

Line Capacity vs. Speed for Same Speed Railways

Volume 2

Summary

The initial edition of this article was never foreseen as a mere volume 1, but effectively it has outgrown its natural scope, and a volume 2 is clearly required, to handle the more advanced aspects of Same Speed Railways.

The problem became acute when I added the very involved (surprisingly so) subject of change of line speed, while maintaining line capacity at the same level. I felt that this completely unbalanced the article, and was likely to prove confusing to the readership; after all, the whole purpose of the article(s) is to introduce the important subject of line capacity to an intelligent but non-specialist audience.

Accordingly, the entire section on change of line speed has been removed from volume 1, replaced by a stub, and an entirely new exposition of the topic, with a more rational presentation sequence, has been written for volume 2.

This is preceded by the entirely new topic of different switch types for different speed ranges, including a rigorous distinction between speed ranges for which the Extended Train Separation standard is appropriate, and those where the Basic standard must be used.

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Introduction and Background

The (implicit) first volume of this article introduces the concept of the Same Speed Railway, and explains how the line capacity is determined. This, it notes, has a precise value, (strictly, a precise **maximum** value,) for a particular line speed; it is in fact a **property** of the model.

Initially, capacity was all that the article was concerned with. But the seemingly ridiculous values that the model offers caused me increasing unease, so that eventually, (beginning in April 2020,) I found myself obliged to consider the timetabling of Same Speed railways, from the initial point of interest of how to combine non-stop and stopping services, allowing overtaking at stations, without losing line capacity. This introduced the Capacity Slot Model, which I had originally developed earlier, for another purpose, but which showed itself to be of fundamental importance in timetabling, and then introduced the concepts of the Slot Stream Advance and Sweet-Speeds into the public domain. I believe I have demonstrated that the promised capacity values are in fact perfectly legitimate, if still astonishing.

The main reason for writing a separate volume 2 is a consequence of dealing with the topic of variation of line speed while maintaining constant line capacity. I found this an extraordinarily difficult section to write, mainly because I sensed that the whole focus of the article was being compromised: having explained the concept of Sweet-Speeds, the very few values of line speed which enable a usable timetable to be constructed, I was now showing how to circumvent this, to any speed of choice, provided less than the Sweet-Speed for that capacity. The readers must have found it confusing. (I myself found it confusing.) I decided that the treatment of this astonishingly complicated subject was unsatisfactory, and have removed it from volume 1. A completely new and, I believe, greatly improved exposition has been written for volume 2.

I began the original article very much from the standpoint of High Speed railways. It was only later that I realised that Medium and Low Same Speeds shared most or all of the same properties. But the tendency persisted, still to think primarily in High Speed terms, (often quite unconsciously). For this second volume, I will ensure that all speed ranges are treated equally. As a consequence of this, a whole range of switch types is now included, allowing the Extended Train Separation Distance standard to be used over as wide a speed range as possible, the Basic standard being restricted to only the lowest speeds, where it is unavoidable.

Volume 2 is specifically for the more advanced issues of Same Speed, initially the two topics just described. Further advanced topics will be added as they are recognised and investigated.

It is assumed that the reader is familiar with the contents of volume 1. If that is not the case, then don't waste your time with the present article until you have command of the earlier one.

The Speed Ranges of Same Speed Railways

Volume 1 recognised High, Medium and Low speed ranges, and set the boundary between High and Medium at the Buffer-end Speed, since that is the speed at which the following train, still travelling at full line speed, is precisely the absolute minimum distance behind the diverging train, and also the point where the switch has just been reset to point to the direct (main) line, so that the diverging train is no longer in the path of the following train. This is perfectly correct, and entirely rational, but is, nonetheless, a prime example of thinking in High Speed terms exclusively, as deprecated above.

The original treatment considered only one type of switch, the Ultra-High-Speed Swing Nose Crossing with Manganese Cradle. This is fine for High Speed lines, but a ridiculous overkill for Medium and Low Speed ones. With the recognition that the Same Speed model applies to **all** speed ranges, a whole series of switch types is now proposed, allowing the benefits of the Extended TSD standard to be extended to the widest possible range of speeds. (There **is** a minimum speed, below which TSD(e) is not available, and recourse must be had to the basic standard, TSD(b). That is not much of a problem, in practice.)

Generally speaking, if the TSD(e) standard applies, there is no point whatever in using a switch whose turnout limit speed is higher than the line speed – it would work, of course, but offers performance (and price!) way beyond what is required. (If the Basic TSD standard applies, then the turnout limit speed of the switch type in use **must** be equal to or exceed the line speed, since the trains are already the minimum distance apart, and no deceleration is possible on the main line.) It was not recognised as such, originally, but having just the one switch type available effectively restricts the Extended TSD standard to just the High Speed range; that was, indeed, taken as a definition of that range.

All Same Speed railways are designed for a particular chosen capacity. A particular model of switch is selected with a turnout limit speed below that of the Sweet-Speed which corresponds to that capacity, but not too far below. In all speed ranges for which TSD(e) is available, trains decelerate on the main line down to the turnout limit speed as they reach the switch, continuing their steady deceleration across the switch and on the diverging track. The basic standard TSD(b) allows no deceleration on the main line; the trains are already the minimum permissible distance apart, and the line speed is less than or equal to the turnout limit speed of the switch type in use. The consequence of this is that a diverging train must travel a distance equal to the buffer length for that switch type onto the diverging track, and the switch be reset behind it, before it can even begin its deceleration.

This means that the boundary between speed ranges is now indeed the turnout limit speed for the higher speed range, as one would naturally expect, and this is determined by the switch type in actual use.

Switch Types:

This present section derives the Capacity vs. Speed relationships for four different switches from Vossloh Cogifer. (I must stress that I am in no contractual relationship with Vossloh Cogifer, but since they were so helpful in supplying me with the relevant technical data, their reward is that their products get quoted!) These are:

1. The UHS switch dealt with so far:

Turnout Limit Speed $v_t = 63.889\text{m/s}$ / 230kph / 142.95mph

Length of moving switch parts 194.5m

(Maximum) time to reset switch 4s (NR standard for swing nose crossings).

2. Switch Type HV:
Turnout Limit Speed $v_t = 40.225\text{m/s} / 144.81\text{kph} / 90\text{mph}$
Length of moving switch parts 89.693m
(Maximum) time to reset switch 5s (NR standard for switch lengths SG – J).
3. Switch Type GV:
Turnout Limit Speed $v_t = 31.286\text{m/s} / 112.63\text{kph} / 70\text{mph}$
Length of moving switch parts 65.136m
(Maximum) time to reset switch 5s (NR standard for switch lengths SG – J).
4. Switch Type FV
Turnout Limit Speed $v_t = 22.347\text{m/s} / 80.45\text{kph} / 50\text{mph}$
Length of moving switch parts 49.816m
(Maximum) time to reset switch 4s (NR standard for switch lengths up to F).

To these I add a fifth switch type, for reasons which will very quickly become apparent:

5. Switch Type EV
Turnout Limit Speed $v_t = 17.878\text{m/s} / 84.36\text{kph} / 40\text{mph}$
Length of moving switch parts 40.457m
(Maximum) time to reset switch 4s (NR standard for switch lengths up to F).

(The speeds in red are the values specified in the source documentation.)

Let's call the speed ranges corresponding to the first four types of switch High Speed, Medium Speed, Low Speed and Slow Speed.

With the notation:

- v_l = line speed (m/s)
- v_t = turnout limit speed (m/s)
- b = buffer length (m)
- v_b = buffer-end speed (m/s)
- a_d = (uniform) rate of deceleration (m/s^2)
- c_l = line capacity (tps – multiply by 3600 to get tph)

then we have the following results (which are constants for a particular switch):

b = train length + length of moving switch parts + distance travelled (decelerating) in time to reset switch
(train length value taken as 400m: 16 or 2*8 carriages of 25m each)

$$v_b = \sqrt{v_t^2 - 2a_d b}.$$

The buffer length is rounded up to the next multiple of 5.

So we have the following, constant values:

1. UHS switch type	$v_{tH} = 63.889\text{m/s}$	$b_H = 830\text{m}$	$v_{bH} = 57.02\text{m/s}$
2. Switch Type HV	$v_{tM} = 40.225\text{m/s}$	$b_M = 655\text{m}$	$v_{bM} = 31.03\text{m/s}$
3. Switch Type GV	$v_{tL} = 31.286\text{m/s}$	$b_L = 575\text{m}$	$v_{bL} = 20.10\text{m/s}$
4. Switch Type FV	$v_{tS} = 22.347\text{m/s}$	$b_S = 475\text{m}$	$v_{bS} = 4.94\text{m/s}$
5. Switch Type EV	$v_{t5} = 17.878\text{m/s}$	results not available – see later.	

The standard formulae for Train Separation Distance (TSD) are:

$$\begin{aligned} \text{Basic, TSD(b)} &= v_l^2/2a_d + b \\ \text{Extended, TSD(e)} &= v_l^2/2a_d + b + e \\ \text{where } e &= (v_l - v_b)^2/2a_d \quad (\text{the separation distance extension}). \end{aligned}$$

Finally, line capacity = line speed / TSD, basic or extended, i.e.

$$c_l = v_l/(v_l^2/2a_d + b) \quad \text{or}$$

$$c_l = v_l/(v_l^2/2a_d + b + e)$$

as appropriate. Multiply result by 3600 for tph.

If v_{cmax} is the line speed at which the line capacity, calculated on TSD(b), is at its (theoretical) maximum,

$$\text{then } v_{cmax} = \sqrt{(2a_d b)} \text{ m/s} \quad \text{and thus } c_{max} = \sqrt{(a_d/2b)} \text{ tps (*3600 for tph);}$$

or, calculated on TSD(e),

$$\text{then } v_{cmax} = \sqrt{(v_b^2/2 + a_d b)} \text{ m/s} \quad \text{and thus } c_{max} = a_d/[2\sqrt{(v_b^2/2 + a_d b)} - v_b] \text{ tps (*3600 for tph).}$$

So the values for the four switch types are:

1. UHS switch type	$v_{cmax} = 28.81 \text{ m/s}$	$c_{max} = 62.48 \text{ tph}$	
2. Switch Type HV	$v_{cmax} = 25.59 \text{ m/s}$	$c_{max} = 70.33 \text{ tph}$	
3. Switch Type GV	$v_{cmax} = 23.98 \text{ m/s}$	$c_{max} = 75.07 \text{ tph}$	
	$v_{cmax} = 22.12 \text{ m/s}$	$c_{max} = 74.53 \text{ tph}$	(NB calculated on TSD(e).)
4. Switch Type FV	$v_{cmax} = 21.79 \text{ m/s}$	$c_{max} = 82.59 \text{ tph}$	
	$v_{cmax} = 15.80 \text{ m/s}$	$c_{max} = 67.51 \text{ tph}$	(NB calculated on TSD(e).)

(These have been calculated, where appropriate, for both cases, since both appear on the graph.)

There now follow (after the remarks, below, on switch type EV) four graphs of line capacity vs. speed, for the four switch types. Elucidatory commentary follows.

Switch Type EV and Below:

The results are not available for switch type EV. What this actually means is that the Extended Train Separation Distance standard is no longer available. This relies on trains decelerating on the main line down to the turnout limit speed by the time they reach the switch, and continuing their deceleration during and after crossing the switch, down to the buffer-end speed, by the time they reach a distance equal to the buffer length beyond the switch points, at which point the following train, still travelling at line speed, is precisely the minimum distance, TSD(b), behind the first train, but the switch has reset, so the diverging train is no longer in the path of the following train. (Pause for breath.) The buffer-end speed is given by $v_b = \sqrt{(v_t^2 - 2a_d b)}$.

As can be seen for switch type FV, this is already dangerously low. The buffer length b varies semi-linearly with speed; the quadratic v_t^2 decreases much faster, and very soon the quantity whose square root is required goes negative. Hence the Extended TSD standard is no longer applicable. This does **not** mean capacity calculations are no longer possible. What it **does** mean is that at these low speeds, the **Basic** Train Separation Distance standard applies.

TSD(b) means that there can be no deceleration on the main line. For this standard, therefore, the turnout limit speed for the applicable switch type must indeed be greater than or equal to the line speed (thus contradicting the second paragraph of the current section, for this special case). A diverging train must

travel the buffer distance beyond the switch points at constant line speed, with the following train precisely TSD(b) behind it, throughout. Only at that instant, when the switch has reset behind it, so that it is no longer in the path of the following train, can it begin its deceleration.

Whereas with the switch types dealt with above, their speed range of applicability is greater than or equal to their turnout limit speed, the switch types used with the basic TSD standard have speed range of less than or equal to their turnout limit speed. Switch type FV thus has two speed ranges of applicability, its above details being for TSD(e). In all the following switch types, the maximum time to reset the switch is 4sec. I deal with this by including a component equal to the distance travelled in 4s at the switch's turnout limit speed. This will be a slight over-estimate in most cases, but no matter.

Further switches from Vossloh Cogifer:

6. Switch Type DV:
Turnout Limit Speed $v_t = 13.408\text{m/s} / 48.27\text{kph} / 30\text{mph}$
Length of moving switch parts 29.174m.
7. Switch Type CV:
Turnout Limit Speed $11.174\text{m/s} / 40.23\text{kph} / 25\text{mph}$
Length of moving switch parts 24.877m.
8. Switch Type BV:
Turnout Limit Speed $8.939\text{m/s} / 32.18\text{kph} / 20\text{mph}$
Length of moving switch parts 21.337.

So we have the following, constant values:

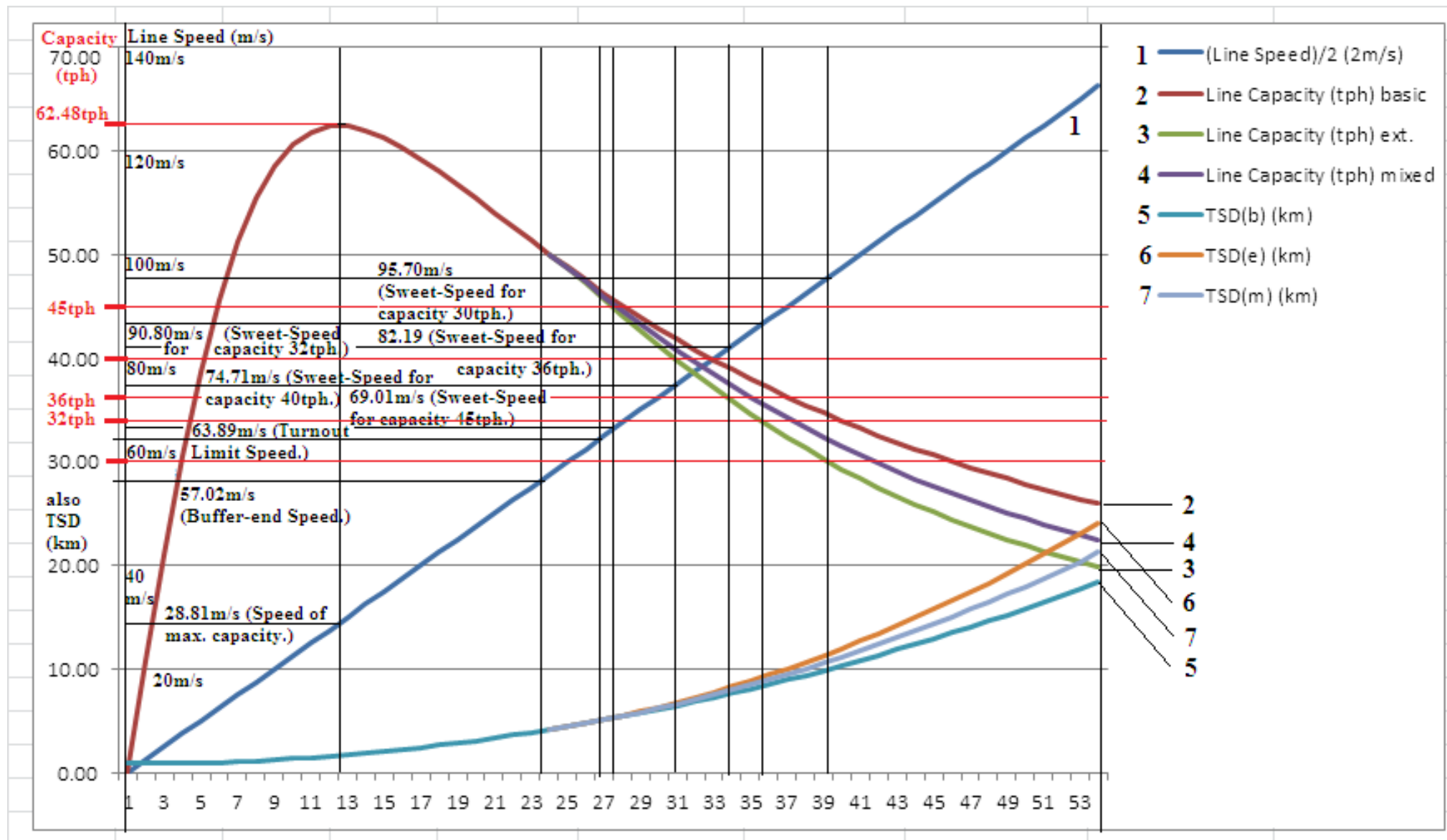
- | | | |
|-------------------|--------------------------|-------------------|
| 4. Switch Type FV | $v_t = 22.347\text{m/s}$ | $b = 540\text{m}$ |
| 5. Switch Type EV | $v_t = 17.878\text{m/s}$ | $b = 515\text{m}$ |
| 6. Switch Type DV | $v_t = 13.408\text{m/s}$ | $b = 485\text{m}$ |
| 7. Switch Type CV | $v_t = 11.174\text{m/s}$ | $b = 470\text{m}$ |
| 8. Switch Type BV | $v_t = 8.939\text{m/s}$ | $b = 460\text{m}$ |

