	6876	128	4125	**343**	110			433	499	539
11500	65.7	219	7188	131	4313	**350**	115			435	502	542
12000	67.1	224	7501	134	4500	**358**	120			437	504	545
12500	68.5	228	7813	137	4688	**365**	125			438	506	548
13000	69.8	233	8126	140	4875	**372**	130			439	509	551
13500	71.2	237	8438	142	5063	**379**	135			440	511	554
14000	72.5	242	8751	145	5250	**386**	140			441	513	556
14500	73.7	246	9063	147	5438	**393**	145			442	515	558
15000	75.0	250	9376	150	5625	**400**	150			443	516	561
15500	76.2	254	9688	152	5813	**407**	155			443	518	563
16000	77.5	258	10001	155	6000	**413**	160			444	519	565
16500	78.7	262	10313	157	6188	**420**	165			444	521	567
17000	79.8	266	10626	160	6375	**426**	170			445	522	569
17500	81.0	270	10938	162	6563	**432**	175			445	523	570
18000	82.2	274	11251	164	6750	**438**	180			445	525	572
18500	83.3	278	11563	167	6938	**444**	185			445	526	573

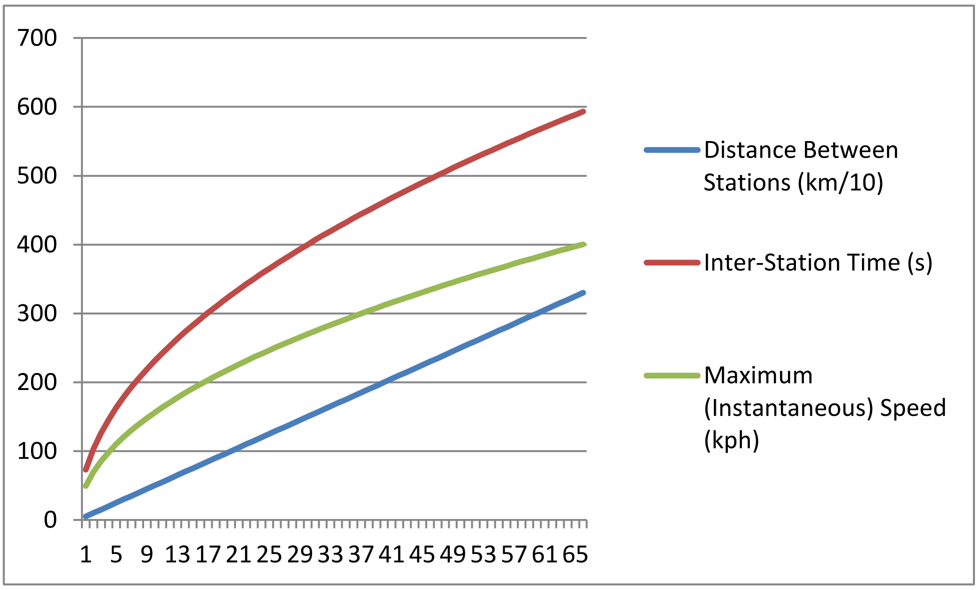
19000	84.4	281	11876	169	7125	**450**	190		527	575
19500	85.5	285	12188	171	7313	**456**	195		527	576
20000	86.6	289	12501	173	7500	**462**	200		528	578
20500	87.7	292	12814	175	7688	**468**	205		529	579
21000	88.7	296	13126	177	7875	**473**	210		530	580
21500	89.8	299	13439	180	8063	**479**	215		530	581
22000	90.8	303	13751	182	8250	**484**	220		531	582
22500	91.9	306	14064	184	8438	**490**	225		531	583
23000	92.9	310	14376	186	8625	**495**	230		532	584
23500	93.9	313	14689	188	8813	**501**	235		532	585
24000	94.9	316	15001	190	9000	**506**	240		532	586
24500	95.9	320	15314	192	9188	**511**	245		533	586
25000	96.8	323	15626	194	9375	**516**	250		533	587
25500	97.8	326	15939	196	9563	**522**	255		533	588
26000	98.7	329	16251	197	9750	**527**	260		533	588
26500	99.7	332	16564	199	9938	**532**	265		533	589
27000	100.6	335	16876	201	10125	**537**	270			589
27500	101.6	339	17189	203	10313	**542**	275			590
28000	102.5	342	17501	205	10500	**547**	280			590
28500	103.4	345	17814	207	10688	**551**	285			591
29000	104.3	348	18126	209	10875	**556**	290			591
29500	105.2	351	18439	210	11063	**561**	295			591
30000	106.1	354	18752	212	11250	**566**	300			591
30500	107.0	357	19064	214	11438	**570**	305			592
31000	107.8	359	19377	216	11625	**575**	310			592
31500	108.7	362	19689	217	11813	**580**	315			592
32000	109.5	365	20002	219	12000	**584**	320			592
32500	110.4	368	20314	221	12188	**589**	325			592
33000	111.2	371	20627	222	12375	**593**	330			592



It is instructive to plot a slightly different result, taking the same set of inter-station distances, assume they are adjacent stations and plot the inter-station time (same as before) and the maximum speed achieved (instantaneously) between stations, thus they actually **are** adjacent stations for any line speed exceeding that value.

s (m)	Vmax (m/s)	t (s)	s (km/10)	Vmax (kph)	Vmax (mph)
500	13.7	73	5	49.30	30.64
1000	19.4	103	10	69.71	43.33
1500	23.7	126	15	85.38	53.06
2000	27.4	146	20	98.59	61.27
2500	30.6	163	25	110.23	68.51
3000	33.5	179	30	120.75	75.05
3500	36.2	193	35	130.42	81.06
4000	38.7	207	40	139.43	86.65
4500	41.1	219	45	147.89	91.91
5000	43.3	231	50	155.88	96.88
5500	45.4	242	55	163.49	101.61
6000	47.4	253	60	170.76	106.13
6500	49.4	263	65	177.74	110.46
7000	51.2	273	70	184.45	114.63
7500	53.0	283	75	190.92	118.66
8000	54.8	292	80	197.18	122.55
8500	56.5	301	85	203.25	126.32
9000	58.1	310	90	209.14	129.98
9500	59.7	318	95	214.87	133.54
10000	61.2	327	100	220.45	137.01
10500	62.7	335	105	225.90	140.40
11000	64.2	343	110	231.21	143.70
11500	65.7	350	115	236.41	146.93
12000	67.1	358	120	241.50	150.09
12500	68.5	365	125	246.48	153.19
13000	69.8	372	130	251.36	156.22
13500	71.2	379	135	256.14	159.19
14000	72.5	386	140	260.84	162.12
14500	73.7	393	145	265.46	164.99
15000	75.0	400	150	270.00	167.81
15500	76.2	407	155	274.46	170.58
16000	77.5	413	160	278.85	173.31
16500	78.7	420	165	283.18	176.00
17000	79.8	426	170	287.44	178.64
17500	81.0	432	175	291.63	181.25
18000	82.2	438	180	295.77	183.82
18500	83.3	444	185	299.85	186.36
19000	84.4	450	190	303.87	188.86
19500	85.5	456	195	307.85	191.33
20000	86.6	462	200	311.77	193.77
20500	87.7	468	205	315.64	196.17
21000	88.7	473	210	319.47	198.55
21500	89.8	479	215	323.25	200.90
22000	90.8	484	220	326.99	203.22

22500	91.9	490	225	330.68	205.52
23000	92.9	495	230	334.34	207.79
23500	93.9	501	235	337.95	210.04
24000	94.9	506	240	341.53	212.26
24500	95.9	511	245	345.07	214.46
25000	96.8	516	250	348.57	216.64
25500	97.8	522	255	352.04	218.79
26000	98.7	527	260	355.47	220.93
26500	99.7	532	265	358.87	223.04
27000	100.6	537	270	362.24	225.14
27500	101.6	542	275	365.58	227.21
28000	102.5	547	280	368.89	229.27
28500	103.4	551	285	372.17	231.30
29000	104.3	556	290	375.42	233.32
29500	105.2	561	295	378.64	235.33
30000	106.1	566	300	381.84	237.31
30500	106.9	570	305	385.01	239.28
31000	107.8	575	310	388.15	241.24
31500	108.7	580	315	391.27	243.17
32000	109.5	584	320	394.36	245.10
32500	110.4	589	325	397.43	247.00
33000	111.2	593	330	400.47	248.90



Adjacent Junctions

[This section is left in situ as it is still valid within its assumptions. But the more general case of Propinquant Adjacent Junctions has now been recognised and investigated (see next section).]

A similar effect to adjacent stations occurs when there are adjacent junctions. There aren't many practical instances of this, but they **do** exist. It is thus important that they be analysed.

We are of course speaking of **divergent / convergent** junctions. Nuthall South and North junctions on the HS3 main line, for example, are straight-ahead; the junctions have no effect on the through trains. Likewise Awsworth and Strelley junctions have no effect on through traffic passing between HS7 and the Nottingham loop of HS3.

So the situation being considered here is where a junction diverges from one route, then, after a short stretch of intermediate track, converges on another route. Trains decelerate from line speed to the turnout limit speed on the first route, before the divergent junction, and accelerate back up to the lime speed on the second route, following the convergent junction. Between the two they maintain the turnout limit speed – it is not worth trying to go any faster over this short distance. An essential point to stress is that trains are travelling at full line speed either side of the junction pair. There are, in fact, only five instances of junction pairs in the entire HS network, and, of these, only two are adjacent junctions, in the present sense. (The other three pairs all have one junction that is in fact a propinquant junction, where the situation is quite different.) Adjacent Junctions are a generalisation of the single junction case (where the inter-junction distance reduces to zero). The point of this concept is to allow the effect of the junction(s) to be included as a single time penalty value, in any relevant route section, for the shorter journey times spreadsheets, where only inter-station sections, or sections involving a change of line speed, are considered. For the longer version of the spreadsheets, which considers all immediate (passing) points of interest, all junctions will be points of interest in any case, and the calculations performed explicitly for them. (Of course, the junction time penalty approach was developed when inter-station sections were all that was considered, but things are a lot more complicated now.)

Assume that s_j is the distance between the two junctions. The train decelerates from line speed (360kph) to turnout limit speed (230kph) in a distance of 5918m and a time of 72s. (Refer to the Junction Effects table on p13 for the various values.) It then travels the distance s_j metres, plus a further 400metres beyond the second junction, to ensure that the entire train has cleared it, at 230kph, thus in a time of $(s_j + 400)/63.9$ s. Finally, it accelerates back up to line speed in a distance 9864m in 120s. Thus we have a total deceleration/steady/acceleration distance of $5918 + (s_j + 400) + 9864 = (16182 + s_j)$ metres, in a time of $72 + (s_j + 400)/63.9 + 120 = (198 + s_j/63.9)$ secs. Travelled at full line speed, that distance would take $(16182 + s_j)/100$ secs, so the time penalty for the double junction is $[(198 + s_j/63.9) – (16182 + s_j)/100]$, $= [36 + (37.11/6389)*s_j] = (36 + s_j*5.81*10^{-3})$ sec.

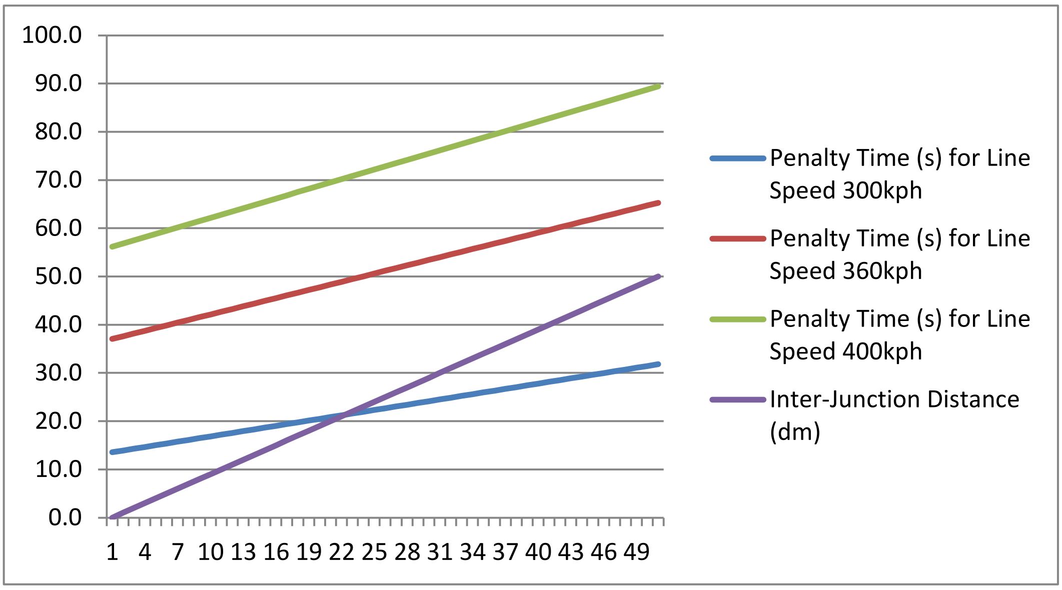
The odd distance units, deci-kilometers – 100-meters, are clearly to get the best spread on the line chart.