Converging at a Switch

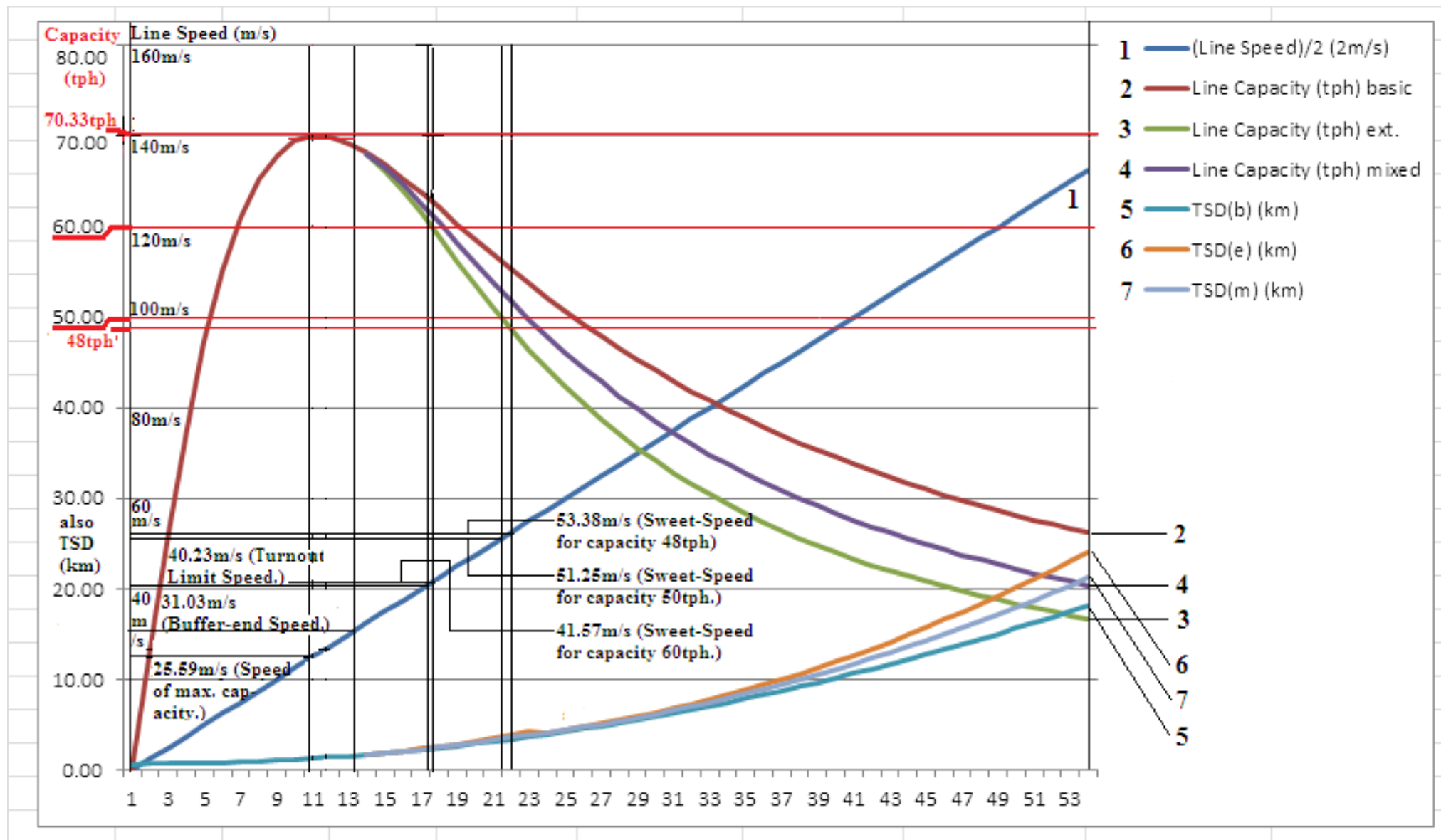
All previous considerations have dealt with the behaviour of trains when **diverging** at a switch. For completeness, the **converging** behaviour is now specified. This is in fact very simple and straightforward.

In the most usual case, where the train is re-joining the main line after calling at a station, and is travelling with constant acceleration, the train must reach the TLS (the Turn-**in** Limit Speed, in this case?) only when the train, in its entirety, has cleared all moving parts of the switch, thus when the **back-end** of the train has just cleared the switch points, onto the main line.

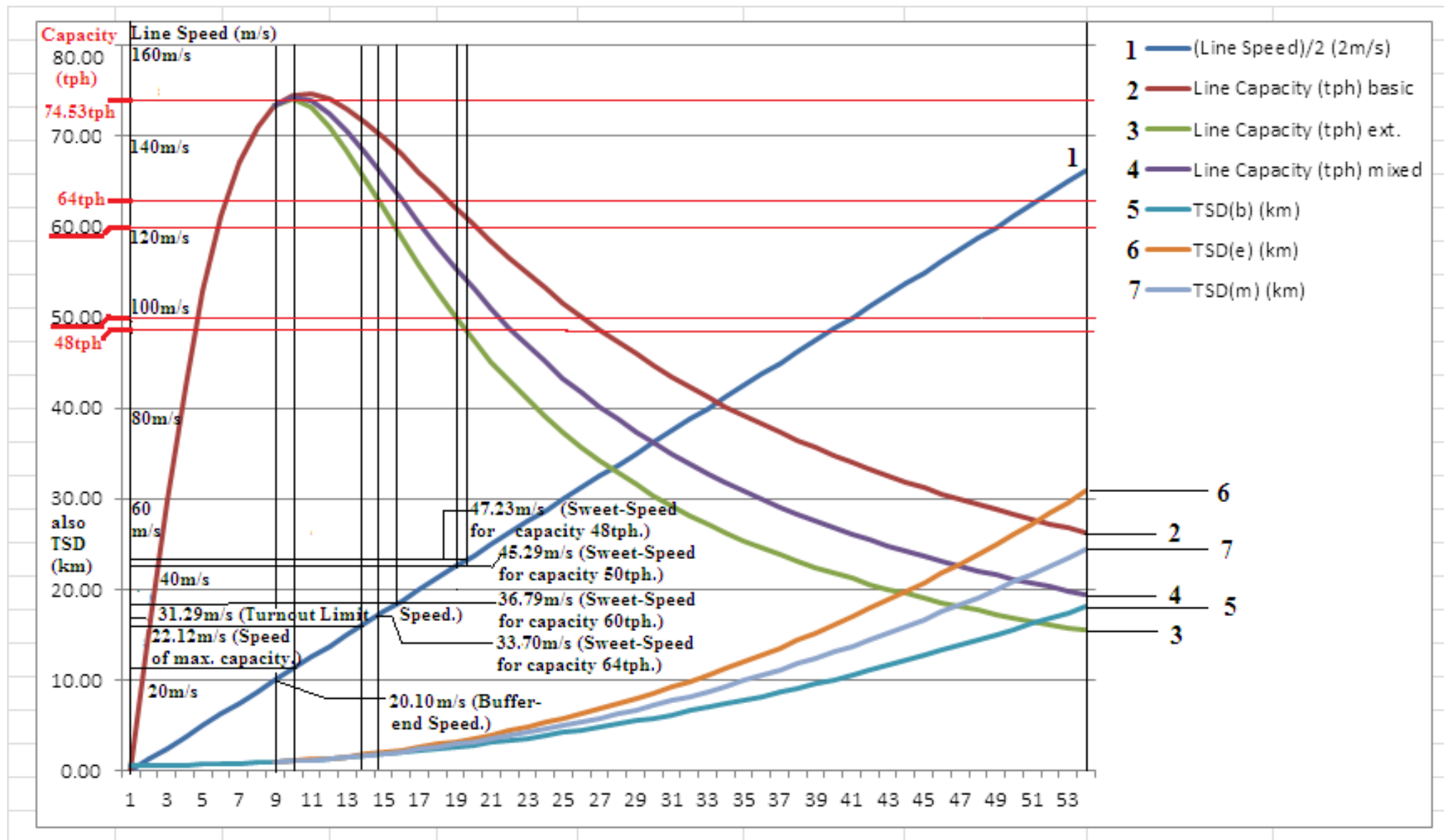
In the other case, when the train is joining the main line from a subordinate merging route, (i.e. the merging trains must perform the required speed changes to cross the switch,) the merging train must decelerate to the TLS by the time the front end reaches the moving parts of the switch, before the switch points, and continue at that steady speed until the back-end of the train has just cleared the switch points, onto the main line (thus, again, when the train, in its entirety, has cleared all moving parts of the switch, as before, but with a different speed profile).



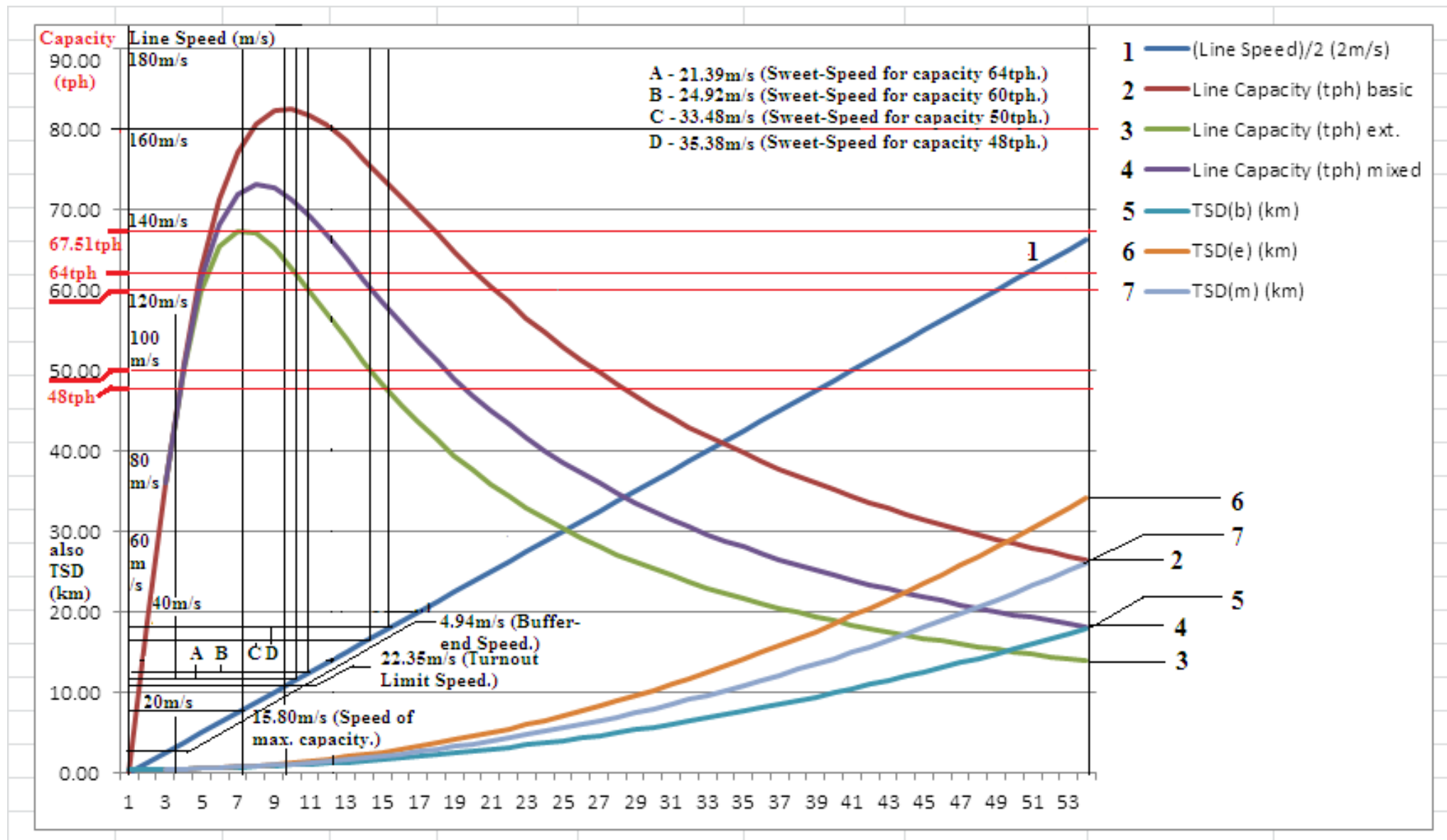
Capacity vs. Line Speed for the High Speed range, using Vossloh Cogifer Ultra High Speed Swing Nose Crossing with Manganese Cradle.



Capacity vs. Line Speed for the Medium Speed range, using Vossloh Cogifer switch type HV.



Capacity vs. Line Speed for the Low Speed range, using Vossloh Cogifer switch type GV..



Capacity vs. Line Speed for the Slow Speed range, using Vossloh Cogifer switch type FV.

Four characteristics stand out very clearly from the graphs:

1. **All** the available capacities (except 64tph) **could** be operated in all four speed ranges. (64tph is actually above the theoretical maximum capacity for the High Speed range, and its Sweet Speed for the Medium Speed range is below the turnout limit speed.)
2. The Sweet-Speeds corresponding to the available capacities are **very** strongly dependent on the switch type chosen. (Note that the available capacities, i.e. those that enable a usable, clock-face timetable, are the same for all speed ranges, except for 64tph, as noted above. This is unsurprising, as they are determined solely by integer arithmetic. But the Sweet Speeds corresponding to those capacities do depend intimately on the parameters of the speed range.)
3. For all speed ranges below Medium, the extended Train Separation Distance standard, TSD(e) applies to all speeds at or in excess of that of the theoretical maximum capacity. (And even for Medium, this is very nearly true. High Speed is the only range with a significant amount of **only** TSD(b) above the Maximum capacity. Refer to the graph on p.8.)
4. As the switch turnout limit speed decreases, the (theoretical) maximum capacity (calculated on TSD(b) increases, and the speed to which it corresponds decreases. The hump in the graph thus becomes higher and steeper. .

The summary tables of capacities and Sweet-Speeds are now given, for the four switch types. These apply for the case of a mixture of non-stop and overtaking services, and as usual, my recommendations are highlighted in red. But note that I recommend only for capacities relevant to the speed range. Specifically this means that recommendations for the High Speed range are given only for capacities of 45,40,36,32 and 30tph. For the other ranges, however, recommendations are given for all capacities greater than or equal to 48tph, provided the Sweet-Speed exceeds the turnout limit speed for that switch. Thus capacities 48, 50 and 60 are added for switch type HV, but not for 64tph since its Sweet-Speed is less than the TLS for that switch. Likewise 64tph is added to the Low Speed range, but not 72tph. Finally, 72tph disappears altogether from the Slow Speed range, as it is now over the maximum capacity, as are 75 and 80tph. The reason for this is that there are several considerations to be taken into account when deciding which capacity to select, Most important is the Sweet-Speed itself but also of note are the minimum inter-station distances and the station wait times. 48tph may well be appropriate for all three speed ranges, offering the ideal speed of 120mph for routes such as the WCML, ECML, GWML and MML, which already have significant stretches of 125mph. Likewise, 105mph could be ideal for routes such as Birmingham to Bristol and the West Country, which has superb alignments, but has never been properly developed as an express route, rather than cross-country. Finally, 80mph could bring palpable enhancement to country lines.

Note further that these are genuine Sweet Speeds. They are the maximum speed possible for that capacity in that speed range. The slot time is constant for a particular capacity, in **any** speed range, but the slot **length** depends on the line speed, and is at its maximum, equal to TSD(e), when that line speed is equal to the Sweet Speed.

(I am being necessarily pedantic here, because it is possible to operate at **any** line speed **less than the Sweet Speed**, while maintaining the same line capacity. In such a case the slot length is simply the distance travelled in the slot time at that lower speed, and while it is less than the value for the Sweet Speed, it is nonetheless **larger** than TSD(e) for that (non-sweet) speed. The slot length is at its maximum for that capacity and speed range, and is precisely equal to TSD(e), when the line speed is equal to the Sweet Speed. This sounds wrong, but the key fact here is that the slot length, calculated as slot time

multiplied by line speed, depends linearly on the speed, but the Train Separation Distance depends on the square on the line speed. It thus both increases and decreases faster than the linear slot length.

(This is conceptually very difficult stuff, and it has taken me several months to get at ease with it.)

UPS Switch type:

Line Capacity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
60	60	38.37	138.12	85.83	3.92 / 2.44	4 / 5 / 6	138 / 198 / 258	4 / 5 / 6
50	72	57.58	207.29	128.81	8.84 / 5.49	5	206	6
48	75	61.15	220.13	136.79	9.97 / 6.19	4 / 6 / 8	212 / 287 / 362	5 / 7½ / 10
45	80	69.01	248.44	154.38	11.68 / 7.26	5	216	6⅔
40	90	74.71	268.94	167.12	14.88 / 9.24	5 / 8	251 / 521	7½ / 12
36	100	82.19	295.90	183.87	18.02 / 11.19	4 / 6	181 / 381	6⅔ / 10
32	112.5	90.80	326.87	203.11	21.98 / 13.65	4 / 8	208 / 433	7½ / 15
30	120	95.70	344.51	214.08	24.42 / 15.17	5 / 6	345 / 465	10 / 12
25	144	110.57	398.04	247.34	32.60 / 20.24	5	425	12
24	150	114.14	410.92	255.34	34.74 / 21.57	6 / 8	596 / 896	15 / 20

Switch Type HV:

Line Capacity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
64	56.25	37.70	135.71	84.53	3.79 / 2.36	4 / 8	124 / 349	3¾ / 7½
60	60	41.57	149.66	93.00	4.61 / 2.86	4 / 5 / 6	129 / 189 / 249	4 / 5 / 6
50	72	51.25	184.49	114.64	7.00 / 4.35	5	223	6
48	75	53.38	192.15	119.40	7.60 / 4.72	4 / 6 / 8	158 / 308 / 458	5 / 7½ / 10
45	80	56.79	204.43	127.03	8.60 / 5.34	5	249	6⅔
40	90	63.24	227.66	141.47	10.66 / 6.63	5 / 8	281 / 551	7½ / 12
36	100	69.37	249.73	155.18	12.83 / 7.98	4 / 6	215 / 415	6⅔ / 10
32	112.5	76.74	276.27	171.67	15.70 / 9.76	4 / 8	245 / 695	7½ / 15
30	120	81.05	291.79	181.31	17.52 / 10.89	5 / 6	384 / 504	10 / 12
25	144	94.47	340.09	211.33	25.48 / 15.84	5	468	12
24	150	97.76	351.93	218.68	25.48 / 15.84	6 / 8	639 / 939	15 / 20

48tph is a superb capacity, second in my affections only to 32tph.

Switch Type GV:

Line Capacity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
72	50	26.90	96.86	60.19	1.93 / 1.20	4 / 6 / 8	128 / 228 / 328	3½ / 5 / 6⅔
64	56.25	33.70	121.31	75.38	3.03 / 1.88	4 / 8	135 / 360	3¾ / 7½
60	60	36.79	132.46	82.31	3.61 / 2.24	4 / 5 / 6	142 / 202 / 262	4 / 5 / 6
50	72	45.29	163.04	101.31	5.47 / 3.40	5	239	6
48	75	47.23	160.04	105.66	5.95 / 3.70	4 / 6 / 8	174 / 324 / 474	5 / 7½ / 10
45	80	50.38	181.37	112.70	6.77 / 4.21	5	266	6⅔
40	90	56.42	203.12	126.21	8.49 / 5.28	5 / 8	300 / 570	7½ / 12
36	100	62.23	224.03	139.21	10.33 / 6.42	4 / 6	234 / 434	6⅔ / 10
32	112.5	69.28	249.41	154.98	12.80 / 7.96	4 / 8	265 / 715	7½ / 15
30	120	73.43	264.35	164.26	14.38 / 8.94	5 / 6	404 / 764	10 / 12
25	144	86.43	311.16	193.35	19.92 / 12.38	5	490	12
24	150	89.64	322.69	200.51	21.43 / 33.32	6 / 8	661 / 961	15 / 20

Switch Type FV:

Line Capacity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
80	45	28.09	101.12	62.84	2.10 / 1.31	4 / 5 / 8	105 / 150 / 285	3 / 3¾ / 6
75	48	34.05	122.58	76.17	3.09 / 1.92	5	149	4
72	50	37.25	134.09	83.32	3.70 / 2.30	4 / 6 / 8	101 / 201 / 301	3½ / 5 / 6⅔
64	56.25	21.39	77.01	47.85	1.22 / 0.76	4 / 8	168 / 393	3¾ / 7½
60	60	24.92	89.71	55.74	1.66 / 1.03	4 / 5 / 6	174 / 234 / 294	4 / 5 / 6
50	72	33.48	120.53	74.90	2.99 / 1.86	5	271	6
48	75	35.38	127.38	79.15	3.34 / 2.07	4 / 6 / 8	206 / 356 / 506	5 / 7½ / 10
45	80	38.44	138.40	86.00	3.94 / 2.45	5	297	6⅔
40	90	44.30	159.49	99.11	5.23 / 3.25	5 / 8	332 / 602	7½ / 12
36	100	49.94	179.78	111.71	6.65 / 4.13	4 / 6	267 / 467	6⅔ / 10
32	112.5	56.79	204.45	127.05	8.60 / 5.35	4 / 8	299 / 749	7½ / 15
30	120	60.83	219.01	136.09	9.87 / 6.13	5 / 6	438 / 558	10 / 12
25	144	73.54	264.76	164.52	14.42 / 8.96	5	524	12
24	150	76.68	276.06	171.54	15.68 / 9.75	6 / 8	696 / 996	15 / 20

Note the discontinuities in speed, inter-station distance and station wait times between the capacity values 72 and 64. The maximum capacity value when operating with the extended TSD standard, TSD(e), is 67.51m/s. Capacities 72, 75 and 80tph simply do not exist for that standard; the values quoted in the table are for TSD(b), hence the discontinuities. This should be perfectly clear from the Capacity vs. Line Speed graph for switch type FV.

The equivalent results for pure metros are:

Switch Type HV:

Line Capacity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
64	56.25	16.46	59.26	36.83	0.72 / 0.45	2 / 4	69 / 181	1 ⁷ / ₈ / 3 ³ / ₄
60	60	14.35	51.65	32.10	0.55 / 0.34	2 / 3 / 4 / 5 / 6	82 / 142 / 202 / 262 / 322	2 / 3 / 4 / 5 / 6
50	72	10.68	38.81	23.90	0.30 / 0.19	2	116	2.4 = 2m24s
48	75	10.09	36.33	22.57	0.27 / 0.17	2 / 3 / 4	123 / 198 / 273	2 ¹ / ₂ / 3 ³ / ₄ / 5

Switch Type GV:

Line Capacity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
72	50	17.93	64.54	40.11	0.86 / 0.53	2 / 3 / 4	52 / 102 / 152	1 ² / ₃ / 2 ¹ / ₂ / 3 ¹ / ₃
64	56.25	13.43	48.34	30.04	0.48 / 0.30	2 / 4	77 / 189	1 ⁷ / ₈ / 3 ³ / ₄
60	60	11.97	43.10	26.78	0.38 / 0.24	2 / 3 / 4 / 5 / 6	88 / 148 / 208 / 268 / 328	2 / 3 / 4 / 5 / 6
50	72	9.15	32.93	20.47	0.22 / 0.14	2	120	2.4 = 2m24s
48	75	8.67	31.21	19.39	0.20 / 0.12	2 / 3 / 4	127 / 202 / 277	2 ¹ / ₂ / 3 ³ / ₄ / 5

Switch Type FV:

Line Capacity (tph)	Slot time (sec)	Line Speed (m/s)	Line Speed (kph)	Line Speed (mph)	Minimum Inter Station Distance (km / miles)	Slot Stream Advance (integer Slots)	Station Wait Time (sec)	Clock-Face Timetable (every ↓ min)
64	56.25	11.67	42.02	26.11	0.36 / 0.23	2 / 4	81 / 194	1 ⁷ / ₈ / 3 ³ / ₄
60	60	10.02	36.07	22.42	0.27 / 0.17	2 / 3 / 4 / 5 / 6	93 / 153 / 213 / 273 / 333	2 / 3 / 4 / 5 / 6
50	72	7.46	26.85	16.68	0.15 / 0.09	2	124	2.4 = 2m24s
48	75	7.06	25.41	15.79	0.13 / 0.08	2 / 3 / 4	131 / 206 / 281	2 ¹ / ₂ / 3 ³ / ₄ / 5

These are not indicated explicitly on the graphs, but correspond to the point where the horizontal red line for that capacity intersects the **ascending** side of the graph.

Change of Line Speed for a Same Speed Railway

Basic Principles:

While the line speed of a Same Speed Railway remains constant within each section of the route, it is possible that the line speed may be different in adjacent sections. In such a situation, trains travelling from the higher speed section into the lower speed one must perform all their deceleration in the higher speed section, crossing the section boundary only when they have reached the lower speed. (Going in the reverse directions, trains would cross the section boundary, and begin their acceleration once completely inside the higher speed section. There is no problem with this; the problems are all concerned with deceleration.)

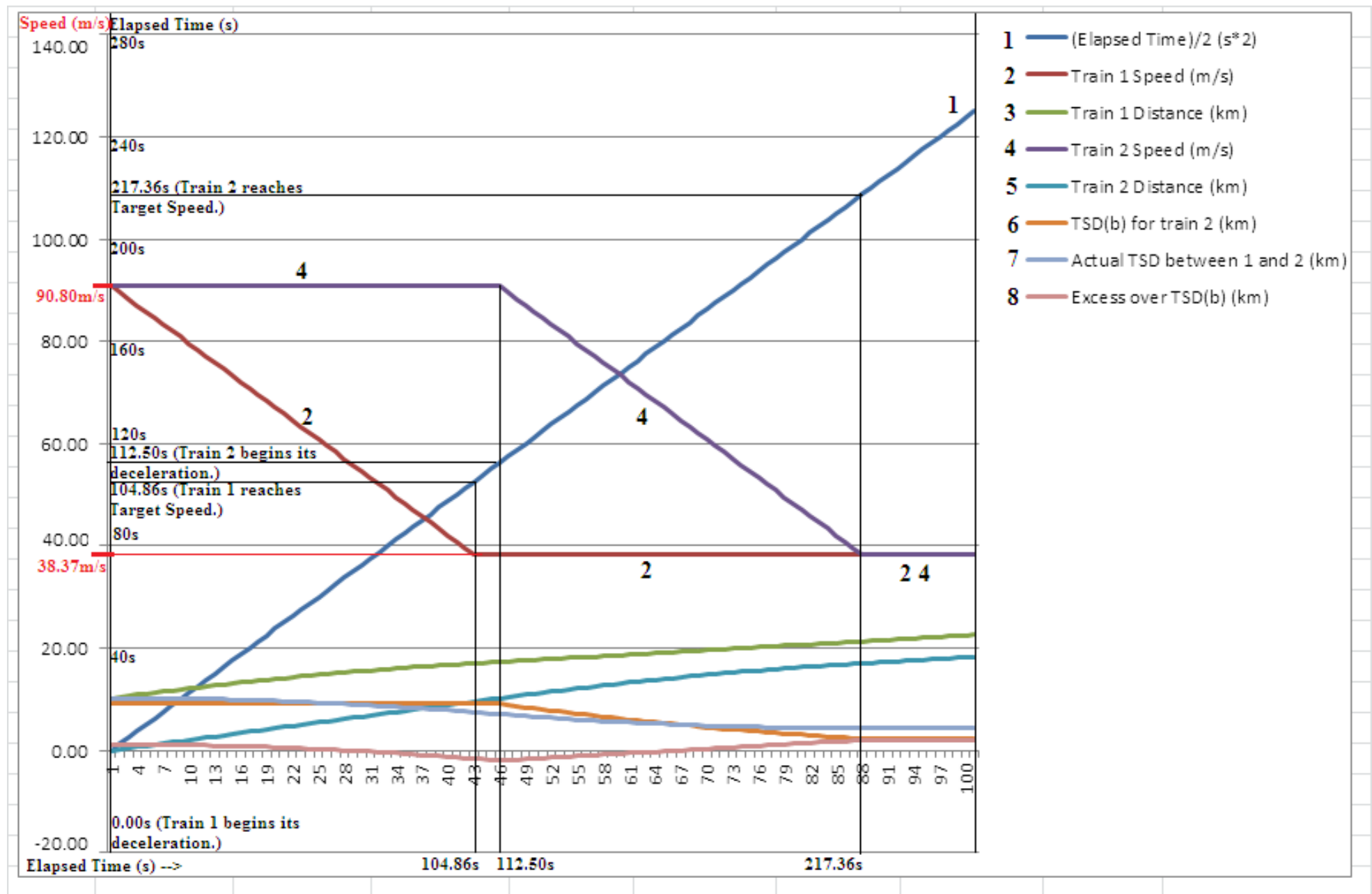
For new-build High Speed routes, (or at least for the High Speed trunk portions thereof,) this really shouldn't be an issue. All sections should be implemented for the same line speed, and trains should perform the entire journey at that line speed. Note that this includes provision for intermediate station calls, with or without overtaking. What this section is about is the global deceleration of the entire traffic stream, with trains and their containing capacity slots decelerating in lockstep together, the capacity slot(s) changing in size during the process. On completion of the deceleration, the traffic stream continues with the new values of line speed and capacity slot. Note further that the deceleration begins, for all trains, at a particular, fixed location, and is complete by a second, fixed location.

For Same Speed Railways in the Medium Speed range, this is a much more serious issue. These will normally be conversions to Same Speed standards of conventional routes, and will almost certainly have line speed variations as a legacy of their origins, which it will often not be practicable to correct. Even new High Speed routes may have the requirement as a (hopefully!) temporary implementation issue, where sections of conventional route are incorporated in order to get some services running as soon as possible, rather than wait for the whole thing to be complete before opening.

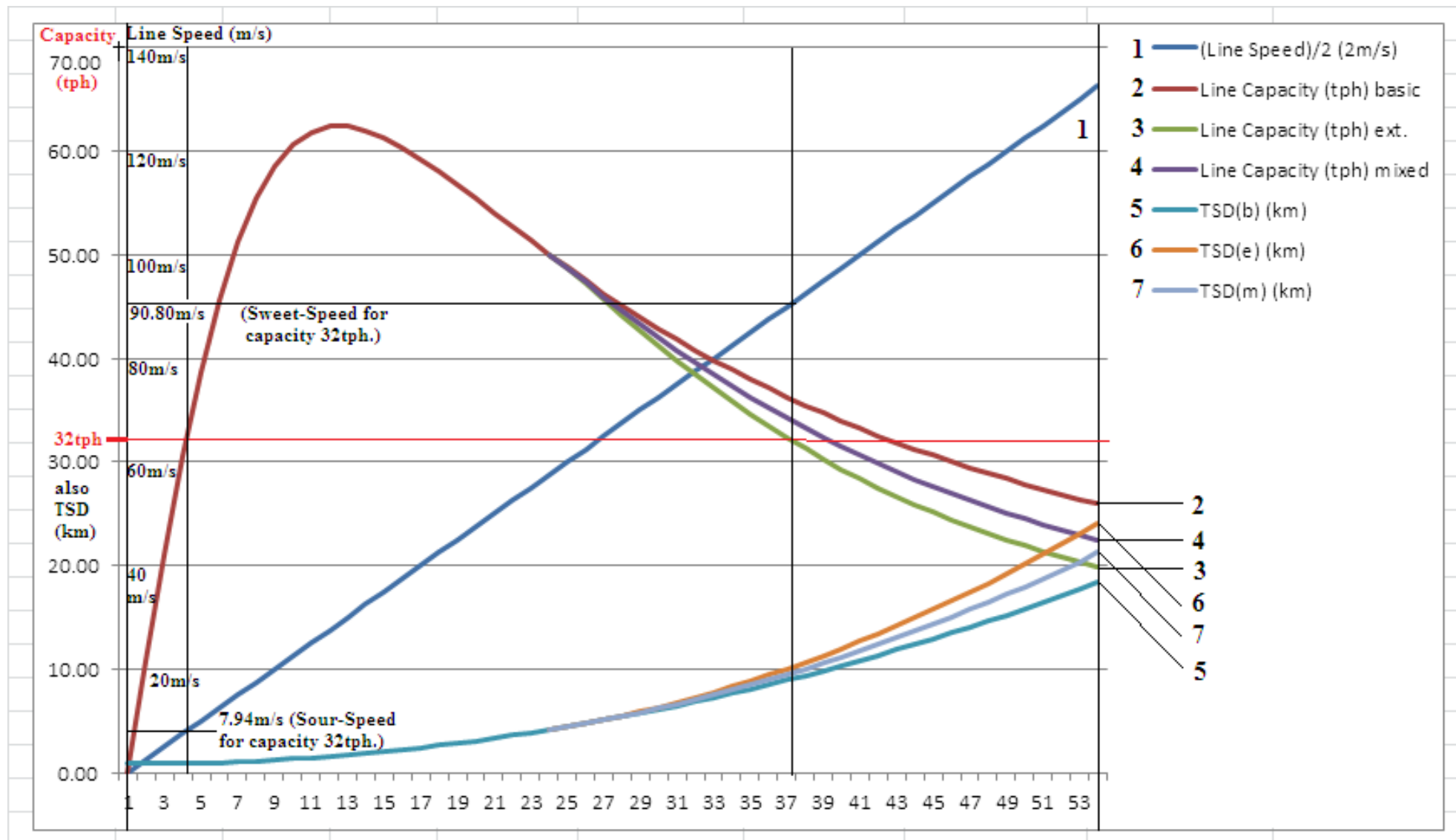
This is an extraordinarily complicated subject, with seemingly endless ramifications. We first need to establish some basic principles.