The only two pairs of true adjacent are (together with the single junction limit):

Junction Pairs	Distance Apart (m)	Time Penalty (s) (line speed 360kph)	Time Penalty (s) (line speed 300kph)
Kenyon South and West	1400	45	
Kenyon West and North	1070	43	
Single route junction	0	37	14

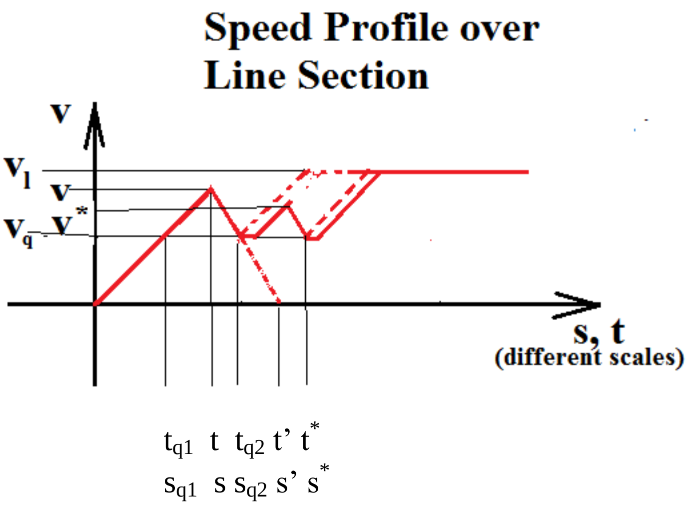
Sj (m)	Line Speed 300kph			Line Speed 360kph			Line Speed 400kph		
	Dist. (m)	Time (s)	Penalty Time (s) for Line Speed 300kph	Dist. (m)	Time (s)	Penalty Time (s) for Line Speed 360kph	Dist. (m)	Time (s)	Penalty Time (s) for Line Speed 400kph
0.0	8033.7	110.0	13.6	16181.9	198.9	37.0	22437.0	258.1	56.2
100.0	8133.7	111.5	13.9	16281.9	200.4	37.6	22537.0	259.7	56.8
200.0	8233.7	113.1	14.3	16381.9	202.0	38.2	22637.0	261.2	57.5
300.0	8333.7	114.7	14.7	16481.9	203.5	38.7	22737.0	262.8	58.2
400.0	8433.7	116.2	15.0	16581.9	205.1	39.3	22837.0	264.4	58.8
500.0	8533.7	117.8	15.4	16681.9	206.7	39.9	22937.0	265.9	59.5
600.0	8633.7	119.4	15.8	16781.9	208.2	40.4	23037.0	267.5	60.2
700.0	8733.7	120.9	16.1	16881.9	209.8	41.0	23137.0	269.1	60.8
800.0	8833.7	122.5	16.5	16981.9	211.4	41.6	23237.0	270.6	61.5
900.0	8933.7	124.1	16.8	17081.9	212.9	42.1	23337.0	272.2	62.2
1000.0	9033.7	125.6	17.2	17181.9	214.5	42.7	23437.0	273.8	62.8
1100.0	9133.7	127.2	17.6	17281.9	216.1	43.3	23537.0	275.3	63.5
1200.0	9233.7	128.7	17.9	17381.9	217.6	43.8	23637.0	276.9	64.2
1300.0	9333.7	130.3	18.3	17481.9	219.2	44.4	23737.0	278.5	64.8
1400.0	9433.7	131.9	18.7	17581.9	220.8	44.9	23837.0	280.0	65.5
1500.0	9533.7	133.4	19.0	17681.9	222.3	45.5	23937.0	281.6	66.2
1600.0	9633.7	135.0	19.4	17781.9	223.9	46.1	24037.0	283.2	66.8
1700.0	9733.7	136.6	19.8	17881.9	225.5	46.6	24137.0	284.7	67.5
1800.0	9833.7	138.1	20.1	17981.9	227.0	47.2	24237.0	286.3	68.2
1900.0	9933.7	139.7	20.5	18081.9	228.6	47.8	24337.0	287.9	68.8
2000.0	10033.7	141.3	20.9	18181.9	230.2	48.3	24437.0	289.4	69.5
2100.0	10133.7	142.8	21.2	18281.9	231.7	48.9	24537.0	291.0	70.1
2200.0	10233.7	144.4	21.6	18381.9	233.3	49.5	24637.0	292.5	70.8
2300.0	10333.7	146.0	22.0	18481.9	234.9	50.0	24737.0	294.1	71.5
2400.0	10433.7	147.5	22.3	18581.9	236.4	50.6	24837.0	295.7	72.1
2500.0	10533.7	149.1	22.7	18681.9	238.0	51.2	24937.0	297.2	72.8
2600.0	10633.7	150.7	23.1	18781.9	239.5	51.7	25037.0	298.8	73.5
2700.0	10733.7	152.2	23.4	18881.9	241.1	52.3	25137.0	300.4	74.1
2800.0	10833.7	153.8	23.8	18981.9	242.7	52.9	25237.0	301.9	74.8
2900.0	10933.7	155.4	24.2	19081.9	244.2	53.4	25337.0	303.5	75.5
3000.0	11033.7	156.9	24.5	19181.9	245.8	54.0	25437.0	305.1	76.1
3100.0	11133.7	158.5	24.9	19281.9	247.4	54.6	25537.0	306.6	76.8
3200.0	11233.7	160.1	25.2	19381.9	248.9	55.1	25637.0	308.2	77.5
3300.0	11333.7	161.6	25.6	19481.9	250.5	55.7	25737.0	309.8	78.1
3400.0	11433.7	163.2	26.0	19581.9	252.1	56.3	25837.0	311.3	78.8
3500.0	11533.7	164.7	26.3	19681.9	253.6	56.8	25937.0	312.9	79.5
3600.0	11633.7	166.3	26.7	19781.9	255.2	57.4	26037.0	314.5	80.1
3700.0	11733.7	167.9	27.1	19881.9	256.8	57.9	26137.0	316.0	80.8
3800.0	11833.7	169.4	27.4	19981.9	258.3	58.5	26237.0	317.6	81.5

3900.0	11933.7	171.0	27.8	20081.9	259.9	59.1	26337.0	319.2	82.1
4000.0	12033.7	172.6	28.2	20181.9	261.5	59.6	26437.0	320.7	82.8
4100.0	12133.7	174.1	28.5	20281.9	263.0	60.2	26537.0	322.3	83.5
4200.0	12233.7	175.7	28.9	20381.9	264.6	60.8	26637.0	323.9	84.1
4300.0	12333.7	177.3	29.3	20481.9	266.2	61.3	26737.0	325.4	84.8
4400.0	12433.7	178.8	29.6	20581.9	267.7	61.9	26837.0	327.0	85.4
4500.0	12533.7	180.4	30.0	20681.9	269.3	62.5	26937.0	328.5	86.1
4600.0	12633.7	182.0	30.4	20781.9	270.9	63.0	27037.0	330.1	86.8
4700.0	12733.7	183.5	30.7	20881.9	272.4	63.6	27137.0	331.7	87.4
4800.0	12833.7	185.1	31.1	20981.9	274.0	64.2	27237.0	333.2	88.1
4900.0	12933.7	186.7	31.5	21081.9	275.5	64.7	27337.0	334.8	88.8
5000.0	13033.7	188.2	31.8	21181.9	277.1	65.3	27437.0	336.4	89.4



Propinquant Adjacent Junctions

In fact, an even more esoteric situation presents itself, that of Propinquant Adjacent Junctions. This is, in fact, a more accurate treatment of Adjacent Junctions, given in the previous section. Of the five pairs of adjacent junctions so far discovered in the HS network, (and I should be **very** surprised to find any more,) only two cases are truly normal, adjacent junctions, and these are dealt with in the previous section. In the other three cases, one of the junctions is itself a propinquant junction, in the sense described already in the section on Propinquant Junctions: Strelley Junction (of the Strelley / Nuthall South pair) is propinquant to Nottingham, Awsworth Junction (of the Awsworth / Nuthall North pair) is propinquant to Derby and Garforth West Junction (of the Garforth West / East pair) is propinquant to Leeds New Lane. So for these three cases, the assumption of trains approaching that particular junction at line speed is not met – they haven't reached line speed. Also, the previous treatment took constant speed – turnout limit speed – between the junctions; the treatment just given for propinquant junctions rejected doing this because, although the effects are small, they are easily determined from the table given. (In the remaining two cases, both involve Kenyon West Junction, which is just sufficiently far away from Liverpool Lime St. – 29km – not to be propinquant to it – the accelerating / diverging propinquancy limit is 22.6km and the decelerating / converging propinquancy limit is significantly less, at 19.9km.)



In the above diagram, s_{q2} is the distance of the first, accelerating / diverging propinquant junction and s^* is the distance of the second junction of the pair. Both these values are known. The corresponding times are t_{q2} and t^*. These are to be calculated. It is also of interest to know the value of v^*, the maximum speed attained between the junctions.

Referring back to the previous treatment, starting at p.43, we already have the results for everything but the asterisked values. So all we need to calculate are v^* and t^*, knowing s^*.

The calculation is pretty trivial. The first point to note is that we must take the train length into account. The train crosses the first junction at constant speed v_q (230kph = 63.9m/s), which, for the standard train length of 400m takes 6.3s. We must thus subtract 400m from the inter-junction distance, derive the resulting time over this reduced distance, and add in 6.3s for the train length effect.

So if s is the reduced, inter-junction distance, (= s^* - s_{q2} – 400 m). and s_a, s_d are the accelerating and decelerating distances (at constant acceleration rate a_a and deceleration rate a_d) then we know that the acceleration and deceleration times and distances are both in the inverse ration of the acceleration and deceleration rates, i.e. $s_a/s_d = t_a/t_d = a_d/a_a$. $s = s_a + s_d$ so, after a bit of trivial algebra, $s_a = a_d s/(a_a+a_d)$, $s_d = a_a s/(a_a+a_d)$.

The simplest way to proceed is to use the general relationship of distance, speed and time, derived as the final equation at the bottom of p.11: $s = s_0 + v_0(t - t_0) = a(t - t_0)^2/2$.
$s_0 = t_0 = 0$, but $v_0 = v_q$, so $s = v_q t + at^2/2$. Rearranging: $at^2/2 + v_q t - s = 0$.

The general solution of the quadratic equation $ax^2 + bx + c = 0$ is $x = (-b \pm \sqrt{(b^2 - 4ac)})/2a$ as every 14-year-old knows, (or did, when I was 14).

So, given that $t > 0$, $t = (\sqrt{(v_q^2 + 2as)} - v_q)/a$
Thus, for the acceleration portion $t_a = (\sqrt{(v_q^2 + 2a_a s_a)} - v_q)/a_a$ and $s_a = a_d s/(a_a+a_d)$
ad for the deceleration portion $t_d = (\sqrt{(v_q^2 + 2a_d s_d)} - v_q)/a_d$ and $s_d = a_a s/(a_a+a_d)$

We have s = distance between junctions – train length = $s^* - s_{q2} - 400$, but we actually perform the calculations for s itself as the variable, calculate ta and td then
$(t^* - t_{q2}) = t_a + t_d + 6.3$

The end result thus comes in two parts:

- We know s_{q2}, thus we know t_{q2}, from the Propinquant Junction tables.
- We know s^*, thus we know s and from that we know $t^* - t_{q2}$.

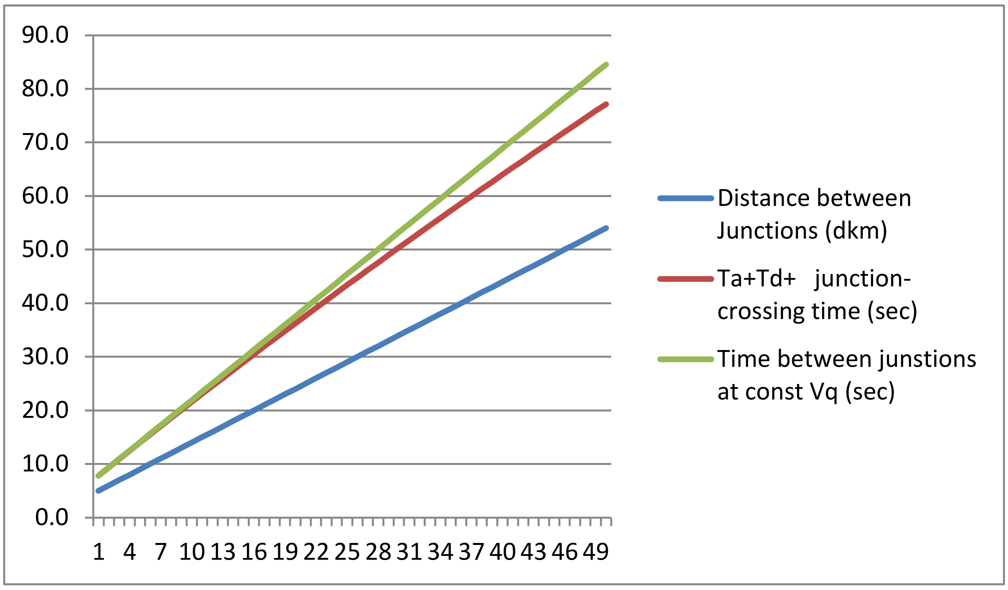
Adding the two times together gives us t^*, the time to travel from the starting station to the second junction of the pair. It has to be done like this as there is absolutely no relationship whatever between the distance to the propinquant junction and the distance between the junctions.