Deceleration behaviour is depicted on Deceleration Graphs, for which the independent variable is elapsed time, and the graphs are focussed on either speed or distance travelled, since different scales are required, to display these legibly and intelligibly. The next page contains a simplified Deceleration / Speed graph. The same graph will be encountered again later, with much extra detail, but, for now, I wish to focus attention on its characteristic lozenge shape. The behaviour of two adjacent trains is illustrated, but it must be clearly understood that these represent a continuous stream of identically-behaving trains. The graph represents the behaviour of trains travelling at the Sweet-Speed, 90.80m/s, for line capacity 32tph, in the High Speed range, and decelerating to a Target Speed of 38.37m/s in the Low Speed range. This latter value has no intrinsic significance, beyond its location in a lower speed range. Remember that the quantities on the horizontal scale are elapsed times, **not** distances; do not even **think** of distances, in the present context. The vertical scale is line speed.

Elapsed times are included, but ignore them for now. Train 1's speed history is defined by graph line 2, and Train 2's by graph line 4. Manifestly, the times taken to decelerate from the Sweet-Speed to the Target Speed for trains 1 and 2 are identical (the inclined sides of the lozenge). Therefore, so are the times taken by train 2 in travelling at Sweet-Speed until it begins its own deceleration, and that taken by train 1 between reaching the Target Speed and train 2 reaching the Target Speed (the horizontal sides of the lozenge). The two trains travel the same amount of time at different speeds. What this means is that they



Deceleration / Speed Graph.



Speed Range for Line Capacity 32tph (Sweet-Speed in HS Range).

are travelling with the same level of capacity, 32tph. They do not travel the same **distances** at different speeds, but they do travel the same **times**. This is the capacity slot time, 112.5s, for 32tph, and is the same for this capacity for all speeds in all speed ranges. (Of course it is; it is equal to 3600 divided by the line capacity in tph.)

This is a fundamental property of Same Speed Railways: they naturally, ordinarily and automatically maintain constant line capacity while reducing line speed. (Strictly speaking, they have the same line capacity after decelerating as they had before; the instantaneous capacity value while one train is decelerating while its successor isn't – yet – is moot.)

One might well describe them as ισο χωρητικότητα (iso-choritikotita), or constant-capacity.

The point about the Same Speed model is that, like any mathematical model, it enables the ready mathematical analysis of such properties, and thus their discovery and elucidation. The diagram on the preceding page is an amended version of the standard Capacity vs. Line Speed graph, with a horizontal red line added for capacity 32tph. This intersects the graph (strictly the graph for the extended train separation standard) at two points, one either side of the (theoretical) maximum capacity. That on the right hand side is the maximum speed at which that capacity is available; that on the left the minimum. But the railway may be operated with that line capacity at **any** speed within that range. For these intermediate speeds, that would not be the optimum capacity; a higher value would always theoretically be possible. But it does enable enormous operational flexibility, in that a given timetable model may be run at any speed. Careful readers will doubtless protest at that claim, quite justifiably, so it is important to elucidate precisely what it means. Constant capacity means that a particular number of trains pass a given point in a specified time, generally one hour. Alternatively stated, a train passes a given point every n seconds, where n is the capacity slot time. As the line speed decreases, the trains get closer together (which, incidentally, provides a conceptual proof of the necessity for a minimum speed – if they travel sufficiently slowly the front end of one train connects with the back end of its predecessor). If the line speed were reduced by 50% at constant capacity, it would be just as busy, in that the same number of trains per hour would pass an intermediate point, but they would all take twice as long to reach their destination. In a sense, capacity and journey time bear the same **sort** of relationship as topology and geometry.

Summarising: for any particular capacity value, up to and including the theoretical maximum, there is a particular range of speeds, identified by the maximum end of the range, at any intermediate speed of which a Same Speed railway could, in theory, be operated. I call these the **Deceleration Ranges**. In practice, there are very, very few capacities and speed ranges for which a viable, usable timetable could be scheduled. These, very few, speed ranges have, as their maxima, the **Sweet-Speeds**. (Their corresponding minima, by analogy, would be the Sour-Speeds?) Some imprecise analogy may well apply to conventional railways, as a concept though not as a property.

Referring back to the Deceleration / Speed graph, train 2 is travelling at a higher speed than train 1 until train 2 itself reaches the target speed. Throughout this entire time, it would continue to get closer to train 1, if they were on the same track. Train 1 has used up most of its extended separation by the time it reaches the turnout limit speed (and all of it by the time it reaches the buffer-end speed,) so the main line must split into two deceleration tracks at the point at which the turnout limit speed is reached. Train 1 diverges onto track 1, and, by the time it has reached the buffer-end speed, the switch has been reset behind it to point to the other deceleration track, so it is no longer in the path of train 2.

This is the second, fundamental aspect of line speed deceleration on Same Speed railways: it **always** involves a pair of deceleration tracks. (As always, I'm dealing with maximum capacity operation, a continuous stream of trains.)

An alternative formulation of this second aspect is that a train cannot simply decelerate at will. Deceleration is possible only at prescribed locations, where the necessary infrastructure of deceleration tracks is available.

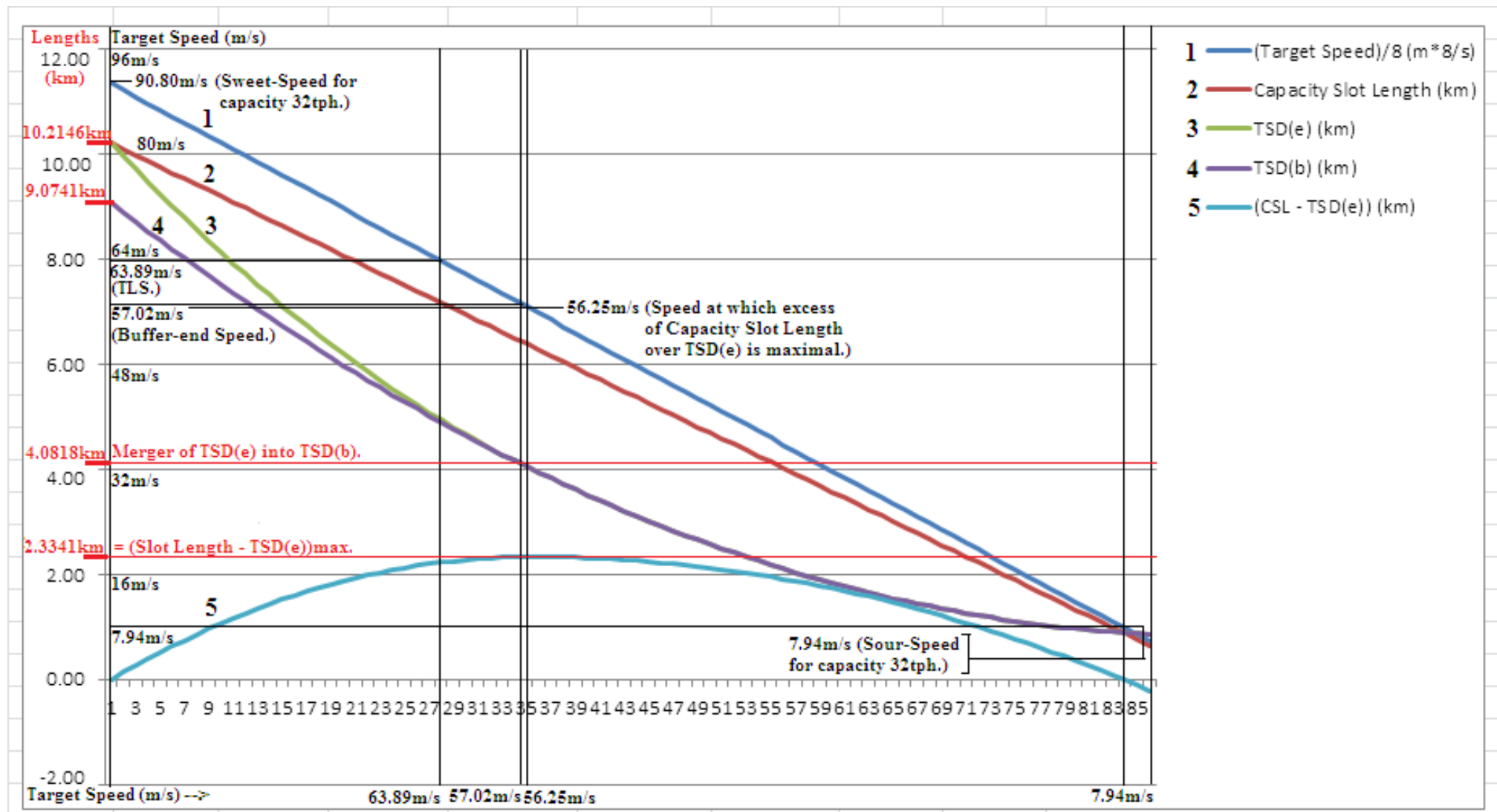
Same Speed Railways are, naturally and, in practice, almost invariably, constant capacity railways. It would, in theory, be possible to change the capacity, but this would involve every train stopping at a particular station, and a whole new timetable take effect there. I can't really see any practical point to this beyond the very special case of trans-metropolitan traverses, such as merging the 32tph HS2 services from the West Midlands and North West, and the 32tph HS4 services from Bristol, South Wales and the West Country, at Old Oak Common, into a 64tph pure metro service via Euston Cross out to Stratford HS South, where they split into 32tph HS1 services to Kent and West Sussex, and 32tph HS11 and HS12 services to North Kent and East Anglia. (Likewise, the 32tph HS3 services from the East Midlands, Yorkshire, the North East and Scotland, and the 32tph HS6 and HS10 services from Humberside, West Yorkshire, Lincolnshire and West Anglia are merged at Pancras Cross into the 64tph HS5 High Speed metro services to Sussex and Hampshire.)

Deceleration Ranges:

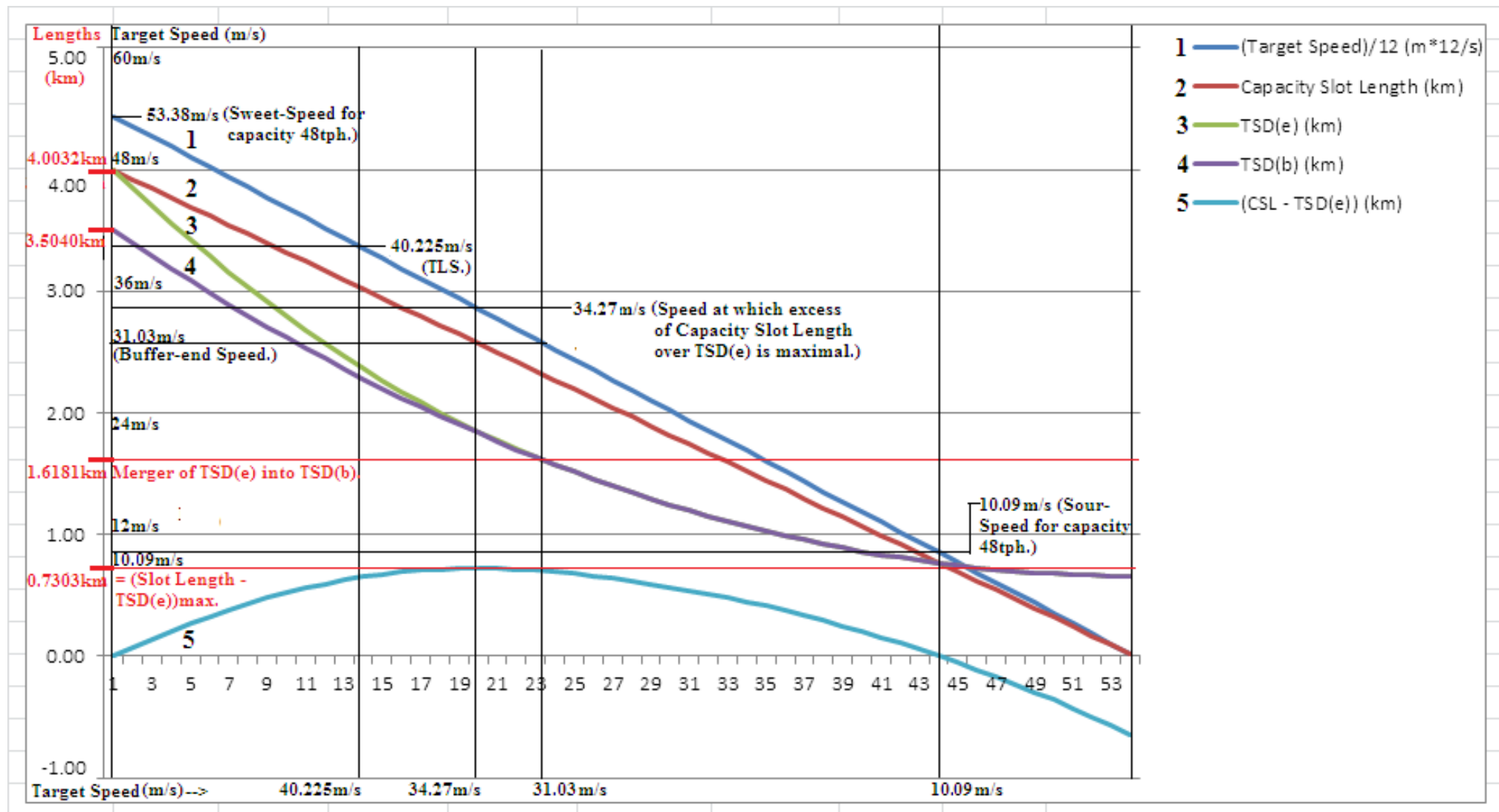
So, while, having fixed on the capacity for a particular Same Speed route, we have a constant value for the capacity time slot, it is worth considering how the capacity distance slot varies, as line speed varies within the available range, the **Deceleration Range**. The next four pages contain illustrative examples for each of the four speed ranges (and switch types). The initial speed is the maximum, the Sweet-Speed for that capacity and speed range / switch type. The other end of the deceleration range (of Target Speeds) is the Sour-Speed, the minimum speed available for that line capacity and speed range / switch type. (The lines on the graph continue beyond the Sour-Speed, but the quantities involved have no physical existence below that minimum speed.)

At the Sweet-Speed maximum, the capacity slot length is equal to TSD(e). At the Sour-Speed minimum, the capacity slot length is equal to TSD(b)), (**usually** – the fourth example, 64tph in the Slow Speed range / switch type FV is an exception; TSD(e) has not yet merged – or ever will – with TSD(b)). Over that entire range, the capacity slot length – a straight line on the graph – is greater than TSD(e)/TSD(b). This difference is semi-parabolic (the constant value of buffer length is involved also); the maximum value is indicated, together with the speed at which it occurs (derived by a simple bit of calculus – no big deal). The value of TSD(e)/TSD(b) at their merge point is likewise included, as is its location – at the buffer-end speed, of course.

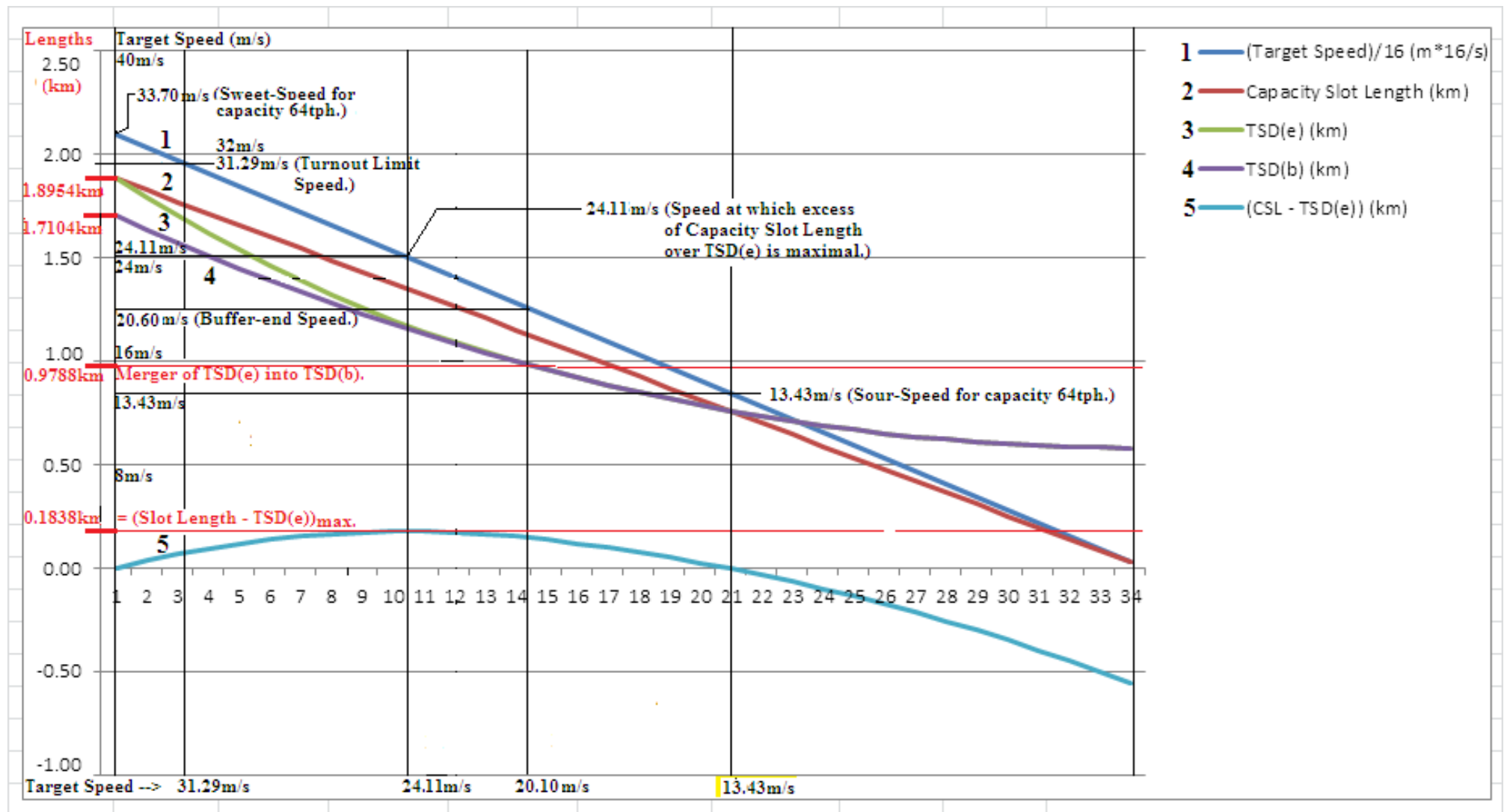
I well recognise that this stuff is very difficult to envisage, and hope these illustrative examples of the deceleration ranges prove helpful. I personally find the deceleration range a very helpful unifying concept in the complex of ideas that the Same Speed model generates and displays.



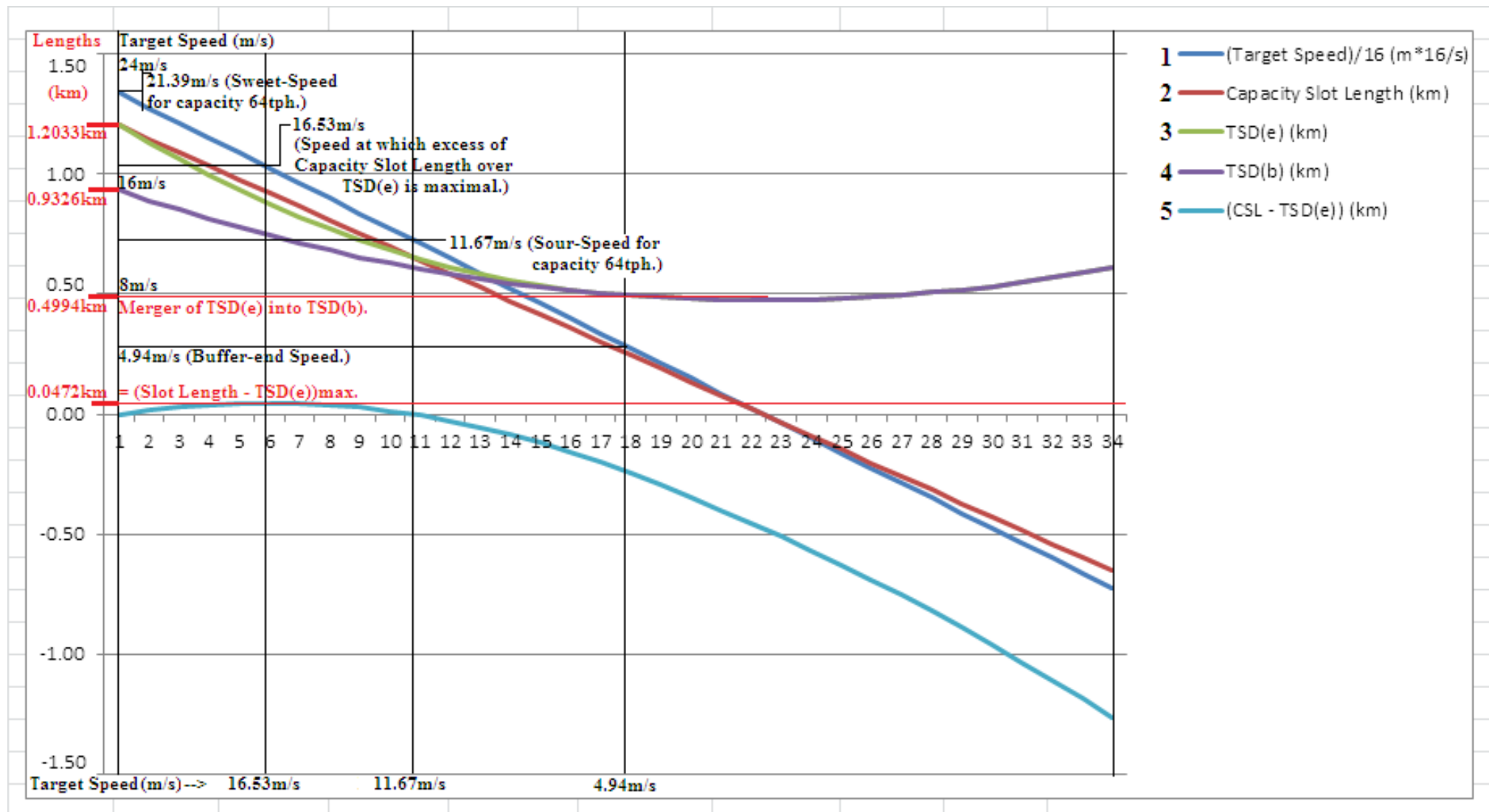
Deceleration Range for High Speed Capacity 32tph, Sweet-Speed 90.80m/s. Sour-Speed 7.94m/s.



Deceleration Range for Medium Speed Capacity 48tph, Sweet-Speed 53.26m/s. Sour-Speed 10.18m/s.



Deceleration Range for Low Speed Capacity 64tph, Sweet-Speed 33.48m/s. Sour-Speed 13.60m/s.



Deceleration Range for Slow Speed Capacity 64tph, Sweet-Speed 33.48m/s. Sour-Speed 13.60m/s.

Summarising for all relevant capacities in all ranges:

Line Capacity (tph)	Slot time (sec)	HS Range Switch Type UHS		MS Range Switch Type HV		LS Range Switch Type GV		SS Range Switch Type FV	
		Sweet-Speed (m/s)	Sour-Speed (m/s)	Sweet-Speed (m/s)	Sour-Speed (m/s)	Sweet-Speed (m/s)	Sour-Speed (m/s)	Sweet-Speed (m/s)	Sour-Speed (m/s)
64	56.25					33.70	13.43	21.39	11.67
60	60	38.37 *	21.63	41.57	14.35	36.79	11.97	24.92	10.02
50	72	57.58	14.41	51.25	10.68	45.29	9.15	33.48	7.46
48	75	61.15	13.49	53.38	10.09	47.23	8.67	35.38	7.06
45	80	69.01	12.25	56.79	9.26	50.38	7.98	38.44	6.50
40	90	74.71	10.43	63.24	7.99	56.42	6.92	44.30	5.64
36	100	82.19	9.13	69.37	7.05	62.23	6.13	49.94	5.00
32	112.5	90.80	7.94	76.74	6.16	69.28	5.37	56.79	4.39
30	120	95.70	7.37	81.05	5.73	73.43	5.00	60.83	4.10
25	144	110.57	6.02	94.47	4.70	86.43	4.11	73.54	3.38
24	150	114.14	5.75	97.76	4.50	89.64	3.94	76.68	3.24

* 38.37m/s may look wrong, but it isn't. Capacity 60tph in the HS range is **only** available for TSD(b) standard. TSD(e) is only available from the Buffer-end Speed of 57.02m/s. (Refer to graph on p.8.) Every other Sweet-Speed in the table is for TSD(e).

Deceleration Tracks:

An interesting question is how long the deceleration tracks need to be. In order to explain this, we must consider precisely how the (notional) distance between the trains – as it would be if they were both on the same track – varies. The main-line bifurcates into two deceleration tracks at the point where the decelerating train reaches turnout limit speed. When train 1 reaches the buffer-end speed, The switch has just been reset behind it, to point to the other deceleration track, so it is no longer in the path of the following train. At this point, train 2 is precisely the basic separation, TSD(b), behind it. As the trains continue, train 2 continues to get closer to train 1, closer than permissible if they were on the same track. This persists until train 2 itself begins its own deceleration. Up until this point, TSD(b) for train 2 has a constant value. After this point, train 2 continues to get closer to train 1, **but** TSD(b), for train 2, since it depends on the square on (instantaneous) line speed, decreases faster still, so that, quite quickly, the **actual** separation distance once again exceeds TSD(b). So although the trains are still getting physically closer together, and continue to do so until train 2 itself reaches the target speed, they are getting further apart relative to the instantaneous value of TSD(b) for train 2. So the deceleration track comes to its end when it is no longer needed, when the actual distance between the trains once again exceeds TSD(b), calculated on the instantaneous speed of train 2. Hopefully, given the explanatory build-up, that **formal** definition of the end of the deceleration track is now intelligible.

It will almost certainly come as a surprise that the length of the deceleration tracks is constant, for a given speed range and switch type. It is fairly simple to demonstrate (see the 'Same Speed Railways' article) that the deceleration track length

$$s_{dt} = v_t^2 / 2a_d + b, = \text{TSD}(b)_{vt}$$

This applies universally (for a given switch and speed range), since v_t , b and a_d are invariant. The length of the deceleration track, for any line capacity and thus (Sweet-) line speed to any target speed is the same, and equal to the Basic Train Separation Distance for the Turnout Limit Speed.

The values for the four speed ranges and switch types are:

- | | |
|--------------------|-----------------------------|
| 1. UHS switch type | $s_{dtH} = 4.9118\text{km}$ |
| 2. Switch Type HV | $s_{dtM} = 2.2731\text{km}$ |
| 3. Switch Type GV | $s_{dtL} = 1.5538\text{km}$ |
| 4. Switch Type FV | $s_{dtS} = 0.9744\text{km}$ |

Despite much thought, I have yet to find an explanation of this surprising result. I would merely draw attention to a similar one (whose justification is, however, immediately clear). The length of the (physical) station loop, is also invariant for a given speed range and switch type, since it is the deceleration to zero and reacceleration length from and to the Turnout Limit Speed.

A further interesting quantity is the time that a train spends on the deceleration track:

$$t_{dt} = v_l/a_d - (v_g^2/2a_d - b)/v_g$$

This does vary with the target speed, v_g , which is not surprising, but with nothing else, which perhaps is.

Actually, the above result for time is not quite correct. It does, correctly, give the time that the **front end** of the train spends on the deceleration track. But we have also to consider the time taken for the train **completely** to clear the end-of-deceleration-track switch. This is, in fact, trivial: it is:

$$400/v_g + 4 \text{ or } 5 \text{ seconds.}$$

400m is the standard train length, so $400/v_g$ sec for the back end to clear the switch, to which should be added the time for the switch to reset behind the train – 4sec for switch types UHS and FV, and 5sec for types HV and GV.

It is of mainly theoretical interest, but the length of deceleration track is **not** constant for operation under the Basic TSD standard, but its form is. Here, the formula is:

$$s_{dt} = (v_l^2/2a_d + b) + b = \text{TSD}(b)_{v_l} + b$$

This is precisely analogous to the result for the Extended TSD standard, above. But note that:

1. this depends on the **line speed**, v_l , rather than the turnout limit speed, v_t , since that is the speed, (which, for the basic standard, is less than or equal to the TLS,) at which the train is travelling when it crosses the switch and enters the deceleration track,
2. the extra buffer-length value reflects the fact that the train passes completely, (i.e. the buffer-length distance, at which point the switch has just been reset behind it,) onto the deceleration track at constant line speed. Only at that point, when it is no longer in the path of the following train, can it even begin its deceleration.

The time it spends on the deceleration track is given by:

$$t_{dt} = b/v_l + v_l/a_d - (v_g^2/2a_d - b)/v_g$$

(plus the end-of-track correction, as explained above).

Again, this is the same as for the Extended TSD standard, excepting only the additional time, b/v_l , spent travelling at constant line speed. (See the ‘Same Speed Railways’ article for the proof.)

Note that the Basic TSD standard cannot apply to the High Speed range (and switch type UHS) **precisely because** it requires that the turnout limit speed is greater than or equal to the line speed; there **is** no switch type faster than UHS.

Although the deceleration track length is no longer constant, it is nonetheless interesting to derive results where the line speed is equal to the TLS of the switch:

- | | |
|--|-----------------------------|
| 2. Medium Speed range but UHS switch type | $s_{dtM} = 5.7418\text{km}$ |
| 3. Low Speed range but Switch Type HV | $s_{dtL} = 2.9281\text{km}$ |
| 4. Slow Speed range but Switch Type GV | $s_{dts} = 2.1288\text{km}$ |
| 5. Extra-Slow Speed range but Switch Type FV | $s_{dtE} = 1.4494\text{km}$ |

These are, of course, the same as the previous set, above, plus the relevant buffer length.

Although this case is indeed of almost purely theoretical interest, it is gratifying (and reassuring) to see how well it agrees with the Extended TSD standard.

Specimen Decelerations:

There are three possible and quite separate types of deceleration to consider:

1. The Target Speed is in a lower speed range than the normal (Sweet-) line speed.
The first train decelerates down to the TLS on reaching the switch, and continues decelerating at uniform rate on the deceleration track until it reaches the target speed, after which it continues at that constant speed until it reaches the end of that track (at which point, as defined earlier, the actual separation distance ahead of the following train is, once again, greater than the instantaneous TSD(b) value for that train. At that point, the deceleration tracks merge to reconstitute the main line and, simultaneously, the section boundary with the following, lower-speed section is reached, so the train passes immediately onto the lower-speed section.
The switch type of the diverging switch is of course that corresponding to the speed range. The type of switch at the end of the deceleration track, where the two tracks merge to re-form the main line, is that type whose TLS is the lowest value which is greater than or equal to the target speed. (Thus, decelerating from HS to MS uses the same, UHS switch type at the end of the deceleration tracks, but HS to LS would use switch type HV for that purpose – the slowest which will do the job, in other words, in line with normal, good engineering practice.
2. The Target Speed is in the **same** speed range as the normal (Sweet-) line speed, albeit lower, of course.
The first train behaves exactly as above until it has reached the Buffer-end Speed. That completes its actual deceleration, and it continues at that speed until it reaches the end of the deceleration track. At this point, now moving back onto the reconstituted main line, it begins its re-acceleration back up to the target speed. When that speed is reached, that is the completion of that train's **net** deceleration, and, simultaneously, the section boundary with the following, lower-speed section, and it continues at that speed onto the following section.
3. For completeness, it is necessary to demonstrate precisely what happens for the Basic TSD standard also. This is almost completely of purely theoretical interest, but it is, I think, necessary. It is also, I think, actually interesting.

Examples are now provided to illustrate all three cases. Two graphs are provided, in each case, with elapsed time in seconds as the independent variable. They present the same data, but scaled differently, to illuminate and focus on train speed and distance travelled, respectively. For the first two cases, the same capacity, 32tph, in the High Speed range, is used. What is actually being demonstrated is the behaviour of the Extended TSD standard, which is the same for all ranges. The third example necessary requires a lower speed, so the capacity of 30tph is used, in the medium speed range but using UHS type switches.

For the first case only, an extra two graphs are provided, illustrating re-acceleration from the target speed back up to the original Sweet-Speed. This is purely to illustrate that that process is automatic and that separation distances remain adequate throughout.

Note particularly that, although only two trains are illustrated in the graphs, they must be understood as representing a constant stream of trains, each of which affects and is affected by both the preceding and the following train.

Case 1 Deceleration between High and Low Speed ranges.

Specimen Example 32tph throughout.

Decelerating from Overall line speed 90.80m/s to Target line speed 38.37m/s.

The speed graph is simple to follow. Train 1 decelerates from the overall line speed to the target line speed, and then continues at that constant speed. Train 2 travels at constant line speed for a distance equal to the capacity distance slot, throughout train 1's deceleration, and also for a short time beyond, getting closer to train 1 until the distance between them is at its minimum, **relative to TSD(b) for train 2**, at which time train 2 has reached the start of deceleration point, and begins its own deceleration. As train 2 decelerates, train 1 travels at constant target line speed. The distance between them continues to decrease – of course it does, train 2 is still travelling faster – but what changes is that the TSD(b) needed between them decreases faster still. Train 2's speed decreases linearly, but TSD(b) decreases quadratically, and quite quickly falls below the actual separation distance between the trains. At this point, the deceleration tracks merge to re-form the main line, and the same point is also the section boundary between the Overall and Target line speeds. This constitutes the completion of the deceleration process, for train 1. TSD(b) continues to decrease faster than does the speed until train 2 reaches the target speed, beyond which both trains continue at that constant speed. The trains travel on different tracks for the time during which their separation distance would be below the minimum TSD(b) for the instantaneous line speed of train 2 – if they were on the same track.