The values for time between junctions are now givern:

Vq	Aa	Ad	Aa+Ad	Aa/ (Aa+Ad)	Ad/ (Aa+Ad)	Vq**2	Time for train to cross junction (s)
63.9	0.3	0.5	0.8	0.4	0.6	4081.8	6.3

S = (Distance between Junctions - 400) (m)	Distance between Junctions (dkm)	Sa (m)	Sd (m)	Ta (s)	Td (s)	Ta+Td+ junction-crossing time (sec)	Time between junstions at const Vq (sec)
100.0	5.0	62.5	37.5	1.0	0.6	7.8	7.8
200.0	6.0	125.0	75.0	1.9	1.2	9.4	9.4
300.0	7.0	187.5	112.5	2.9	1.7	10.9	11.0
400.0	8.0	250.0	150.0	3.9	2.3	12.5	12.5
500.0	9.0	312.5	187.5	4.8	2.9	14.0	14.1
600.0	10.0	375.0	225.0	5.8	3.5	15.5	15.7
700.0	11.0	437.5	262.5	6.7	4.0	17.0	17.2
800.0	12.0	500.0	300.0	7.7	4.6	18.6	18.8
900.0	13.0	562.5	337.5	8.6	5.2	20.1	20.3
1000.0	14.0	625.0	375.0	9.6	5.7	21.6	21.9
1100.0	15.0	687.5	412.5	10.5	6.3	23.1	23.5

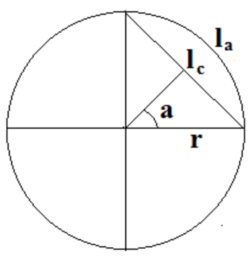
1200.0	16.0	750.0	450.0	11.4	6.9	24.6	25.0
1300.0	17.0	812.5	487.5	12.4	7.4	26.0	26.6
1400.0	18.0	875.0	525.0	13.3	8.0	27.5	28.2
1500.0	19.0	937.5	562.5	14.2	8.5	29.0	29.7
1600.0	20.0	1000.0	600.0	15.1	9.1	30.4	31.3
1700.0	21.0	1062.5	637.5	16.0	9.6	31.9	32.9
1800.0	22.0	1125.0	675.0	16.9	10.2	33.4	34.4
1900.0	23.0	1187.5	712.5	17.8	10.7	34.8	36.0
2000.0	24.0	1250.0	750.0	18.7	11.2	36.2	37.6
2100.0	25.0	1312.5	787.5	19.6	11.8	37.7	39.1
2200.0	26.0	1375.0	825.0	20.5	12.3	39.1	40.7
2300.0	27.0	1437.5	862.5	21.4	12.9	40.5	42.3
2400.0	28.0	1500.0	900.0	22.3	13.4	42.0	43.8
2500.0	29.0	1562.5	937.5	23.2	13.9	43.4	45.4
2600.0	30.0	1625.0	975.0	24.1	14.4	44.8	47.0
2700.0	31.0	1687.5	1012.5	25.0	15.0	46.2	48.5
2800.0	32.0	1750.0	1050.0	25.8	15.5	47.6	50.1
2900.0	33.0	1812.5	1087.5	26.7	16.0	49.0	51.7
3000.0	34.0	1875.0	1125.0	27.6	16.5	50.4	53.2
3100.0	35.0	1937.5	1162.5	28.4	17.1	51.7	54.8
3200.0	36.0	2000.0	1200.0	29.3	17.6	53.1	56.3
3300.0	37.0	2062.5	1237.5	30.1	18.1	54.5	57.9
3400.0	38.0	2125.0	1275.0	31.0	18.6	55.9	59.5
3500.0	39.0	2187.5	1312.5	31.9	19.1	57.2	61.0
3600.0	40.0	2250.0	1350.0	32.7	19.6	58.6	62.6
3700.0	41.0	2312.5	1387.5	33.6	20.1	59.9	64.2
3800.0	42.0	2375.0	1425.0	34.4	20.6	61.3	65.7
3900.0	43.0	2437.5	1462.5	35.2	21.1	62.6	67.3
4000.0	44.0	2500.0	1500.0	36.1	21.6	64.0	68.9
4100.0	45.0	2562.5	1537.5	36.9	22.1	65.3	70.4
4200.0	46.0	2625.0	1575.0	37.7	22.6	66.6	72.0
4300.0	47.0	2687.5	1612.5	38.6	23.1	68.0	73.6
4400.0	48.0	2750.0	1650.0	39.4	23.6	69.3	75.1
4500.0	49.0	2812.5	1687.5	40.2	24.1	70.6	76.7
4600.0	50.0	2875.0	1725.0	41.0	24.6	71.9	78.3
4700.0	51.0	2937.5	1762.5	41.9	25.1	73.2	79.8
4800.0	52.0	3000.0	1800.0	42.7	25.6	74.5	81.4
4900.0	53.0	3062.5	1837.5	43.5	26.1	75.9	83.0
5000.0	54.0	3125.0	1875.0	44.3	26.6	77.1	84.5



The surprising distance units – deci-kilometres = tenths of a km = 100m are, of course, merely to get all quantities displaying on the same scale. The 400m for the train length, over which speed would in any case have to be constant at 230kph, is added back into the distance value to give the true inter-junction distance. As can be seen, the time saved by accelerating and decelerating between junctions as opposed to holding the speed steady at 230kph is nugatory – for inter-junction distances below 2km it is less than 1 sec; even at 5.5km it is less than 8sec.

The main difficulty is in determining the length of the curved track between the junctions. The location of the junctions is known; map references are provided for every junction on the various routes (strictly, for junctions on new infrastructure). The direct, crow-fly distances can thus be measured (to a significantly higher precision than the usual journey section measurements, using dividers on the relevant 1:25000 OS maps). But this just isn't good enough: we can hardly improve the accuracy of the time estimate while introducing a probably greater inaccuracy in the distance measurement.

So I have to make an assumption, and it is that the section between junctions is an arc of a circle (ignoring transition curves). Since, in all but one case, the two lines being linked by these arcs cross essentially at right angles, I further assume that the arcs are a quarter circle - 90°. In the fifth case – between Garforth West and East junctions, which is more oblique – I take an eighth of a circle – 45°. The known, straight-line distance is the chord between the end points on the circle's circumference. A diagram will clarify. ("… And what is the use", said Alice, "of a book with no pictures?")



This illustrates the quarter-circle case: $\mathbf{l_c}$ is the chord length, the straight-line distance between the junctions, and $\mathbf{l_a}$ is the arc length, the distance between the same points, along the circumference of a circle. The radius of the circle is **r** and **a** (I really wanted alpha but the drawing program didn't have it!) is the angle between the radius to one junction and the perpendicular to the chord between the ends of the arc. It is of course 45° in the present case. Its relevance will appear shortly.

By Pythagoras, as usual: the chord length, $l_c^2 = 2r^2$ so $r = l_c/\sqrt{2}$. The arc length l_a is a quarter of the circumference: $l_a = 2\pi r/4$, $= \pi r/2$. $= \pi l_c/2\sqrt{2}$. Thus $l_a = l_c * \pi/2\sqrt{2}$.

This could more generally be stated as $l_c = 2r*\sin(a)$ so $r = l_c/2\sin(a)$. We know the angle, so we know the length of the arc: it is that which subtends an angle 2a at the centre of the circle. If the angle is expressed in radians, then $l_a/2\pi r = a/\pi$ so $l_a = 2r*a$, $= l_c a/2\sin(a)$.

So, in the general case, $l_a = l_c * a/\sin(a)$.

[Sanity check: in the usual case, above, angle $a = \pi/4$ (radians) and $\sin(\pi/4) = \sqrt{2}/2$ ($\pi/4 = 45°$ and $\sin(45°) = \sqrt{2}/2$ – everybody knows that!) so $l_a = l_c * a/\sin(a) = l_c * \pi/2\sqrt{2}$. Q.E.D.]

The other case we're interested in has $l_a = 2\pi r/8 = \pi r/4$. The corresponding angle is now $\pi/16 = 22.5°$. There's no standard formula for sin(22.5°) so $l_a = l_c * \pi/8\sin(22.5°)$

The following table gives full details for the five pairs of adjacent junctions, propinquant or otherwise:

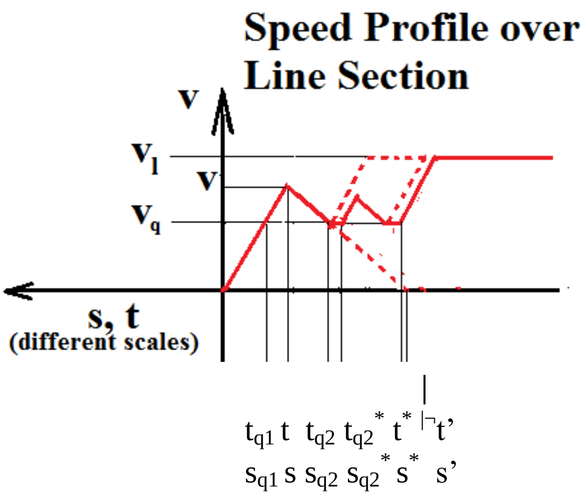
Adjacent Junctions	Lc (km)	La (km)	Inter-Junction Time (sec)
Kenyon West (SJ628961) - Kenyon South (SJ639955)	1.26	1.40	21.6
Kenyon West (SJ628961) - Kenyon North (SJ634968)	0.96	1.07	16.5
Garforth West (SE387342) - Garforth East (SE395341)	0.80	0.82	12.8
Strelley (SK512423) - Nuthall South (SK509425)	0.34	0.38	5.9
Awsworth (SK484444) - Nuthall North (SK514469)	4.25	4.72	68.3

Note that the distance between Strelley and Nuthall South junctions is shorter than the length of the train. The inter- junction time is then simply 380/63.9 = 5.9sec.

Taking the two components for the three pairs of propinquant adjacent junctions:

Station / Junction – Junctions	s_{q2} (km)	t_{q2} (s)	s^*-s_{q2} (km)	t^*-t_{q2} (s)	s^* (km)	t^* (s)
Leeds New Lane – Garforth West Junction	10.0	260.0				
Garforth West Junction – Garforth East Junction			0.8	13.0	10.8	273.0
Nottingham – Strelley Junction	7.0	216.0				
Strelley Junction – Nuthall South			0.4	6.0	7.4	222.0
Derby – Awsworth Junction	15.5	329.0				
Awsworth Junction – Nuthall North Junction			4.7	68.0	20.2	397.0

For the Convergent Propinquant Junction case, for trans decelerating to the station stop, the situation is:



(It's a bit tricky to draw this stuff, as you may well imagine. Refer back to the corresponding diagram on p.52 for elucidation of the precise location / time of the convergent junction s_{q2}^*, t_{q2}^*). The various quantities are all as indicated previously, but with the new entries t^* and s^* for the location / time of the second junction. As compared with the previous calculation, for the divergent, accelerating case, the situation between the junctions is identical, but we now have an extra, constant-speed crossing of a junction (of the propinquant junction itself), to take into account. This is the section between sq2* and sq2 in the diagram. This has already been taken into account in the final refinement to the convergent, deceleration junction on pp.53-54.

Taking the two components for the three pairs of propinquant adjacent junctions:

Station / Junction – Junction	s_{q2}^* (km)	t_{q2}^* (s)	$s^* - s_{q2}^*$ (km)	$t^* - t_{q2}^*$ (s)	s^* (km)	t^* (s)
Leeds New Lane – Garforth West Junction	10.0	212.0				
Garforth West Junction – Garforth East Junction			0.8	13.0	10.8	225.0
Nottingham – Strelley Junction	7.0	172.0				
Strelley Junction – Nuthall South			0.4	6.0	7.4	178.0
Derby – Awsworth Junction	15.5	277.0				
Awsworth Junction – Nuthall North Junction			4.7	68.0	20.2	345.0

Sundry Arc Lengths

There are several other situations where it is desirable to calculate a true quarter-circle arc length between junctions on intersecting routes. We always know the straight-line, crow distance between the junctions, since we always have the map references for them, and the method just explained gives a significantly better estimate of the true distance. These situations are **not** propinquant adjacent junctions, but the line speeds are often different on the two intersecting routes, and it is useful (indeed essential) to know that the connecting arc is long enough to accommodate this change in speed, in **both** directions – otherwise the precise junction locations need to be adjusted. In the table below l_c and l_a are the chord and arc lengths, v_{l1} and v_{l2} the line speeds at junctions 1 and 2, (maximum value 230kph, of course, for diverging / converging trains,) and s_a and s_d are the distances required to accelerate / decelerate between those speeds (at the usual standard, average rates). (In many cases, the speed is the sam – 230kph – for both junctions, so the entire arc is travelled at that speed.)

Further examples will be added as they are recognised.

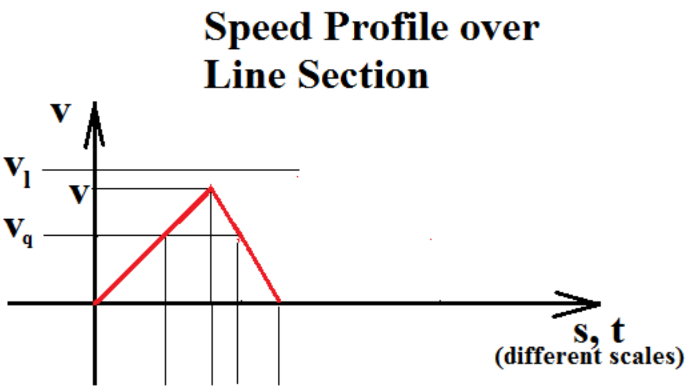
Juntion$_1$ / Map Ref.$_1$	Junction$_2$ / Map Ref.$_2$	l_c (km)	l_a (km)	v_{l1} (kph)	v_{l2} (kph)	s_a (km)	s_d (km)
Swillington Common SE378331	Manston SE372344	1.75	1.94	230	200	1.76	1.00
Wales SK469819	Waleswood SK474835	0.70	0.78	230	225	0.29	0.18
Stadium ST604750	Brentry ST572797	7.0	7.78	230	230	0	0

Odd Situations Not Covered by the Previous, Standard Techniques

The various methods described earlier cover almost all thacases for which we need to calculate journey time. There remain just a few oddities, with few actual instances.

The first is a propinquant junction, diverging or converging, where the junction leads to or from a station loop. So the distance between the junction and the station on the loop is known precisely. (The station-loop-junctions occur in north / south or east / west pairs, reflecting the acceleration and deceleration distances from and to the loop station.)

For the Diverging / Accelerating case:



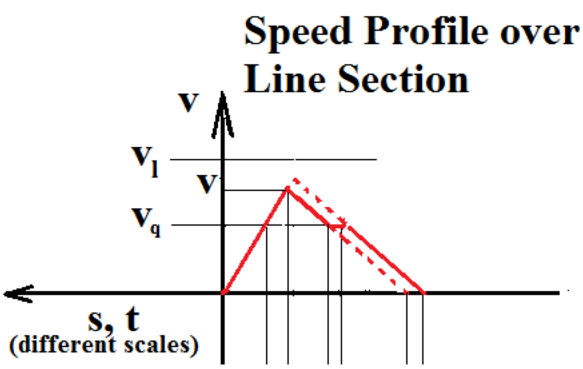
t_{q1} t t_{q2} t’

s_{q1} s s_{q2} s’

The stations are adjacent, distance s’ apart. So the overall start – stop time t’ can be read from the spreadsheet on pp.59-60. The time (t’ – t_{q2}) is simply the time to decelerate from v_q (230kph) to zero, = 128s. Hence t_{q2}. (Alternatively tq2 can simply be read from the Diverging / Accelerating Propinquant Junctions spreadsheet on pp.46-49, and 128s added to get t’. You pay your money and take your pick.)

Westlinton North Junction on HS2’s Scottish Extension is an example of this, where the Longtown (and Riddings) station loop diverges from the main line, just above Carlisle.

For the Converging / Decelerating case:



t_{q1} t t_{q2} t_{q2}^* t’ t^*

s_{q1} s s_{q2} s_{q2}^* s’ s^*

This is slightly more complicated, because of the need to take account of the train crossing the junction at constant speed v_q, Here $s^* - s' = s_{q2}^* - s_{q2} = 400$m, the length of the train, and $t^* - t' = t_{q2}^* - t_{q2} = 6.3$s is the time required for the train to cross the junction at a steady 230kph.

The stations are adjacent, distance s^* apart. But for the overall start – stop time, take the distance s', 400m less, and read the time t' from the spreadsheet on pp.59-60. Add 6.3s to this to get t*, the true time. The time $(t^* – t_{q2}^*)$ is simply the time to accelerate from zero to v_q (230kph, = 213s. Hence t_{q2}^*. (Alternatively t_{q2}^* can simply be read from the Converging / Decelerating Propinquant Junctions spreadsheet (the corrected version) on pp.53-54, and 128s added to get t^*.)

(The above values, as drawn, are, strictly speaking, all negative. Just ignore that. We're all engineers.)

Westlinton South Junction on HS2's Scottish Extension is an example of this, where the Longtown (and Riddings) station loop converges onto the main line, just above Carlisle.

Occasionally we need to calculate the passing time at an arbitrary location, before line speed has been reached (or when deceleration to a station stop has already commenced). We know the distance already travelled from the start, or still to be traveled to the stop. This distance is given by $s = at^2/2$, accordingly $t = \sqrt{(2s/a)}$. To perform the calculation for the next section, during which line speed is reached, but the starting point has a non-zero speed, so none of the previous cases apply, simply combine the two sections and perform the time calculation from the zero-speed start, using the appropriate standard case, and then simply subtract the time just calculated for the first section to get the time for the second section. (There are even a few cases where this has to be done as three sections.)

Finally, there are a (very) few cases where there is nothing for it but to use the completely general solution derived at the bottom of p.11:

$$s = s_0 + v_0(t – t_0) + a(t – t_0)^2/2$$

We can always set $s_0 = t_0 = 0$, but $v_0 \neq 0$. The two cases encountered so far (Leahaugh North Junction – Riccarton North Junction on HS2's Scottish extension, for trains stopping at Newcastleton, and Nuthall South to North Junction on HS3, for trains from Nottingham) involve a converging junction, where $v_0 = 230$kph, so we're taking the starting point where the train has just crossed the junction at constant speed, and is about to start accelerating, but we need the passing time at a known location, before line speed has been reached.

We have $s = v_0t + at^2/2$, or $at^2/2 + v_0t – s = 0$ so, by the standard solution of a quadratic equation:

$$t = (-v_0 \pm \sqrt{(v_0^2 + 2as)})/a \qquad t>0 \text{ of course, so:} \qquad \mathbf{t = (\sqrt{(v_0^2 + 2as)} – v_0)/a}$$

(s in m, v_0 in m/s and a in m/s^2, giving t in s, of course). If $v_0 = 0$ then this reduces to $\mathbf{t = \sqrt{(2s/a)}}$, the time for steady acceleration from zero (or deceleration to zero) over a given distance s.

Appendix C has now covered every single case encountered (so far!) in calculating journey times, most of which, and certainly all the really interesting and esoteric ones, have been recognised since starting to include passing times.

Appendix D – Calculating Journey Times: Adjacent Stations and Propinquant Junctions

Adjacent Stations are defined in appendix C, starting at p.50, with a convenient table of distances and times on p.53. Propinquant Junctions are also defined in appendix C, beginning at p.37, with equivalent tables of distances and times from p.40. Propinquant Adjacent Junctions (even more esoteric, and of which there are very few,) are defined in appendix C as well, starting at p.58. The table of results for these is on page 56. This present appendix lists the examples of all of these actually encountered in the HS network, with the distances and corresponding times extracted from the relevant table. For the two line speeds of principal interest, 187.5mph, 300kph, and 225mph, 360kph, two stations are adjacent if the distance between them is less than 11.5m, 18.5km, or 16.6m, 26.7km respectively. An **accelerating propinquant junction** is one located at least 6.8km (6803 to the nearest metre) from the starting station (as noted earlier, this affects only accelerating trains, so is unidirectional), and at most 14.4km (14437) or 22.6km (22585) respectively; for the two line speeds of interest; the minimum value is of course determined by the turnout limit speed, 230kph, rather than the line speed (propinqant junctions can therefor exist only where the line speed exceeds this value). A **decelerating propinquant junction** is one located at least 4.1km (4082 to the nearest metre) from the destination station (as noted earlier, this affects only decelerating trains, so is unidirectional), and at most 11.7km (11716) or 19.9km 19864) respectively; for the two line speeds of interest; the minimum value is of course determined by the turnout limit speed, 230kph, rather than the line speed (propinqant junctions can therefor exist only where the line speed exceeds this value).

Note that distances stated without a decimal point are on new infrastructure, estimated to the nearest km. (But occasionally, especially for short distances, where it is practicable to measure the distance accurately with dividers on an Ordnance Survey map, these are stated to one place of decimals). Distances stated with a decimal point (and, usually, two places of decimals,) are on existing, classic routes, and are known exactly. There are only five pairs of Propinquuant Adjacent Junctions, and these have just been given (pp. 62-63). It is extremely unlikely that any more will be found.

The contents of this appendix will be updated as the various Route and Service Plans articles are updated, to include passing times in journey time estimates, and take propinquant junctions into consideration. The results will therefore be grouped by route. (Adjacent stations are already known, but propinquant junctions are, in many cases, yet to be identified.)

HS1 Adjacent Stations

Stations	Distance Apart (km)	Start – Stop Time (s)
Rye – Winchelsea	2.93	176
Ore – Hastings	1.45	124
Hastings – St. Leonards	1.26	115

HS2 Adjacent Stations

Stations	Distance Apart (km)	Start – Stop Time |(s)
Euston Cross – Old Oak Common	8	292
Euston – Old Oak Common	9	310
Poulton-le-Fylde – Blackpool North	4.99	231
Oxenholme – Kendal	3.34	189
Manchester Interchange – Manchester HS (Mk2)	8	292
Rugby HS – Coventry HS (Mk3.1)	18	438
Carlisle – Longtown (Mk3.2)	15.93	412
Longtown – Riddings (Mk3.2)	7.6	285
Riddings – Newcastleton (Mk3.2)	16.9	425

HS2 Accelerating Propinquant Junctions

Station – Junction	Junction Distance (km)	Start – Pass Time (s)
Rugby HS – Watford Gap (Mk3.1)	12	286
Carlisle – Westlinton North (Mk3.2)	11.83	284

HS2 Decelerating Propinquant Junctions

Junction - Station	Junction Distance (km)	Start – Pass Time (s)
Watford Gap - Rugby HS (Mk3.1) (*)	12	236
Westlinton South – Carlisle (Mk3.2)	9.13	201

(*) Note that while Rugby HS – Watford Gap Junction is accelerating propinquant at line speed 300kph, the distance is slightly too far to be decelerating propinquant at that line speed, though it would be if the line speed were 360kph (which it isn't, but it's worth making the point anyway).

HS3 Adjacent Stations

Stations	Distance Apart (km)	Start – Stop Time (s)
Pancras Cross / St. Pancras West – West Hampstead	6.3	271
Luton & Dunstable Parkway – Milton Keynes Parkway	18	438
Birmingham New St. - University	4.2	212
Shipley – Bradford Central	4.91	229

HS3 Accelerating Propinquant Junctions

Station – Junction	Junction Distance (km)	Start – Pass Time (s)
Pancras Cross – Scratchwood Junction via West Hampstead Junction (Mk2)	17	346
Northampton Castle – Collingtree East Junction (Mk1A, not Mk2)	6.803 *	213
Northampton Castle – Langborough Junction (Mk1A, not Mk2)	18	358
Northampton Castle – Watford Gap Junction (Mk2)	21	389
Leicester – Stanford Junction (Mk1A, not Mk2)	20	379
Nottingham – Stanford Junction (Mk1A, not Mk2)	22	400
South Yorkshire HL – Ryhill Junction	19	368

HS3 Decelerating Propinquant Junctions

Junction – Station	Junction Distance (km)	Pass – Stop Time (s)
Scratchwood Junction – Pancras Cross via West Hampstead Junction (Mk2)	17	293
Collingtree West Junction – Northampton Castle (Mk1A, not Mk2)	4.082 *	128
Langborough Junction – Northampton Castle (Mk1A, not Mk2)	18	303
Ryhill Jnction – South Yorkshire HL	19	314

HS3 Accelerating Propinquant Adjacent Junctions

Station – Junction / Junction - Junction	Jn. Distance (km)	Start – Pass (s)	Inter-junction Distance (km)	Inter-junction Time (s)
Nottingham – Strelley Junction	7	216		
Strelley Junction – Nuthall South Jn.			0.38	6

HS3 Decelerating Propinquant Adjacent Junctions

Junction – Junction / Junction – Station	Jn. Distance (km)	Pass – Stop (s)	Inter-junction Distance (km)	Inter-junction Time (s)
Nuthall South Jn. – Strelley Junction			0.38	6
Strelley Junction – Nottingham	7	172		

The significance of (Mk1A, not Mk2) is that, at Mk2, there are four tracks south of Garforth East Junction with, in general, new 'relief' lines being added on the outside of the 'main' (original) lines. The exceptions to this are where main and relief lines are on different alignments, such as between Collingtree and Langborough junctions, between Watkin Rd. and Humberstone Rd. junctions, and between Stanford and Nuthall South junctions, where the relief lines pass through Northampton, Leicester and Nottingham stations respectively, and the main lines avoid them. (The other example is between Wales and Garforth East junctions, where the relief lines are via Sheffield, Huddersfield and Leeds, parts of which belong to HS8 and HS9). At Mk1A, the relief lines merge with the main lines at those six junctions, thus being diverging / converging, and services are obliged to decelerate to the turnout limit speed (230kph) to negotiate the junction. At Mk2, the relief lines no longer (in normal service – the junctions are still there for operational flexibility,) have to diverge from / merge with the main lines at those junctions, so special deceleration is no longer necessary.