All this is on the speed graph, but much easier to see on the distance graph, where the speed lines are omitted, and the vertical scale is four times as big. Concentrate on lines 6 (TSD(b) for train 2) and 7 (actual separation distance). These lines cross twice: at elapsed time 67.54s, when train 1 reaches the Buffer-end point, and at 164.86s, where it reaches the end of the deceleration track. (The deceleration track actually begins at 53.82s, where train 1 begins its divergence at the turnout limit speed.)

0.00s Train 1 begins its deceleration.

Train 1, at the start of its deceleration, is 10.2146km, i.e. one capacity slot length, = TSD(e), ahead of train 2. This is the normal separation distance between adjacent trains travelling at line speed. Train 2's distance travelled is taken as zero at this point in time, the origin of the deceleration process.

53.82s Train 1 reaches the bifurcation of the main line into two deceleration tracks.

Train 1 has travelled 4.1623km and its speed has fallen to the Turnout Limit Speed of 63.89m/s, 230kph. In the same time, train 2 has travelled 4.8863km at overall line speed. The separation distance between the trains has reduced to 9.4906km. Train 1 is directed by the switch onto deceleration track 1.

67.54s Train 1 decelerates to the Buffer-end Speed on deceleration track 1.

Train 1 travels the buffer distance, 830m, onto deceleration track 1, at which point it has decelerated to the Buffer-end Speed, 57.02m/s. The switch has moved, behind train 1. to point to deceleration track 2, onto which train 2 will be routed when it reaches the switch. In the same time train 2 has travelled 1.2466km at constant overall line speed. The separation distance between the trains is now 9.0741km, TSD(b) precisely, the absolute minimum for the overall line speed, at which train 2 is still travelling, and will for some time yet. At this instant, train 1 is 830m, the buffer length, constant portion of TSD(b), on track 1 beyond the switch, and train 2 is 8.2441km, the dynamic portion of TSD(b), before the switch, still on the main line but effectively on deceleration track 2, since that is where it will be routed. Train 1 is thus no longer in the path of train 2.

104.86s Train 1 reaches the target line speed.

Train 1 travels a further 1.7798km and reaches the target line speed of 38.37m/s. It will maintain that as a constant speed from this point on, for the time being on deceleration track 1. In the same time, train 2 has travelled 3.3881km at constant overall line speed.

112.50s Train 2 begins its deceleration.

Train 1 travels a further 0.2931km at constant target line speed. In the same time, train 2 travels 0.6936km at constant overall line speed. Train 1 has thus travelled a distance 7.0652km from the point at which it began its deceleration, at which point it was 10.2146km ahead of train 2, and, in the same time, train 2 has travelled 10.2146km, so the separation distance between the trains is now 7.0652km, 2.0089km **less than** the minimum, TSD(b), of 9.0741km.

This is where it gets challenging. The preceding graphs illustrate the interactions between just two trains. But the actuality is a continuous sequence of trains, and each train interacts simultaneously with the train in front, **and the train behind**.

Train 2 now performs **precisely**, relative to the following, **train 3**, as train 1 in the preceding paragraphs did, relative to train 2. Specifically, the elapsed times, distances travelled, and train separation distances are identical.

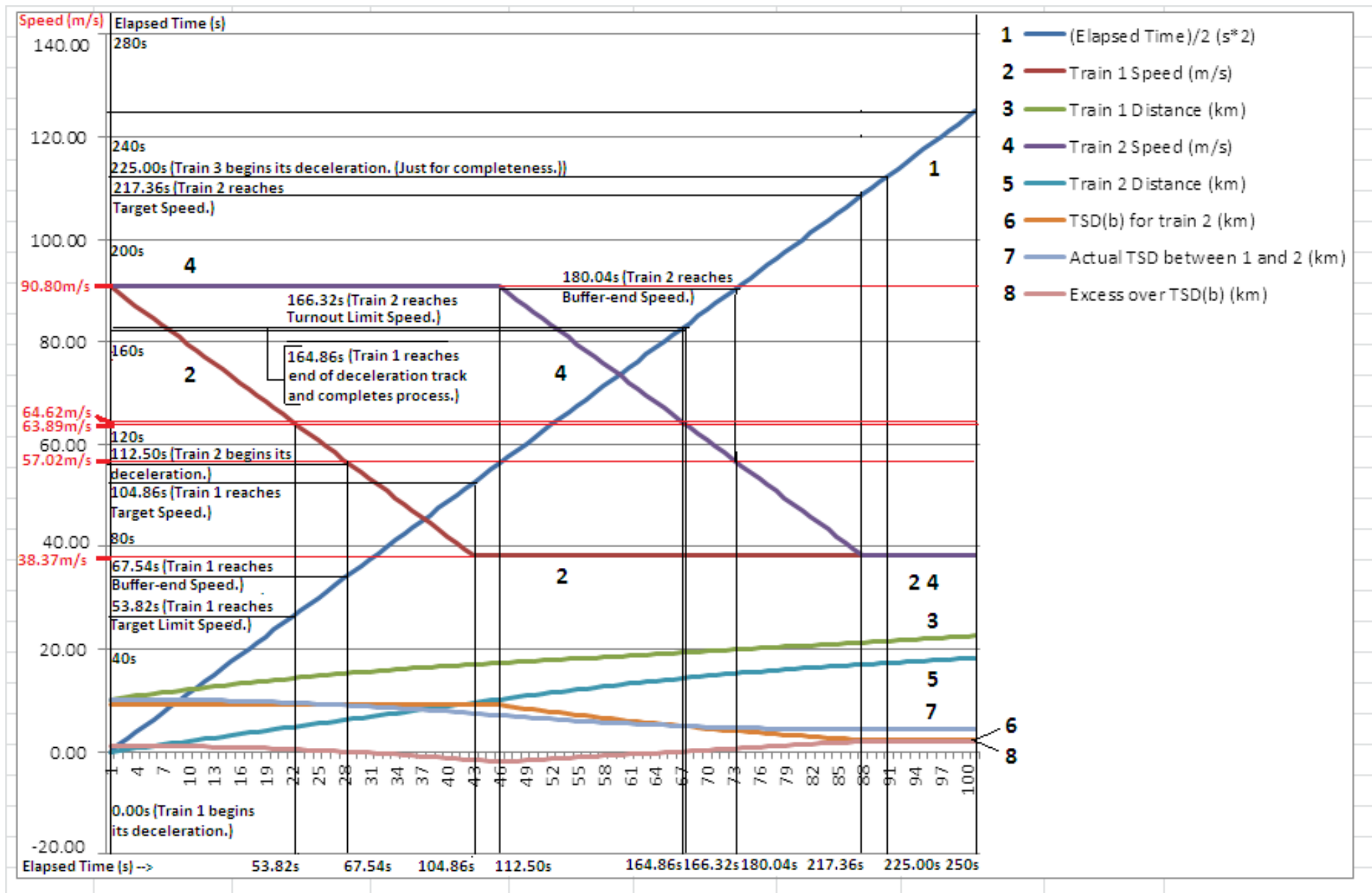
Train 2, at the start of its deceleration, is 10.2146km ahead of train 3. It is also its minimum distance, 7.0652km, relative to the required TSD(b), behind train 1.

164.86s Train 1 reaches end of deceleration track (TSD = TSD(b) for train 2, precisely)

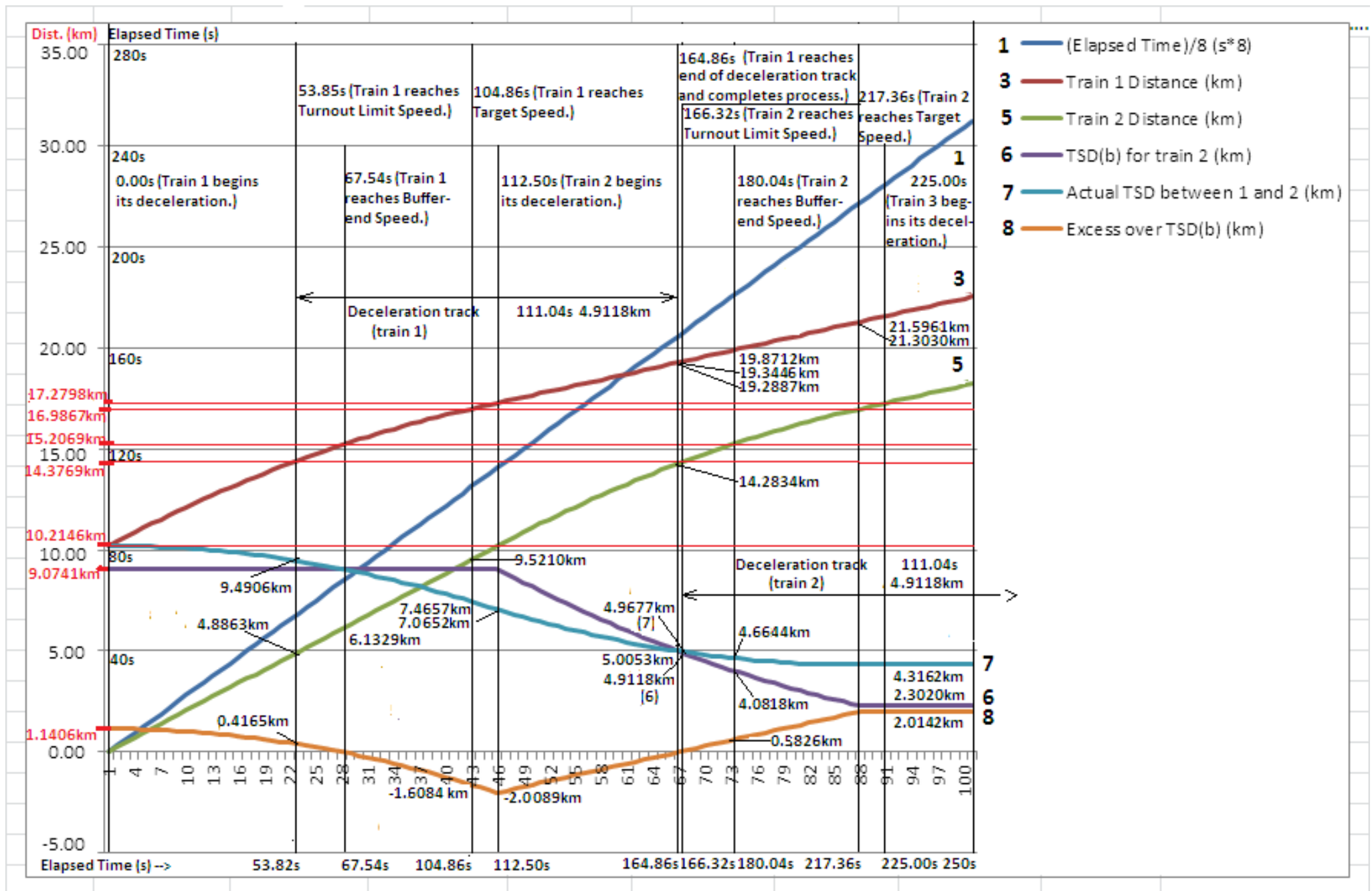
Train 1 has travelled 2.0089km at constant target line speed since train 2's start of deceleration, and has thus cleared the deficit in separation distance at that location, and reached the point at which it is precisely TSD(b) (calculated for train 2's instantaneous speed of 64.62m/s) ahead of train 2, which has, in the same time decelerated from 90.80m/s to 64.62m/s, and travelled a distance of 4.0688km.

Although the TSD is still decreasing, and will continue to decrease until train 2 reaches the target speed, its excess over the instantaneous TSD(b) will continue to increase until train 2 reaches the target speed. This (i.e. train 1's location) is precisely the point at which the deceleration tracks are no longer required, and therefore should merge to re-constitute the main line.

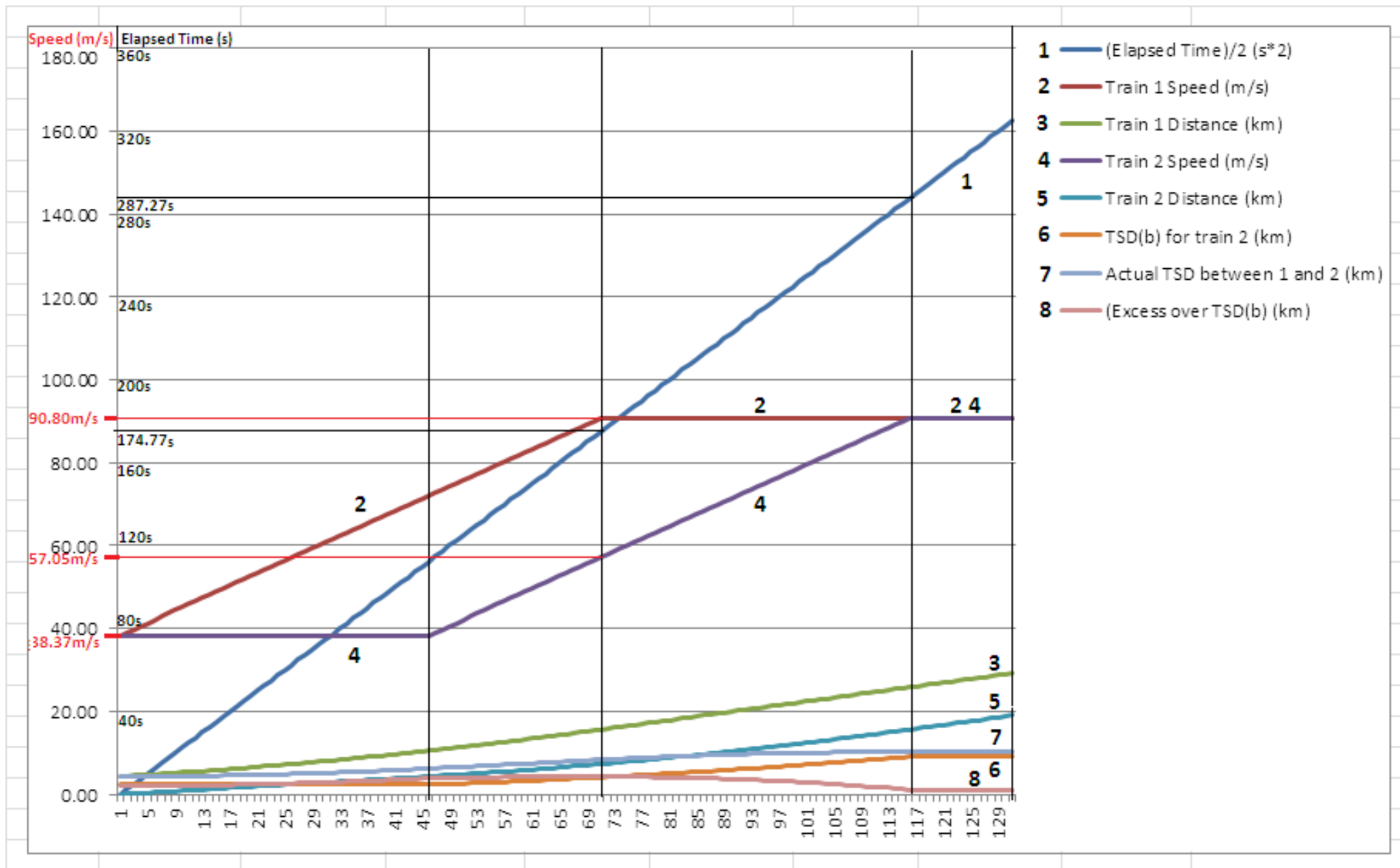
Train 1 has thus completed the deceleration process.