* Collingtree East Junction is in fact borderline accelerating propinquant – it is at precisely the location that a train, accelerating from a Northampton stop, reaches the turnout limit speed – s_{q1} in the propinquant junctions exposition, in appendix C – its location being prescribed thereby. Likewise Collingtree West Junction is borderline decelerating propinquant. The actual speed profile of services diverging / converging at those junctions is therefore unchanged between Mk1A and Mk2.

Nottingham – Stanford Junction likewise differs at Mk1A and Mk2, but here the situation is subtly different. The southbound HS Metro services, which call at both Nottingham and Leicester, become free of the deceleration requirement at Mk2, exactly as just described. However a new UHS service is introduced at Mk2, Derby – Portsmouth and Southsea calling at Nottingham, then non-stop to Pancras Cross. This switches to the main line at Stanford Junction, via the original track junction, which has of course been left in place, is actually used by this single service, and for the **southbound** service is accelerating propinquant. (For the northbound service it is not propinquant at all – the distance too great for the decelerating case.)

Note that, while Nottingham – Stanford Junction and Leicester – Stanford Junction are both accelerating propinquant (at Mk1A), the distances of Stanford Junction from both Nottingham and Leicester are just a little too far for either Stanford Junction – Nottingham or Stanford Junction – Leicester to be decelerating propinquant. Likewise Northampton Castle = Watford Gap Junction (at Mk2) is an accelerating propinquant junction, but Watford Gap Junction is just a little too far from Northampton to be decelerating propinquant.

HS4 Adjacent Stations

Stations	Distance Apart (km)	Start – Stop Time (s)
Paddington – Old Oak Common	5	231
Cardiff – Cardiff (Rhoose) Airport	15	400
Cardiff (Rhoose) Airport – Bridgend (Mk2.1 and later, via Ewenny Jns.)	24	506
Port Talbot Parkway – Swansea HS	13	372
Thatcham - Newbury	5.7	246
Stroud - Stonehouse	4.4	217
Caldecot – Severn Tunnel Junction	1.2	112
Southampton Airport Parkway – Southampton Central	7.1	275
Bournemouth Central – Bournemouth West	6	253
Exeter Central – Exeter St. David’s	1.1	106

There are no junction special effects for HS4, and no special effects at all for HS5 or HS6.

HS7 Adjacent Stations

Stations	Distance Apart (km)	Start – Stop Time (s)
Nottingham – Derby	26	527
Bristol Parkway HS – Bristol Temple Meads HS (Mk2)	8	292
Tiverton Parkway - Cullompton	6.7	267
Dawlish - Teignmouth	4.4	217

HS7 Accelerating Propinquant Junctions

Station – Junction	Junction Distance (km)	Start – Pass Time (s)
Bristol Parkway – Westerleigh Junction (Mk1A) Limit 48kph.	7.4	258
Bristol Temple Meads – Brentry Junction (Mk2)	9	246

HS7 Decelerating Propinquant Junctions

Junction – Station	Junction Distance (km)	Pass – Stop Time (s)
Westerleigh Junction – Bristol Parkway (Mk1A) Limit 48kph	7.4	264
Brentry Junction – Bristol Temple Meads |(Mk2)	9	205

HS7 Accelerating Propinquant Adjacent Junctions

Station – Junction / Junction - Junction	Jn. Distance (km)	Start – Pass (s)	Inter-junction Distance (km)	Inter-junction Time (s)
Derby – Awsworth Junction	15.5	329		
Awsworth Junction – Nuthall North Jn.			4.7	68

HS7 Decelerating Propinquant Adjacent Junctions

Junction – Junction / Junction – Station	Jn. Distance (km)	Pass – Stop (s)	Inter-junction Distance (km)	Inter-junction Time (s)
Nuthall North Jn. – Awsworth Junction			4.7	68
Awsworth Junction – Derby	15.5	277		

HS8 Adjacent Stations

Stations	Distance Apart (km)	Start – Stop Time (s)
Bolton – Manchester Victoria LL	17	426
Manchester Victoria LL – Manchester HS	0.5	73

HS8 Adjacent Junctions

Junction – Junction	Inter-Junction Distance (km)	Penalty Time (s)
Kenyon West –– Kenyon South	1.40	45
Kenyon West – Kenyon North	1.07	43

HS8 Accelerating Propinquant Junctions

Station – Junction	Junction Distance (km)	Start – Pass Time (s)
Bolton – Gibb Farm Junction	8	231
Manchester HS – Guide Bridge HS Junction	8	231
Nottingham – Edwalton Junction	6.8	213

Note that, at 6,8km, Edwalton Junction is at the exact minimum for an accelerating, diverging junction, i.e. it is at s_{q1} precisely.

HS8 Decelerating Propinquant Junctions

Station – Junction	Junction Distance (km)	Pass - Stop Time (s)
Gibb Farm Junction – Bolton	8	185
Guide Bridge HS Junction – Manchester HS	8	185
Edwalton Junction – Nottingham	6.8	174

Note that while Edwalton Junction is propinquant both accelerating and decelerating for HS8, it isn't propinquant at all for HS3, since HS3's services do not diverge / converge there. Note also that while Kenyon West – South and West – North are true adjacent junctions, they aren't actually propinquant, since Kenyon West junction is just too far far from Liverpool Lime St. to be accelerating propinquant, and much too far away to be decelerating propinquant. The above transit times were derived in the analysis of propinquant adjacent junctions, and are valid for deriving passing times, the above are the only two pairs of adjacent junctions for which an overall time penalty is valid, as derived in appendix C, from p.55.

HS8 Diverging / Converging Route and Track Junctions

Junction	Time Penalty (s)
Ladybower	14
Thurlby	14

HS9 Adjacent Stations

Stations	Distance Apart (km)	Start – Stop Time (s)
Yarm – Eaglescliffe	4.1	209
Eaglescliffe - Stockton	4.9	229
Eaglescliffe – Thornaby	4.8	226
Thornaby – Middlesborough	5.2	235
Malton – Rillington Junction	7.1	275
Seamer – Scarborough	4.7	224

HS9 Accelerating Propinquant Junctions

Station – Junction	Junction Distance (km)	Start – Pass Time (s)
Manchester HS – Guide Bridge HS Junction	8	231

HS9 Decelerating Propinquant Junctions

Junction – Station	Junction Distance (km)	Pass – Stop Time (s)
Guide Bridge HS Junction – Manchester HS	8	185

HS9 Accelerating Propinquant Adjacent Junctions

Station – Junction / Junction - Junction	Jn. Distance (km)	Start – Pass (s)	Inter-junction Distance (km)	Inter-junction Time (s)
Leeds New Lane – Garforth West Junction	10.0	260		
Garforth West Junction – Garforth East Junction			0.8	13

HS9 Decelerating Propinquant Adjacent Junctions

Junction – Junction / Junction – Station	Jn. Distance (km)	Pass – Stop (s)	Inter-junction Distance (km)	Inter-junction Time (s)
Garforth East Junction – Garforth West Junction			0.8	13
Garforth West Junction – Leeds New Lane	10.0	212		

There are no special effects at all for HS10, HS11 or HS12.

HS13 Adjacent Stations

Stations	Distance Apart (km)	Start – Stop Time (s)
Newcraighall HS – Edinburgh Waverley HS	7.2	277
Edinburgh Waverley HS – Edinburgh Haymarket HS	2.1	150
Edinburgh Haymarket HS – Edinburgh Airport	9	310
Glasgow Bellgrove – Glasgow St. Enoch	1.8	138
Glasgow St. Enoch – Glasgow Airport	12	358
Glasgow Bellgrove – Glasgow Airport (direct)	15	400
Glasgow Airport – Glasgow Airport Parkway	2	146
Glasgow Airport Parkway – Erskine Parkway South	5	231
Erskine Parkway South – Erskine Parkway North	2	146

Erskine Parkway North – Dalmuir	3	179
Dalry - Kilmarnock	17.5	432

HS14 Accelerating Propinquant Junctions

Station – Junction	Junction Distance (km)	Start – Pass Time (s)
Stirling – Kinnaird Junction (Mk2)	10	260
Stirling – Bankhead Junction	14	311
Perth – Stanley Junction (Mk2)	11.5	280

HS14 Decelerating Propinquant Junctions

Junction - Station	Junction Distance (km)	Pass - Stop Time (s)
Kinnaird Junction – Stirling (Mk2)	10	217
Perth – Stanley Junction (Mk2)	11.5	230

Stanley Junction is propinquant only at Mk2, when the line speed above Perth has been raised to 300kph, and only for Inverness services, of course.

Note that while Stirling – Bankhead Junction is (just) propinquant accelerating at line speed 300kph, the distance is much (c.3km) too far for it to be propinquant decelerating at that speed (though it would be for line speed 360kph). For the decelerating case, (Bankhead Junction to Stirling,) a simple junction time penalty is appropriate, thus:

HS14 Diverging / Converging Route and Track Junctions

Junction	Time Penalty (s)
Bankhead	14

Appendix E – Document Version History

This article is one of the oldest on the website. It existed before the website was created – original publication date of the website was 13.06.2015, and at that point the article's version was 2.1. The contents were the original body of the article plus appendix A.

The first significant change came with v3.0, published 19.02.2016, which added appendix B, totally changing the character of the article, making it the fundamental technical authority for the totality of the proposals. Initially, this dealt just with line capacity. V3.1 (23.02.2016) introduced the treatment of Junction Effects.. It probably introduced Adjacent Stations also, (now shifted to appendix C, pp.50-54), since v3.2 (25.02.2016) made a correction to an error in the calculations thereof. V3.3 (26.02.2016) introduced the extended train separation distance standard, under the heading The Effect of Junctions: Advanced Capacity Aspects; Extended TSD (now at pp.15-19). V3.4 (06.03.2016) introduced the table and line chart for double (adjacent) junctions, now shifted to appendix C, pp.55-57. This was evidently a very intensive period of activity of around a month or so, since presumably a lot of work was carried out before the initial release of the appendix. V3.5 followed a little later (27.03.2016) and was just an error correction to the Adjacent Junctions section. Things were then quiet for the next nine months, until following (actually some time later, but as a consequence of) the EU referendum, the decision was taken to scrap GC-gauge, and update all the HS articles accordingly. (This update took around a year in total.) Thus:

Same Speed Railways Mk1A

Following the referendum on EU membership and the decision to disengage from the EU, several changes have been made to the plans for HS rail, most importantly, abandoning GC-gauge, and building all new infrastructure to standard UK loading gauge. This has very little impact on the routes proposed, but significant impact on the service plans. In certain cases it is now proposed to include sections of classic route in the HS route, rather than building exclusively new throughout. (Note that this is different from the previous proposals to run classic compatible services on classic lines, **beyond** the HS route; this actually incorporates classic sections, upgraded as appropriate, in the HS route itself.

As an indirect consequence of this change, Appendix B of the present article has been recast to provide results for line speeds of 200, 225 and (just for the hell of it) 400 kph, (125, 140 and 250 mph,) as well as 300 and 360 kph, (187.5 and 225 mph) used previously. In addition, it is now assumed that the very latest pointwork, (as at 2014 – it may well have improved since, but those are the latest data that I have,) is used throughout, allowing for a maximum turnout speed of 230kph (143.75mph). for the higher line speeds, and that divergence from lines with line speeds of 200 and 225 kph may be performed at full line speed.

Because of the significant changes introduced at Mk1A, the latest versions of all the Mk1 articles (v3.5 in the present case) have been preserved, available in an archive section on the website. V3.5 of the present article contains a lot of explanatory material in appendix B which is not really essential, (in effect, I was learning this stuff as I wrote it,) and is now omitted at v4.0 (06.12.2016). The exposition is thus now considerably more terse. The references to the original source material are given, on p.10, for the benefit of those techno-freaks, who simply **must** have the really hard stuff.

Apart from the preceding, explanatory remarks, all changes in the present article at v4.0 are confined to appendix B.

V4.1 (27.12.2016) adds a new table (now on pp.54-55) to simplify the calculations for adjacent stations. This **assumes**, for inter-station distances up to 3300m, that they **are** adjacent stations, and gives the maximum speed attained (instantaneously) between stations before deceleration has to begin. Thus they **genuinely are** adjacent stations if the line speed exceeds this inter-station maximum. This greatly simplifies identifying the adjacent pairs, and the inter-station journey time is simply read from the table.

V4.2 (09.11.217) adds a new appendix C, explaining how the journey times are calculated in practice, in particular, how the process is automated in Excel® spreadsheets. As a consequence of the Mk1A changes, journey time calculations in the various Route and Service Plans articles now have to take into account multiple changes of line speed within the journey, usually between station stops. In fact, this is very easily done, and experience with updating the articles led to deriving *passing times* for every location of interest, thus every point of change of line speed, (generally a junction,) and intermediate stations where a particular service is non-stop (though other services do stop). (Such times are indicated in the spreadsheet by red, italicised text, as indicated above.) This gives all the data needed, in principle, to **construct real timetables**. This has actually been done for the last article to be updated, the Scottish routes HS13 and HS14. HS14 has certain characteristics, in particular the fundamental importance of Perth and the service interconnections there, in defining the relationships between all services. Other routes may not be so accommodating, in allowing this exercise to be performed manually, but that will be the next update exercise.

V4.3 (07.12.2017) adds the new concept of Propinquant Junctions to appendix C (pp.37-48). It also adds new appendix D, to contain (eventually) all the values calculated for the special cases of Adjacent Stations, Propinquant Junctions and other such, whose times have to be added explicitly into the Journey Time spreadsheets.

V4.4 (01.01.2018) adds a few more values to appendix D

V4.5 (12.01.2018) adds the capacity slot calculation for intermediate stations to app. B (now at pp.26-27).

V4.6 (15.01.2018) adds a rigorous exposition of the capacity slot model to app. B (pp.23-26). Also introduces the present appendix E.

V5.0 (13.08.2018) carries out a reorganisation, moving the remaining journey time aspects from app. B to app. C, (so B is now exclusively about capacity, and C about journey times). The main body of the article (now dwarfed by its appendices) is edited, mainly to remove remarks which are now recognised to be misleading (particularly any lingering suggestion the 'high speed is all about capacity'!)

V5.1 (19/08/2018) adds further comments at the end of the Capacity Slot Model section (pp.25-26), concerning the behaviour at origin and destination, and how trains are initially dispatched. It also adds a new appendix F, considering how capacity is affected by varying the deceleration rate.

V5.2 (07.10.2018) refines the treatment of Adjacent Junctions and Propinquant Adjacent Junctions, and adds a few more timings to appendix D.

V5.3 (02/11/2018) corrects a diagram (p.34), and a few typos.

V5.4 (25.12.2018) adds a few more adjacent stations (for HS4 and HS7) to appendix D.

V5.5 (03.06.2019) deals with Westerleigh and Brentry Propinquant Junctions. (Westerleigh is the first such on classical track to be recognised.)

V5.6 (17.06.2019) adds a few items to reflect more closely the treatment of the 'Line Capacity vs Speed' article, whose v2.0 issue has been recast into logical, headed sections for enhanced intelligibility.

V6.0 (07.04.2020) recasts the introduction to start everything from the Same Speed **model**.

V6.1 (24.04.2020) adds new section 'Optimum Mixture of Non-stop and Stopping Trains'; to Appendix B.

V6.2 (28.05.2020) extends and refines 'Optimum Mixture …' pointing out how it affects station wait times.

V6.3 (04.06.2020) expounds the true relationship between capacity slots and scheduling of station stops, demonstrating how this can be without any deterioration in line capacity, though the resulting schedules, while strictly regular, are not so in a clock-face sense.

V7.0 (26.10.2020) really does expound the true relationship between capacity and timetabling. This is revolutionary stuff. (V6.3 was an intermediate state; not the full story at all.) It also includes first thoughts on resilience.

V7.1 (15.01.2021) complements the previous version by considering how the speeds of different sections of a Same Speed Railway may be varied – a surprisingly difficult topic.

V8.0 (02.07.2021) gives the full treatment ofvarying the line speed, while maintaining constant capacity. This has been a (totally unanticipated) extraordinarily difficula topic, and has taken an extraordinarily long time – over 6 months.

Appendix F – Effect of Varying the Deceleration Rate

Line capacity is determined by the train separation distance, the basic form of which, TSD(b), depends directly on the square on the line speed (together with a constant component, whose influence rapidly diminishes as line speed increases), The extended train separation distance, TSD(e), is used for speeds above the turnout limit speed (230kph); this is described in detail in the section 'The Effect of Junctions', beginning at p.15.

The only other quantity affecting capacity is the deceleration rate. The numerical results presented earlier all take a deceleration rate of 0.5m/s^2 (and an acceleration rate of 0.3m/s^2). It is instructive (or at least interesting) to consider how different values of deceleration rate (treated as a parameter) would affect the capacity results. Significant alteration of deceleration rate is not something that can readily be provided, so this is presented here as a mind experiment, rather than as a practical proposal, and kept entirely separate from the real-world stuff, earlier.

Since TSD(b) = $v^2/2a + 700$ m and capacity c = $v/(v^2/2a = 700)$ tps = $3600v/(v^2/2a+700)$ tph, then we can state immediately that:

- as a –> 0 then TSD(b) –> ∞ and so c –> 0.
- as a –> ∞, then TSD(b) –> 700 and so c –> v/700 tps = 3600v/700 = 5.143v tph.

What these mean is that if trains are unable to decelerate then they can't run at all, so capacity is zero, whereas, as a becomes very large, then, in the limit, the train can make an instantaneous stop, the separation distance is constant at 700m, and capacity thus reaches a fixed maximum. All of which is ridiculous, of course, but fun to contemplate.

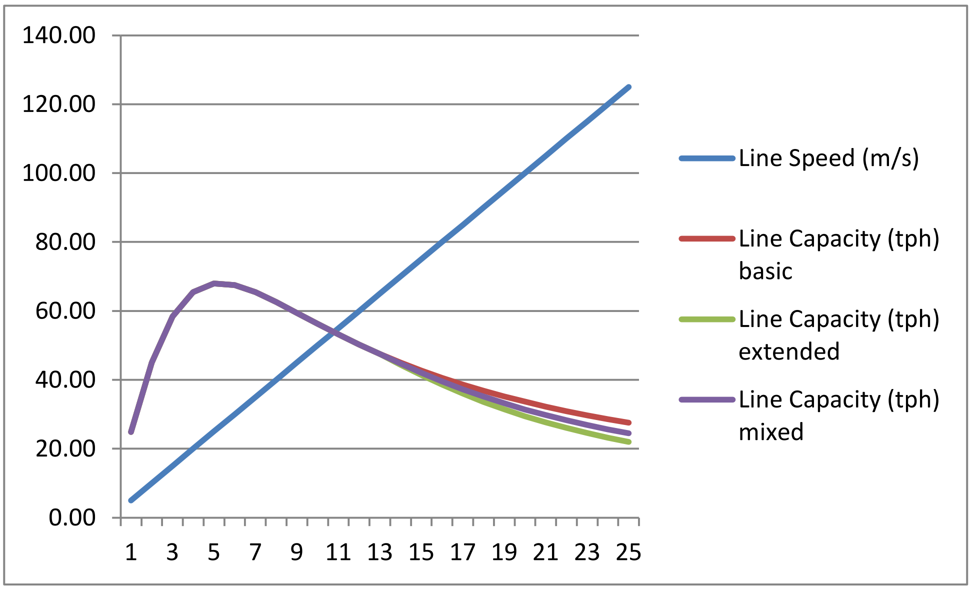
There follow spreadsheets and charts corresponding to the real capacity results on p.23* for values of a from 0.5 to 1.0 m/s^2 in increments of 0.1. (These look slightly different from the one on p.23* as I have subsequently discovered a very simple and efficient method of generating them by, effectively, a single copy and paste operation. Send me an email if you want to know how.) Capacity is shown to be significantly dependent on deceleration rate, which is unsurprising; a pity we can't, in practice, take much advantage of this. (* This has now been changed to the new standard, but still feel free to email me!)

A spreadsheet with a pair of charts then follows illustrating how various quantities of interest vary with deceleration rate. (Note also that the deceleration rate is stated in **cm/s^2**, to make sure it can use the same y-axis scale.) These may look superficially like straight lines but, apart from the graph of the deceleration rate itself, they are all (very slightly) curved, the curvature gradually decreasing as .deceleration rate increases. Note that the excess capacity of a line speed of 100mph over that of 225mph continues to increase as an absolute quantity, but, as both values become larger, the 225mph value does gradually increase as a proportion of the 100mph value.

The final spreadsheet, with three charts, displays how the capacity for a line speed of 100mph varies over three deceleration ranges, the same numeric range, but with units of cm/s^2, dm/s^2 and m/s^2 respectively, so that the final value of one becomes the first nonzero value of the next. It does indeed show how the capacity reaches a fixed maximum, of 231.43tph, as the deceleration rate becomes very high (preposterously, ridiculously and unimaginably high, in fact). All good fun, but quite silly, really.

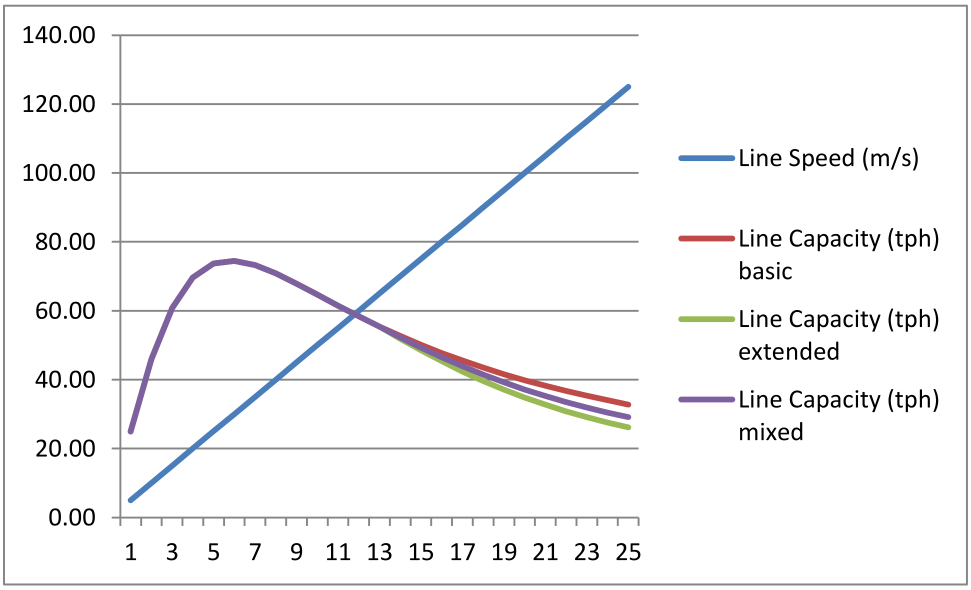
Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph) basic	Line Capacity (tph) extended	Line Capacity (tph) mixed
5.00	18.00	11.19	24.83	24.83	24.83
10.00	36.00	22.37	45.00	45.00	45.00
15.00	54.00	33.56	58.38	58.38	58.38
20.00	72.00	44.75	65.45	65.45	65.45
25.00	90.00	55.94	67.92	67.92	67.92
30.00	108.00	67.12	67.50	67.50	67.50
35.00	126.00	78.31	65.45	65.45	65.45
40.00	144.00	89.50	62.61	62.61	62.61
45.00	162.00	100.68	59.45	59.45	59.45
50.00	180.00	111.87	56.25	56.25	56.25
55.00	198.00	123.06	53.15	53.15	53.15
60.00	216.00	134.24	50.23	50.23	50.23
65.00	234.00	145.43	47.51	47.51	47.51
70.00	252.00	156.62	45.00	44.40	44.70
75.00	270.00	167.81	42.69	41.42	42.05
80.00	288.00	178.99	40.56	38.60	39.56
85.00	306.00	190.18	38.61	35.99	37.25
90.00	324.00	201.37	36.82	33.59	35.13
95.00	342.00	212.55	35.17	31.41	33.18
100.00	360.00	223.74	33.64	29.44	31.40
105.00	378.00	234.93	32.24	27.65	29.77
110.00	396.00	246.12	30.94	26.03	28.27
115.00	414.00	257.30	29.73	24.56	26.90
120.00	432.00	268.49	28.61	23.23	25.64
125.00	450.00	279.68	27.57	22.01	24.48