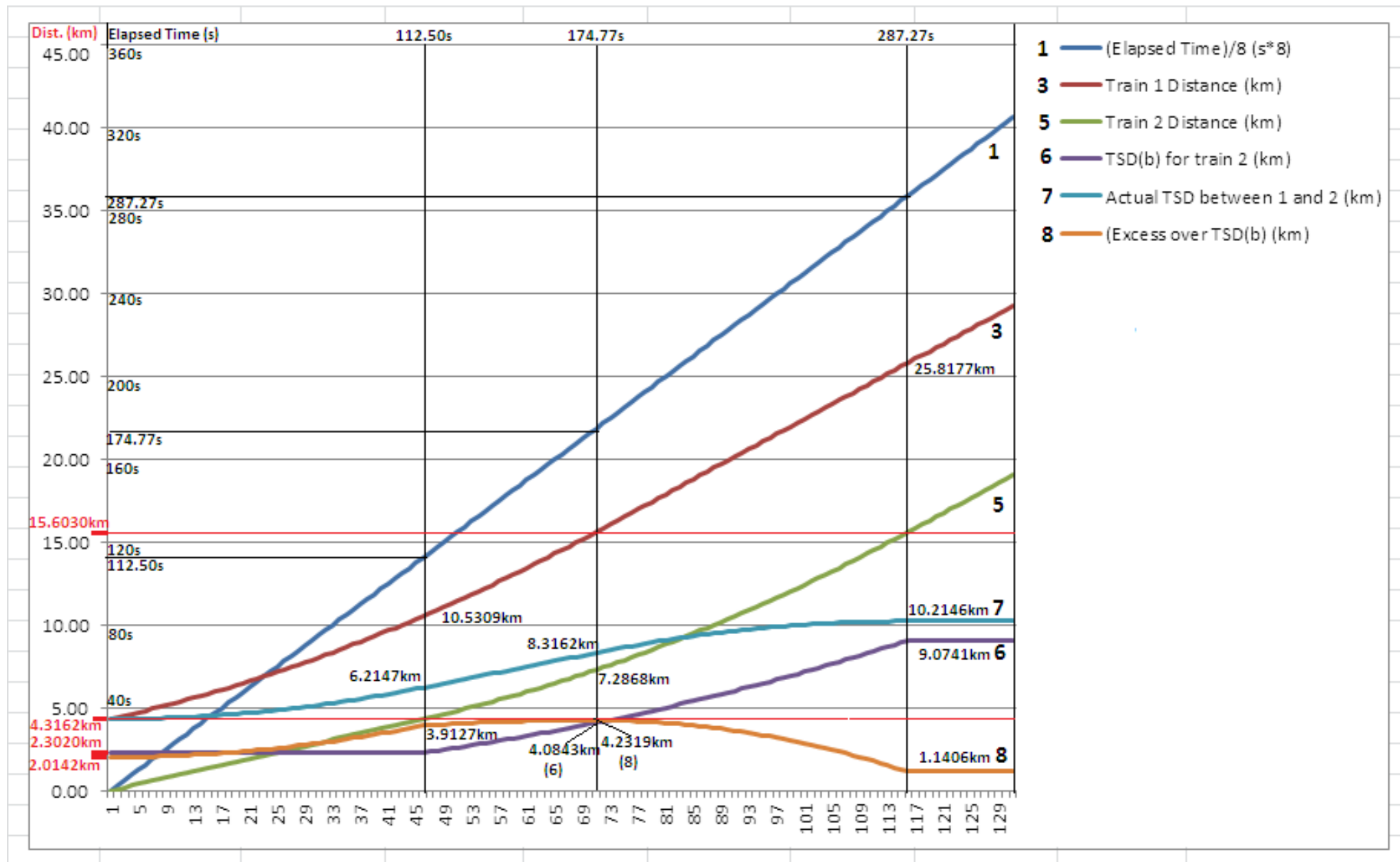
Deceleration Graphs at constant capacity 32tph, to a time scale unit of 2sec, to spread out on the page. Annotated, focussed on speed.



Deceleration Distance Graphs at constant capacity 32tph, to a time scale unit of 8sec, enlarging the distance graphs by octuplet-scale. Annotated.



Re-acceleration Graphs at constant capacity 32tph, to a time scale unit of 2sec, to spread out on the page. Annotated.



Re-acceleration Distance Graphs at constant capacity 32tph, to a time scale unit of 8sec, enlarging the distance graphs by octuplet-scale. Annotated.

The elapsed time taken to travel the deceleration track, for the present case, is 111.04s. The elapsed time periods when the trains are on the deceleration tracks are indicated on the distance graph. In the present example, the elapsed time at which train 1 reaches the end of deceleration track and train 2 reaches the turnout limit speed are almost identical. This is purely fortuitous and has no particular significance.

166.32s Train 2 reaches the bifurcation of the main line into two deceleration tracks.

Train 2 has travelled 4.1623km from the start of deceleration, and its speed has fallen to the Turnout Limit Speed of 63.89m/s, 230kph. In the same time, train 3 has travelled 4.8863km at overall line speed. The separation distance between the trains has reduced to 9.4906km. Train 2 is directed by the switch onto deceleration track 2.

180.04s Train 2 decelerates to the Buffer-end Speed on deceleration track 2.

Train 2 travels the buffer distance, 830m, onto deceleration track 2, at which point it has decelerated to the Buffer-end Speed, 57.02m/s. The switch has moved, behind train 2, to point to deceleration track 1, onto which train 3 will be routed when it reaches the switch. In the same time train 3 has travelled 1.2466km at constant overall line speed. The separation distance between the trains is now 9.0741km, TSD(b) precisely, the absolute minimum for the overall line speed, at which train 3 is still travelling, and will for some time yet. At this instant, train 2 is 830m, the buffer length, constant portion of TSD(b), on track 2 beyond the switch, and train 3 is 8.2441km, the dynamic portion of TSD(b), before the switch, still on the main line but effectively on deceleration track 1, since that is where it will be routed. Train 2 is thus no longer in the path of train 3.

Also in the same time, train 1, (which has of course already completed its deceleration process,) travels 0.5266km at constant target line speed. The separation between trains 1 and 2 is now 4.6644km, as compared with TSD(b) for train 2 of 4.0818km.

217.36s Train 2 reaches the target line speed.

Train 2 travels a further 1.7798km and reaches the target line speed of 38.37m/s. It will maintain that as a constant speed from this point on, for the time being on deceleration track 2. In the same time, train 3 has travelled 3.3881km at constant overall line speed.

Also in the same time, train 1 travels 1.4318km at constant target line speed. The distance between trains 1 and 2 is now 4.3162km, and will remain constant at that value as long as they are both travelling at the target line speed.

225.00s Train 3 begins its deceleration.

Train 2 travels a further 0.2931km at constant target line speed. In the same time, train 3 travels 0.6936km at constant overall line speed. Train 2 has thus travelled a distance 7.0652km from the point at which it began its deceleration, at which point it was 10.2146km ahead of train 3, and, in the same time, train 32 has travelled 10.2146km, so the separation distance between the trains is now 7.0652km,

Also in the same time, train 1 also travels 0.2931km at constant target line speed (of course it does!). The distance between trains 1 and 2 is now 4.3163km, (still!).

277.36s Train 2 reaches end of deceleration track (TSD = TSD(b) for train 3, precisely)

Train 2 has travelled 2.0089km at constant target line speed since train 3's start of deceleration, and has thus cleared the deficit in separation distance at that location, and reached the point at which it is precisely

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TSD(b) (calculated for train 3's instantaneous speed of 64.62m/s) ahead of train 3, which has, in the same time decelerated from 90.80m/s to 64.62m/s, and travelled a distance of 4.0688km.

Although the actual separation of the trains is still decreasing, and will continue to decrease until train 3 reaches the target speed, its excess over the instantaneous TSD(b) will continue to increase until train 3 reaches the target speed. This (i.e. train 2's location) is precisely the point at which the deceleration tracks are no longer required, and therefore now merge to re-constitute the main line.

Train 2 has thus completed the deceleration process.

The switch type used at the beginning of the deceleration tracks is UHS, the standard for this speed range. But the target line speed in this example is sufficiently low – in the Low Speed range – to allow switch type HV to be used at the end of the tracks, where they merge to re-constitute the main-line.

There is nothing much to say about the re-acceleration graphs, save that they demonstrate that the original state is automatically restored, and the separation between the trains is comfortably in excess of the TSD(b) value for the second train throughout.

Every case of line speed reduction for Same Speed railways, (providing that they are operating to full capacity,) necessitates a pair of deceleration tracks. This should come as no surprise; it is simply one of the necessary costs of achieving the high capacities that they make available.

One final aspect of constant-capacity operation at various line speeds needs to be demonstrated: to show that overtaking remains possible. It is, but there are subtle differences from the constant Sweet-Speed case. The following example should be compared with that in the section 'Timetabling Considerations and Sweet-Speeds', in volume 1. We consider overtaking at line speed 38.37m/s, the target speed.

Line capacity = 32tph.	Slot time = 112.5sec.	Slot length = 4.3162km.
Line speed = 38.37m/s = 138.12kph = 85.83mph.		
Deceleration time = 76.73sec = 0.6821 slots	Deceleration distance = 1.4720km = 0.3410 slots	
Acceleration time = 127.89sec = 1.1368 slots	Acceleration distance = 2.4533km = 0.5684 slots	
Station Loop Travelling time = 204.62sec = 1.8189 slots (Virtual and physical station loops identical.)		
Station Loop distance = 3.9253km = 0.9094 slots		

By the time the stopping train has travelled the length of the station loop, its empty slot, which it gave up on entering the loop, has travelled 1.8189 slots (time or distance) on the main line. It is thus 0.9094 slots (time or distance) beyond the end of the station loop. In order to make this 1 slot exactly, it must travel a further 0.0906 slots (time or distance). This implies that the train must wait for 0.0906 **time** slots = 10.19sec at the station.

We now follow the progress of slot stream and stopping train.

Time Slot 1:

The empty slot advances 1 slot along the main line. It is now 1 slot beyond start of loop, but has also (already!) passed the end of loop. It is 0.0906 slots beyond end of loop.

The slot containing the next stopping train arrives at start of loop.

The train decelerates for 0.6821 time slots, and reaches the station, where it waits for 0.0906 time slots. It then accelerates for 0.2274 time slots. It thus has 0.9094 time slots of acceleration still to do.

During Time Slot 2:

The empty slot advances a further 0.9094 slots along the main line. It is now 1.9094 slots beyond start of loop, and the (corrected) slot stream advance is **1 slot exactly beyond end of loop.**

The second empty slot advances 0.9094 slots along the main line. It is now 0.9094 slots beyond start of loop. **It therefore coincides precisely with end of loop.**

The train performs its final 0.9094 time slots of acceleration. **It therefore coincides precisely with end of loop and is travelling at line speed.**

(The second stopping train decelerates for 0.6821 time slots, and reaches the station, where it waits for 0.0906 time slots. It then accelerates for 0.2274 time slots. It thus has 0.9094 time slots of acceleration still to do – but so what?)

The timetabling is thus essentially the same as for the constant Sweet-Speed case, but the numerical results are very different.

The slot stream advance is only 1 slot! We still need it to be 4, to be consistent with the original Sweet-Speed. (The number and types of capacity sub-streams, and thus the slot stream advance must stay the same for as long as the capacity does.) An extra 3 time slots must therefore be added to the station wait time also, giving 3.0906 time slots = 347.69sec = 5min47.69sec.

Operating to constant capacity at lower speeds does have a cost, in efficiency. This has two aspects: the trains are further apart than they would be if operating to the capacity corresponding to that actual speed, and the station wait times are inflated.

Case 2 Deceleration within High Speed range.

Specimen Example 32tph throughout.

Decelerating from Overall line speed 93.60m/s to Target line speed 74.71m/s.

The speed graph is rather more complicated than for the first example, but still simple to follow. The main point to note is that the elapsed time range is 40% greater – 350s vs 250s. Accordingly, the scales adopted use units of 4s and 16s, as compared with 2s and 8s, to get the best available spread over the page.

Train 1 decelerates from the overall line speed to an intermediate target line speed of 57.02m/s, the Buffer-end Speed, and then continues at that constant speed until it reaches the end of the deceleration track, where the two tracks merge to reconstitute the main line, and train 1 begins its re-acceleration up to the final Target speed. When it reaches that speed, then its net deceleration process is complete, and it maintains that constant speed for as long as required.

Train 2 travels at constant line speed for a distance equal to the capacity distance slot, exactly the same as for the first example, throughout train 1's deceleration, (which is smaller than for the first example, since the intermediate target speed is higher than the (final) target speed of that example,) and thus extends for a longer time beyond, getting closer to train 1 until the distance between them is at its minimum, **relative to TSD(b) for train 2**, at which time train 2 has reached the start of deceleration point, and begins its own deceleration.

As train 2 decelerates, train 1 travels at constant target line speed. The distance between them continues to decrease in absolute terms, but increases relative to the (instantaneous) value of TSD(b) for train 2. When the separation distance reaches TSD(b) for train 2, then train 1 has reached the end of the deceleration track, as mentioned above. Train 1 begins its re-acceleration, while train 2 continues its deceleration until it reaches the Turnout Limit Speed, at the bifurcation of the main line into deceleration tracks, and is directed onto track 2, on which it continues its deceleration down to the Buffer-end Speed, at which point the switch has been reset to direct the next, train 3, onto track 1.

Having reached the Buffer-end Speed, the intermediate target speed, train 2 continues at that speed until it reaches the end of the deceleration track, and re-accelerated up to the (final) Target Speed, on reaching which it has completed its net deceleration.

0.00s Train 1 begins its deceleration.

Train 1, at the start of its deceleration, is 10.2146km, i.e. one capacity slot length, = TSD(e), ahead of train 2. This is the normal separation distance between adjacent trains travelling at line speed. Train 2's distance travelled is taken as zero at this point in time, the origin of the deceleration process.

53.82s Train 1 reaches the bifurcation of the main line into two deceleration tracks.

Train 1 has travelled 4.1623km and its speed has fallen to the Turnout Limit Speed of 63.89m/s, 230kph. In the same time, train 2 has travelled 4.8863km at overall line speed. The separation distance between the trains has reduced to 9.4906km. Train 1 is directed by the switch onto deceleration track 1.

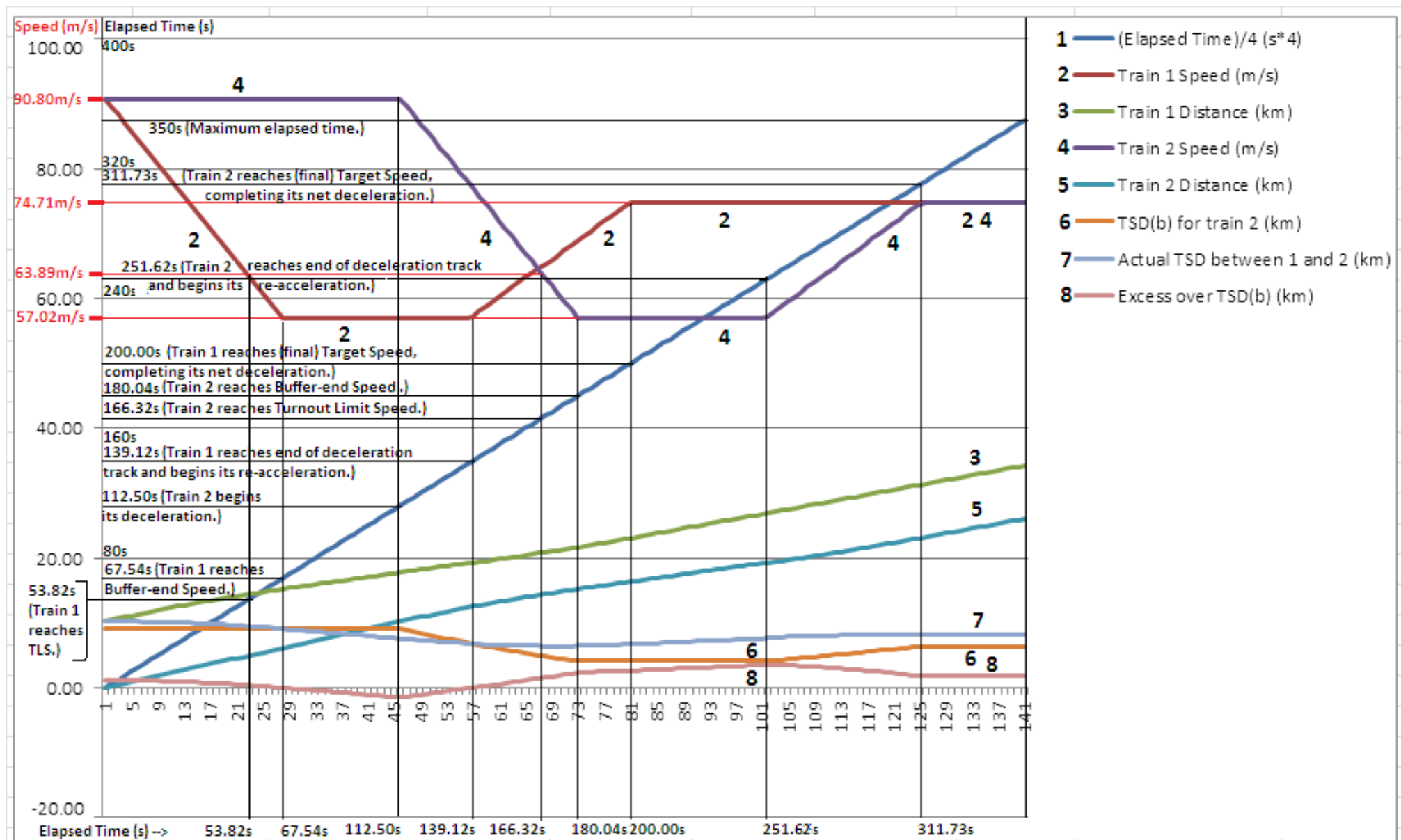
67.54s Train 1 decelerates to the Buffer-end Speed on deceleration track 1.