Line Capacity vs. Speed for Deceleration Rate of 0.5 m/s2**



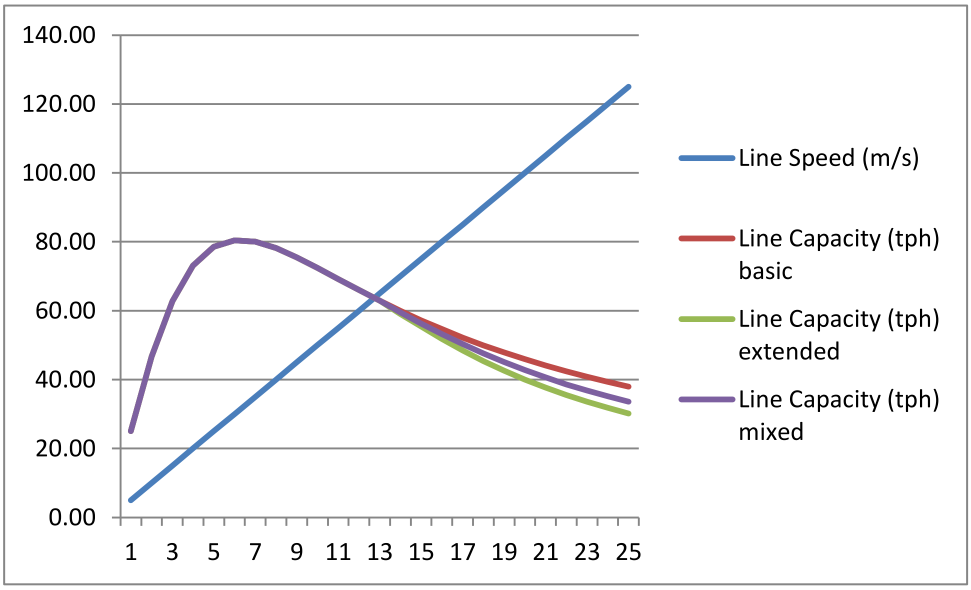
Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph) basic	Line Capacity (tph) extended	Line Capacity (tph) mixed
5.00	18.00	11.19	24.97	24.97	24.97
10.00	36.00	22.37	45.96	45.96	45.96
15.00	54.00	33.56	60.85	60.85	60.85
20.00	72.00	44.75	69.68	69.68	69.68
25.00	90.00	55.94	73.72	73.72	73.72
30.00	108.00	67.12	74.48	74.48	74.48
35.00	126.00	78.31	73.22	73.22	73.22
40.00	144.00	89.50	70.82	70.82	70.82
45.00	162.00	100.68	67.85	67.85	67.85
50.00	180.00	111.87	64.67	64.67	64.67
55.00	198.00	123.06	61.47	61.47	61.47
60.00	216.00	134.24	58.38	58.38	58.38
65.00	234.00	145.43	55.44	55.44	55.44
70.00	252.00	156.62	52.68	51.93	52.30
75.00	270.00	167.81	50.12	48.56	49.33
80.00	288.00	178.99	47.73	45.35	46.51
85.00	306.00	190.18	45.53	42.36	43.89
90.00	324.00	201.37	43.49	39.60	41.45
95.00	342.00	212.55	41.60	37.08	39.21
100.00	360.00	223.74	39.85	34.80	37.15
105.00	378.00	234.93	38.23	32.72	35.26
110.00	396.00	246.12	36.72	30.83	33.52
115.00	414.00	257.30	35.32	29.12	31.92
120.00	432.00	268.49	34.02	27.56	30.45
125.00	450.00	279.68	32.80	26.14	29.09

Line Capacity vs. Speed for Deceleration Rate of 0.6 m/s2**



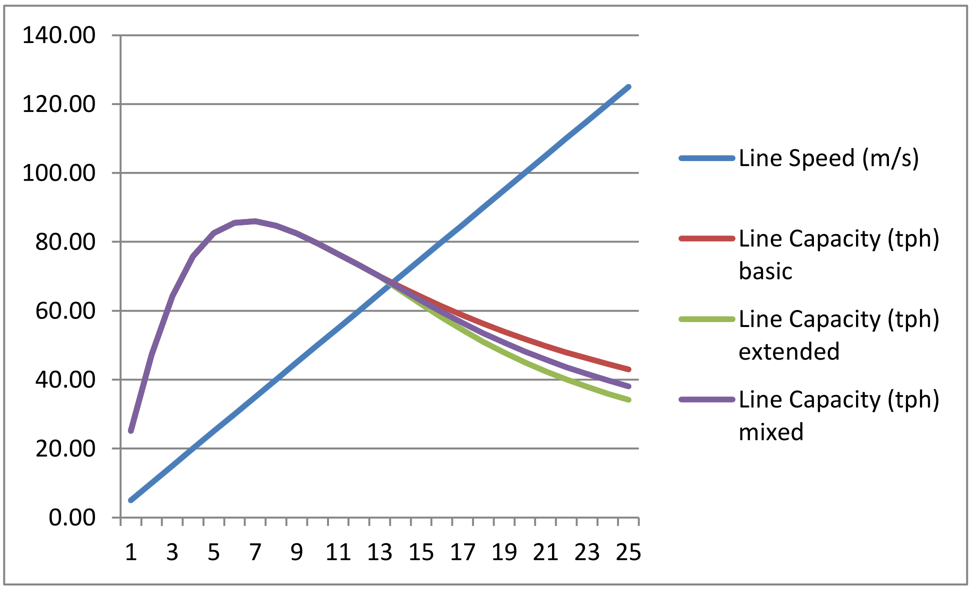
Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph) basic	Line Capacity (tph) extended	Line Capacity (tph) mixed
5.00	18.00	11.19	25.07	25.07	25.07
10.00	36.00	22.37	46.67	46.67	46.67
15.00	54.00	33.56	62.74	62.74	62.74
20.00	72.00	44.75	73.04	73.04	73.04
25.00	90.00	55.94	78.50	78.50	78.50
30.00	108.00	67.12	80.43	80.43	80.43
35.00	126.00	78.31	80.00	80.00	80.00
40.00	144.00	89.50	78.14	78.14	78.14
45.00	162.00	100.68	75.47	75.47	75.47
50.00	180.00	111.87	72.41	72.41	72.41
55.00	198.00	123.06	69.21	69.21	69.21
60.00	216.00	134.24	66.03	66.03	66.03
65.00	234.00	145.43	62.94	62.94	62.94
70.00	252.00	156.62	60.00	59.09	59.54
75.00	270.00	167.81	57.23	55.38	56.29
80.00	288.00	178.99	54.63	51.82	53.19
85.00	306.00	190.18	52.21	48.49	50.28
90.00	324.00	201.37	49.96	45.40	47.57
95.00	342.00	212.55	47.86	42.58	45.06
100.00	360.00	223.74	45.90	40.00	42.75
105.00	378.00	234.93	44.08	37.65	40.61
110.00	396.00	246.12	42.39	35.51	38.65
115.00	414.00	257.30	40.80	33.57	36.83
120.00	432.00	268.49	39.32	31.80	35.16
125.00	450.00	279.68	37.94	30.18	33.62

Line Capacity vs. Speed for Deceleration Rate of 0.7 m/s2**



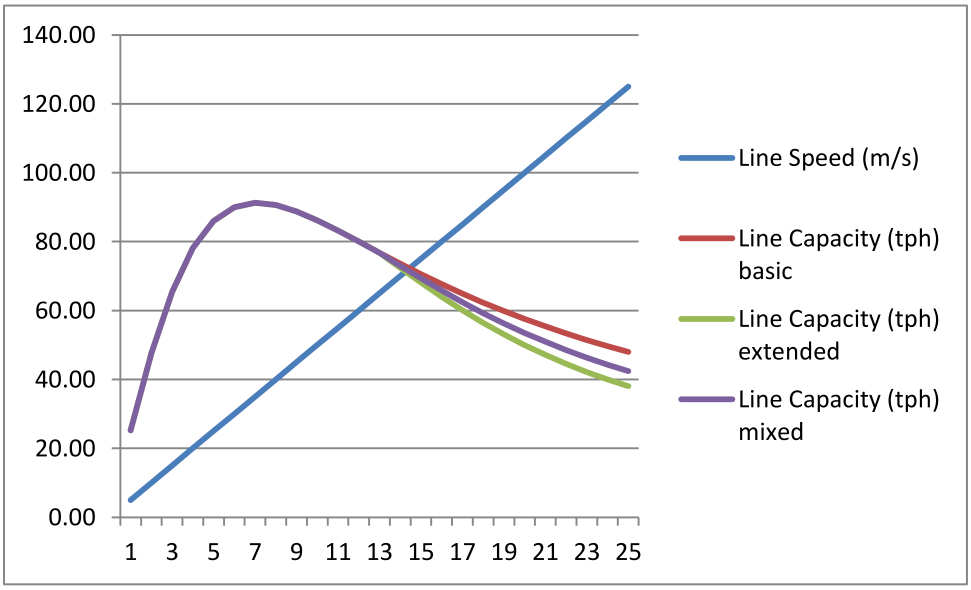
Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph) basic	Line Capacity (tph) extended	Line Capacity (tph) mixed
5.00	18.00	11.19	25.15	25.15	25.15
10.00	36.00	22.37	47.21	47.21	47.21
15.00	54.00	33.56	64.24	64.24	64.24
20.00	72.00	44.75	75.79	75.79	75.79
25.00	90.00	55.94	82.52	82.52	82.52
30.00	108.00	67.12	85.54	85.54	85.54
35.00	126.00	78.31	85.97	85.97	85.97
40.00	144.00	89.50	84.71	84.71	84.71
45.00	162.00	100.68	82.42	82.42	82.42
50.00	180.00	111.87	79.56	79.56	79.56
55.00	198.00	123.06	76.43	76.43	76.43
60.00	216.00	134.24	73.22	73.22	73.22
65.00	234.00	145.43	70.05	70.05	70.05
70.00	252.00	156.62	66.98	65.90	66.43
75.00	270.00	167.81	64.05	61.89	62.95
80.00	288.00	178.99	61.28	58.03	59.61
85.00	306.00	190.18	58.67	54.39	56.45
90.00	324.00	201.37	56.23	51.01	53.49
95.00	342.00	212.55	53.94	47.90	50.74
100.00	360.00	223.74	51.80	45.05	48.19
105.00	378.00	234.93	49.80	42.45	45.83
110.00	396.00	246.12	47.93	40.08	43.65
115.00	414.00	257.30	46.18	37.92	41.64
120.00	432.00	268.49	44.54	35.94	39.78
125.00	450.00	279.68	43.00	34.14	38.06

Line Capacity vs. Speed for Deceleration Rate of 0.8 m/s2**



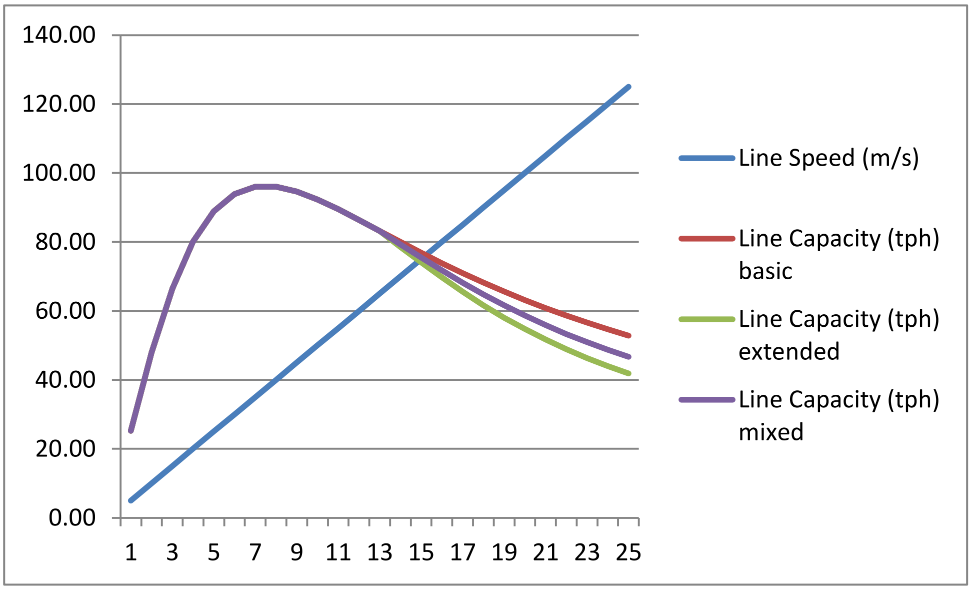
Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph) basic	Line Capacity (tph) extended	Line Capacity (tph) mixed
5.00	18.00	11.19	25.21	25.21	25.21
10.00	36.00	22.37	47.65	47.65	47.65
15.00	54.00	33.56	65.45	65.45	65.45
20.00	72.00	44.75	78.07	78.07	78.07
25.00	90.00	55.94	85.94	85.94	85.94
30.00	108.00	67.12	90.00	90.00	90.00
35.00	126.00	78.31	91.27	91.27	91.27
40.00	144.00	89.50	90.63	90.63	90.63
45.00	162.00	100.68	88.77	88.77	88.77
50.00	180.00	111.87	86.17	86.17	86.17
55.00	198.00	123.06	83.17	83.17	83.17
60.00	216.00	134.24	80.00	80.00	80.00
65.00	234.00	145.43	76.79	76.79	76.79
70.00	252.00	156.62	73.64	72.39	73.01
75.00	270.00	167.81	70.59	68.13	69.34
80.00	288.00	178.99	67.68	63.99	65.78
85.00	306.00	190.18	64.91	60.07	62.40
90.00	324.00	201.37	62.31	56.42	59.22
95.00	342.00	212.55	59.85	53.05	56.25
100.00	360.00	223.74	57.55	49.96	53.49
105.00	378.00	234.93	55.38	47.12	50.92
110.00	396.00	246.12	53.35	44.53	48.55
115.00	414.00	257.30	51.45	42.17	46.35
120.00	432.00	268.49	49.66	40.00	44.31
125.00	450.00	279.68	47.97	38.01	42.42

Line Capacity vs. Speed for Deceleration Rate of 0.9 m/s2**

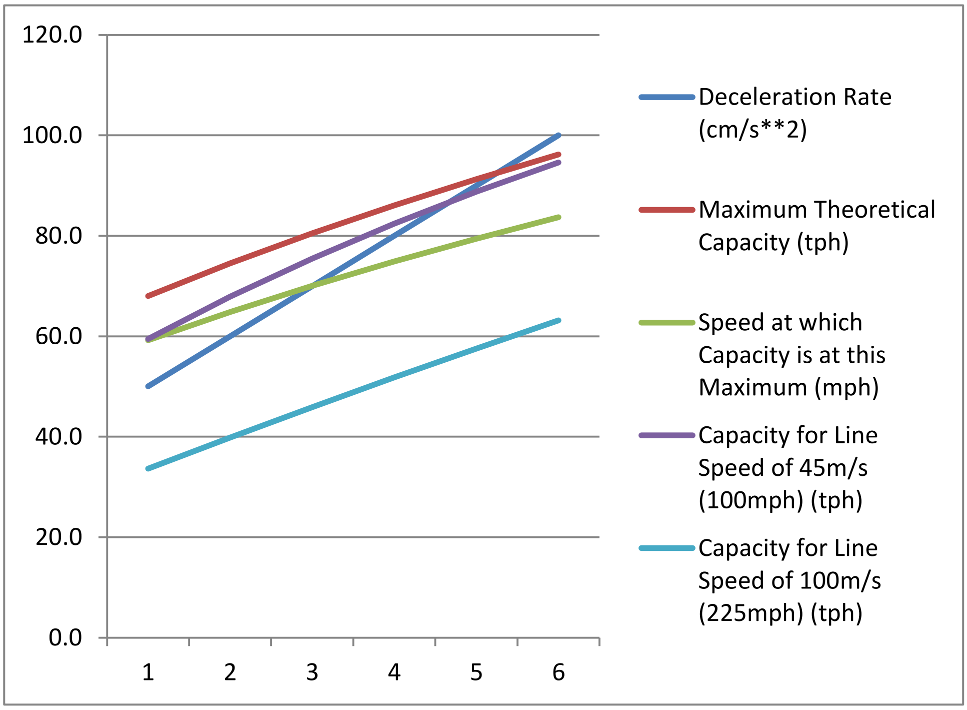


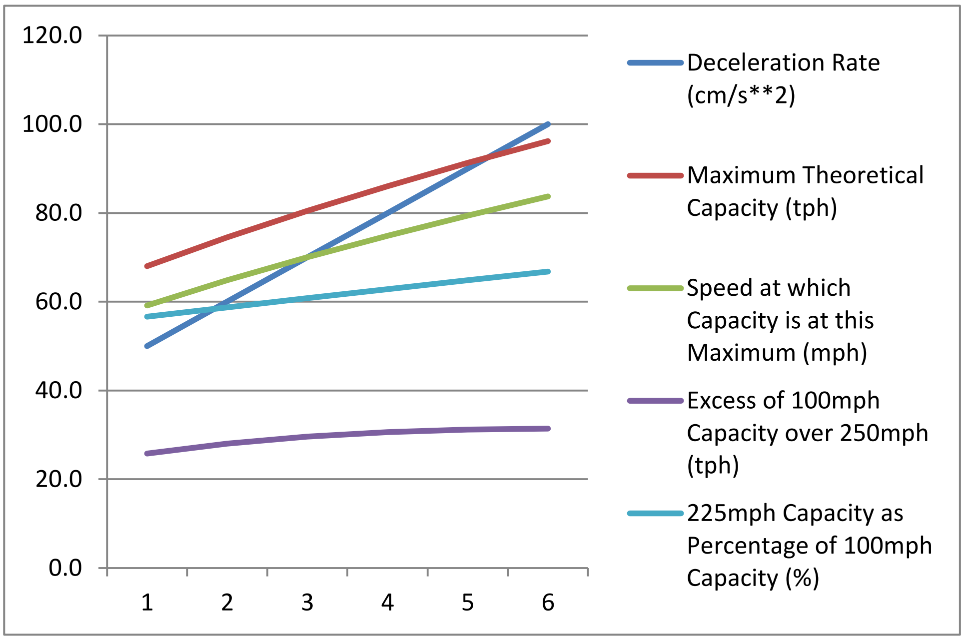
Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph) basic	Line Capacity (tph) extended	Line Capacity (tph) mixed
5.00	18.00	11.19	25.26	25.26	25.26
10.00	36.00	22.37	48.00	48.00	48.00
15.00	54.00	33.56	66.46	66.46	66.46
20.00	72.00	44.75	80.00	80.00	80.00
25.00	90.00	55.94	88.89	88.89	88.89
30.00	108.00	67.12	93.91	93.91	93.91
35.00	126.00	78.31	96.00	96.00	96.00
40.00	144.00	89.50	96.00	96.00	96.00
45.00	162.00	100.68	94.60	94.60	94.60
50.00	180.00	111.87	92.31	92.31	92.31
55.00	198.00	123.06	89.49	89.49	89.49
60.00	216.00	134.24	86.40	86.40	86.40
65.00	234.00	145.43	83.20	83.20	83.20
70.00	252.00	156.62	80.00	78.58	79.28
75.00	270.00	167.81	76.87	74.10	75.46
80.00	288.00	178.99	73.85	69.72	71.73
85.00	306.00	190.18	70.96	65.56	68.15
90.00	324.00	201.37	68.21	61.66	64.77
95.00	342.00	212.55	65.61	58.05	61.60
100.00	360.00	223.74	63.16	54.73	58.64
105.00	378.00	234.93	60.85	51.68	55.89
110.00	396.00	246.12	58.67	48.88	53.33
115.00	414.00	257.30	56.62	46.32	50.95
120.00	432.00	268.49	54.68	43.97	48.74
125.00	450.00	279.68	52.86	41.81	46.69

Line Capacity vs. Speed for Deceleration Rate of 1.0 m/s2**

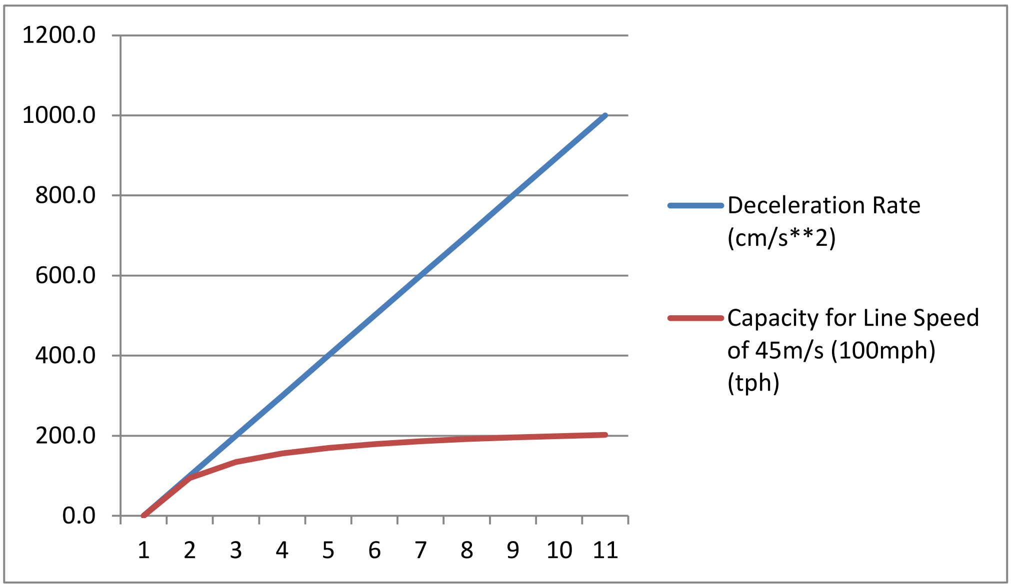


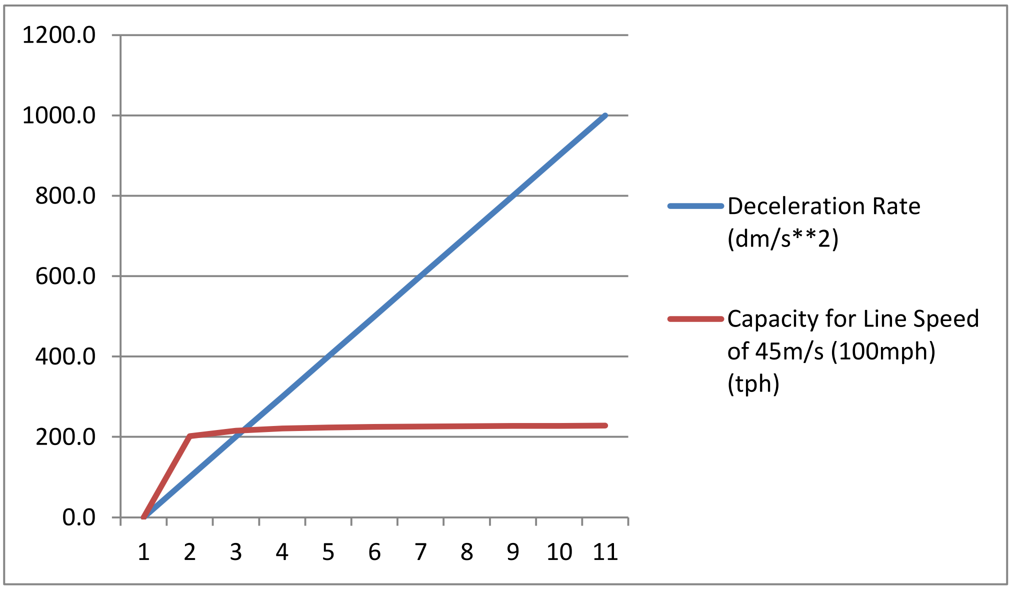
Deceleration Rate (cm/s**2)	Maximum Theoretical Capacity (tph)	Speed at which Capacity is at this Maximum (m/s)	Speed at which Capacity is at this Maximum (mph)	Capacity for Line Speed of 45m/s (100mph) (tph)	Capacity for Line Speed of 100m/s (225mph) (tph)	Excess of 100mph Capacity over 250mph (tph)	225mph Capacity as Percentage of 100mph Capacity (%)
50.0	68.0	26.5	59.2	59.4	33.6	25.8	56.6
60.0	74.5	29.0	64.8	67.9	39.9	28.0	58.7
70.0	80.5	31.3	70.0	75.5	45.9	29.6	60.8
80.0	86.1	33.5	74.9	82.4	51.8	30.6	62.8
90.0	91.3	35.5	79.4	88.8	57.5	31.2	64.8
100.0	96.2	37.4	83.7	94.6	63.2	31.4	66.8

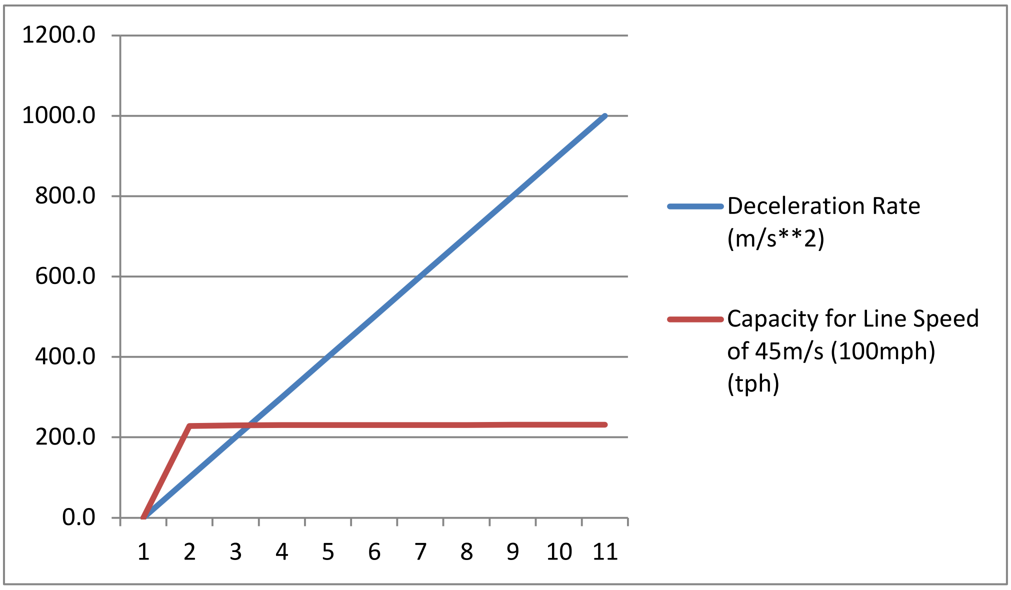




Deceleration Rate (cm/s**2)	Capacity for Line Speed of 45m/s (100mph) (tph)	Deceleration Rate (dm/s**2)	Capacity for Line Speed of 45m/s (100mph) (tph)	Deceleration Rate (m/s**2)	Capacity for Line Speed of 45m/s (100mph) (tph)
0.0	0.0	0.0	0.0	0.0	0.0
100.0	94.6	100.0	202.2	100.0	228.1
200.0	134.3	200.0	215.8	200.0	229.8
300.0	156.1	300.0	220.8	300.0	230.3
400.0	170.0	400.0	223.4	400.0	230.6
500.0	179.5	500.0	224.9	500.0	230.8
600.0	186.5	600.0	226.0	600.0	230.9
700.0	191.8	700.0	226.7	700.0	231.0
800.0	196.0	800.0	227.3	800.0	231.0
900.0	199.4	900.0	227.8	900.0	231.1
1000.0	202.2	1000.0	228.1	1000.0	231.1






1200.0
1000.0
800.0
600.0
400.0
200.0
0.0
1
2
3
4
5
6
7
8
9
10
11
Deceleration Rate (m/s**2)
Capacity for Line Speed of 45m/s (100mph) (tph)