Train 1 travels the buffer distance, 830m, onto deceleration track 1, at which point it has decelerated to the Buffer-end Speed, 57.02m/s. The switch has moved, behind train 1. to point to deceleration track 2, onto which train 2 will be routed when it reaches the switch. In the same time train 2 has travelled 1.2466km at constant overall line speed. The separation distance between the trains is now 9.0741km, TSD(b) precisely, the absolute minimum for the overall line speed, at which train 2 is still travelling, and will for some time yet. At this instant, train 1 is 830m, the buffer length, constant portion of TSD(b), on track 1 beyond the switch, and train 2 is 8.2441km, the dynamic portion of TSD(b), before the switch, still on the main line but effectively on deceleration track 2, since that is where it will be routed. Train 1 is thus no longer in the path of train 2.

Until this point, the behaviour of the two trains has been identical to that for the first example. But now, the buffer-end speed forms the intermediate target speed, so there is no need for train 1 to decelerate further, so it simply continues at that speed.

112.50s Train 2 begins its deceleration.

Train 1 travels a further 2.5620km at constant intermediate target speed. In the same time, train 2 travels 4.0817km at constant overall line speed. These values are much higher for the first example, (c.f. 0.2931km and 0.6936km, respectively,) because the deceleration has been less. Train 1 has now travelled a distance of 7.5543km from the point at which it began its deceleration, at which point it was 10.2146km ahead of train 2, and, in the same time, train 2 has travelled 10.2146km, so the separation distance between the trains is now 7.5543km, 1.5198km **less than** the minimum, TSD(b), of 9.0741km (c.f. 2.0089km in the first example) There is thus less deficit to recover..



Deceleration Graphs at constant capacity 32tph, to a time scale unit of 4sec, to spread out on the page. Annotated, focussed on speed.

Train 2, at the start of its deceleration, is 10.2146km ahead of train 3. It is also its minimum distance, 7.5543km, relative to the required TSD(b), behind train 1.

139.15s Train 1 reaches end of deceleration track (TSD = TSD(b) for train 2, precisely)

This, unsurprisingly, happens a lot earlier – over 25s – than in the first example. But although the elapsed times are very different, the times actually travelled by train 1 when it reaches the end of the deceleration track are identical, at 19.2886km, (from a starting value of 10.2146km when it began its deceleration,) to a rounding error of less than 1 part in a million. The decelerating track distance (from the point at which train 1 reached the TLS) is thus 4.9118km (to rounding error) for both examples.

At this point, (that train 1 has just reached,) the deceleration tracks merge to re-form the main line, and the switch had been pre-set to pass train 1, from deceleration track 1, back onto the main line. Note that, since this is a **converging** junction, the only timing consideration is that the back end of the train must cross the junction before the switch can be reset. Train 1 is travelling at the Buffer-end Speed as it reaches the switch points, and can begin its re-acceleration at that point, as it crosses the switch. Given that the train length is only 400m, and the time taken to reset the switch is 4s, then the time taken for train 1 completely to cross the switch, at constant target or, as here, intermediate target speed, is 630m or 11.05s. Note that this has no effect on the location of the end of deceleration track, but it does mean that an extra 630m or 11.05s is required for train 1 completely to cross the switch, and the switch to be reset to receive the next train off the other deceleration track, train 2 on track 2 in this instance. Note a further strange and surprising effect: for a very short period of time, the main line will be continuous. At elapsed time 150.20s, train 1 has completely re-joined the main line from deceleration track 1, and the converging switch has been reset to accept the next train, train 2, from deceleration track 2. But train 2 has not actually moved on to the deceleration track yet – it is about to do so at elapsed time 166.32s, but until then is still on the main line before the switch, as also is the following train 3. But for the next 16.12s, train 1 is on the main line beyond the deceleration track 2 and trains 2 and 3 are on the main line before the deceleration track, and deceleration track 2 is connected to the main line at both ends. Track 1 on the other hand is completely isolated, not connected at either end.

This is, I believe, perfectly easy to understand, nonetheless it came as quite a surprise to me.

Train 1 now begins its re-acceleration up to the (final) target speed.

166.32s Train 2 reaches the bifurcation of the main line into two deceleration tracks.

Train 2 has travelled 4.1623km from the start of deceleration, and its speed has fallen to the Turnout Limit Speed of 63.89m/s, 230kph. In the same time, train 3 has travelled 4.8863km at overall line speed. The separation distance between the trains has reduced to 9.4906km. Train 2 is directed by the switch onto deceleration track 2, while behind it, the switch is reset to direct the next train, train 3, onto deceleration track 1. There is thus no longer a continuous, through route – train 3 will be sent onto track 1, while train 2 will continue along track 2 until it passes back onto the main line (whereupon the converging switch will be reset to accept the next train, train 3, from track 1; track 1 will then form part of the through route for next 16s.

180.04s Train 2 decelerates to the Buffer-end Speed on deceleration track 2.

Train 2 travels the buffer distance, 830m, onto deceleration track 2, at which point it has decelerated to the Buffer-end Speed, 57.02m/s. The switch has moved, behind train 2, to point to deceleration track 1, onto which train 3 will be routed when it reaches the switch. In the same time train 3 has travelled

1.2466km at constant overall line speed. The separation distance between the trains is now 9.0741km, TSD(b) precisely, the absolute minimum for the overall line speed, at which train 3 is still travelling, and will for some time yet. At this instant, train 2 is 830m, the buffer length, constant portion of TSD(b), on track 2 beyond the switch, and train 3 is 8.2441km, the dynamic portion of TSD(b), before the switch, still on the main line but effectively on deceleration track 1, since that is where it will be routed. Train 2 is thus no longer in the path of train 3.

At the same time, train 1, since reaching the end of the deceleration track, has performed part of its re-acceleration, on the re-merged main line, and has travelled 2.4886km. The separation between trains 1 and 2, back on the same track, is now 6.5703km, as compared with TSD(b) for train 2 of 4.0818km.

200.00s Train 1 reaches the (final) target line speed.

Train 1 accelerates a further 1.2505km and reaches the (final) target line speed of 74.71m/s, and has now completed the net deceleration process. It will maintain that as a constant speed from this point on, indefinitely.

Also in the same time, train 2 travels 6.6828km at constant intermediate target line speed. The distance between trains 1 and 2 is now 4.3163km.

251.62s Train 2 reaches end of deceleration track (TSD = TSD(b) for train 3, precisely)

Train 2 has travelled 4.0817km at constant intermediate target line speed since reaching this speed, reaching the end of deceleration track 2, and beginning its re-acceleration to (final) target speed.

311.73s Train 2 reaches the (final) target line speed.

Train 2 accelerates for 3.7391km and reaches the (final) target speed of 74.71m/s., and has now completed the net deceleration process. It will maintain that as a constant speed from this point on, indefinitely. It is now 8.2111km behind train 1, which will likewise remain constant while both trains maintain this speed.

Case 3 Deceleration between High and Slow Speed ranges.

Specimen Example 60tph throughout, but under Basic TSD standard.

Decelerating from Overall line speed 38.37m/s to Target line speed 25m/s.

This example is purely theoretical; it should never actually be used in practice. If anyone, ever, for whatever reason, (except in a purely virtual, thought-experiment sense, as here,) wished to operate at 60tph in the High Speed range, i.e. with UHS switches, this is how it would have to be done, since the extended TSD standard is not available for that capacity in that range – look at the graph on p.8 to see why. Also, 38.37m/s is well within the turnout limit speed. 60tph is the only usable capacity, in any speed range, for which this is true, which makes it the perfect candidate for the current piece of esoterica.

The speed graph for this case is almost trivially simple: train 1 decelerates from original line speed to target line speed and then continues at that speed indefinitely. Train 2 travels 1 capacity slot distance (= TSD(b) at original line speed) then decelerates similarly. It may be claimed, correctly, that case 1, above, does no more. But for the High Speed case, there is the necessity to perform part of the deceleration on the main line (or, at least, while still in the path of the following train,) so there are several other features of interest, times, speeds and distances. None of that applies here – by definition, indeed. The trains are already travelling the minimum distance apart, at a line speed of less than or equal to the Buffer-end Speed. What is, at first sight, quirky, is that the initial section of the train's 'deceleration' is performed at that constant speed. Train 1 travels a distance equal to the buffer length onto deceleration track 1 at constant line speed, a fixed distance, the capacity slot length, ahead of train 2. This takes the first 21.63s. At this point it is no longer in the path of train 2, the switch having reset behind it to direct train 2 onto deceleration track 2 in due course, and train 1's actual acceleration begins. Note that train 2's begins at elapsed time 60s, the first 21.63s of it being still at constant line speed.

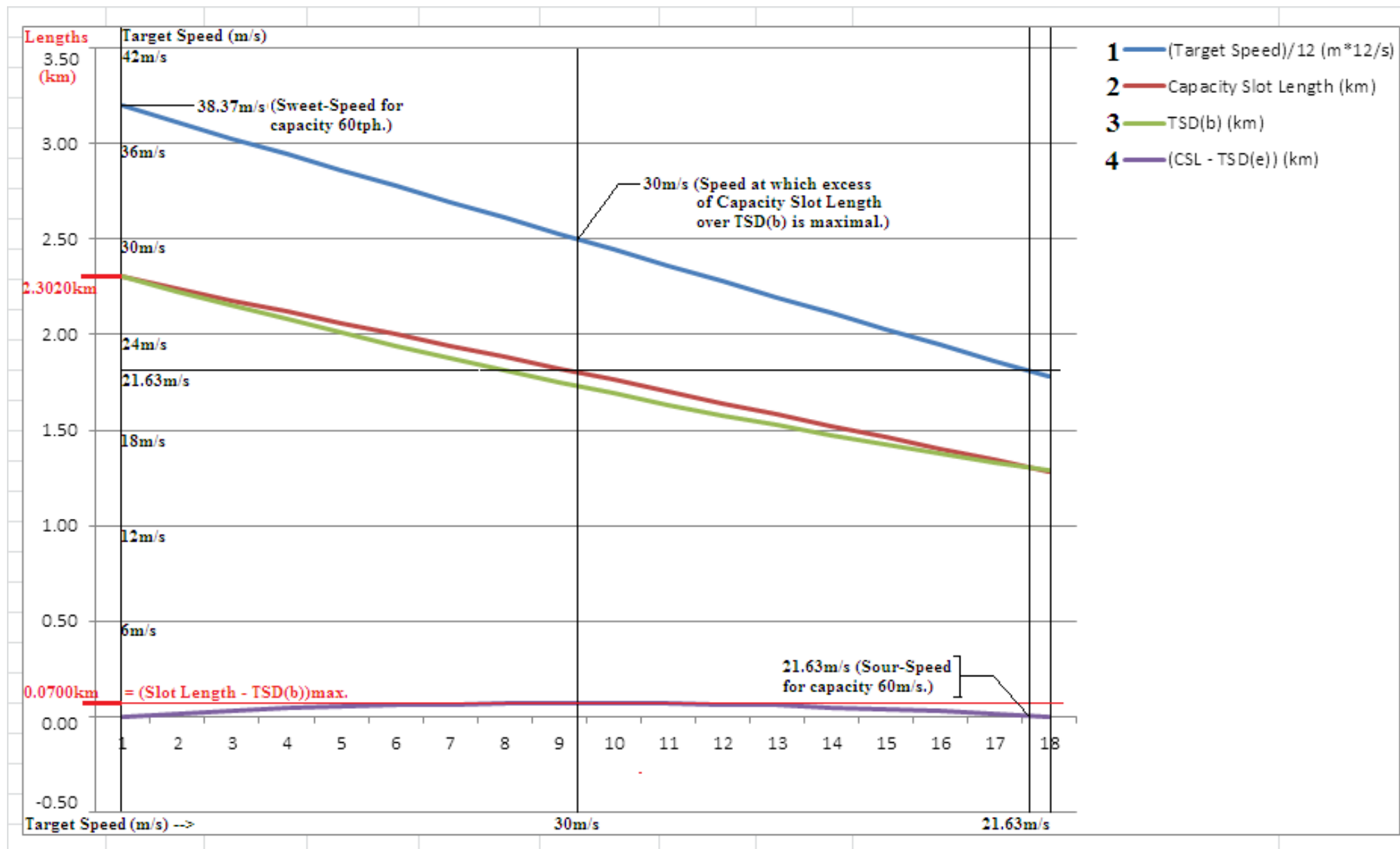
Train 1 travels at constant target speed until it reaches the end of the deceleration track, at which point it re-joins the main line, and its deceleration process is complete.

0.00s Train 1 begins its virtual deceleration, at the bifurcation of the main line into two deceleration tracks.

Train 1, at the start of its deceleration, is one capacity slot length, TSD(b), ahead of train 2. This is the normal separation distance between adjacent trains travelling at constant line speed. At this instant, train 1 is at the bifurcation of the main line into two deceleration tracks. The switch is already pointing to track 1, onto which train 1 will be routed. Train 2's distance travelled is taken as zero at this point in time, the origin of the deceleration process.

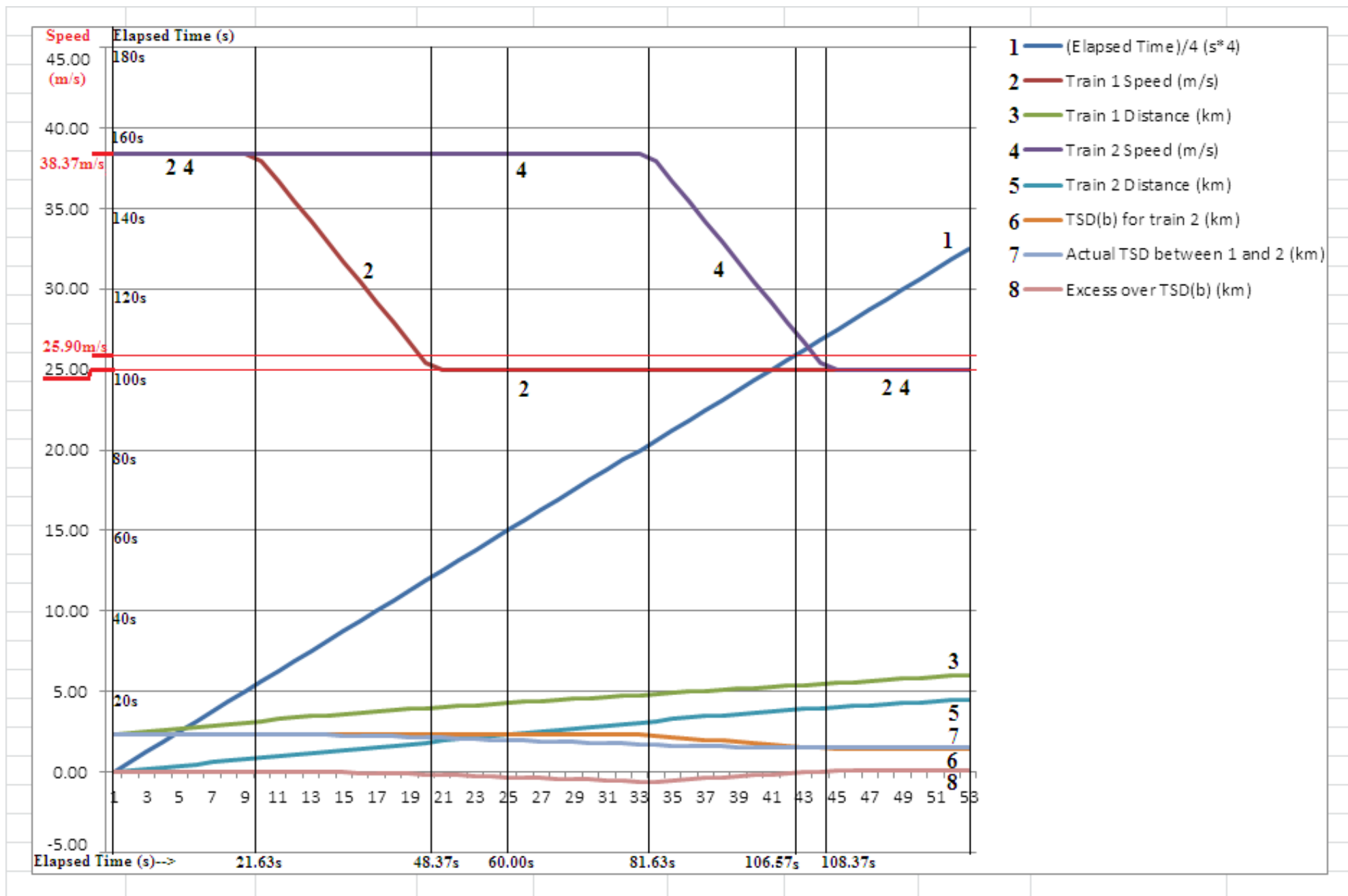
This is operation under the basic TSD standard, so TSD(b) is the separation distance between adjacent trains, i.e. they are already as close to each other as is permissible. No deceleration takes place on the main line.

The bifurcation of the main line takes place at this point, at the very beginning of the deceleration, which is actually a virtual deceleration, initially, since train 1 runs onto the deceleration track at constant line speed, and continues at that speed until it has travelled the buffer length onto that track. A Deceleration Range diagram is provided for this special case, as it is so unlike the previous examples. The speed range itself – 38.27 down to 21.63m/s – is very narrow, and the divergence of TSD(b) from the capacity slot length is likewise small, both features typical of operation at the basic TSD standard.



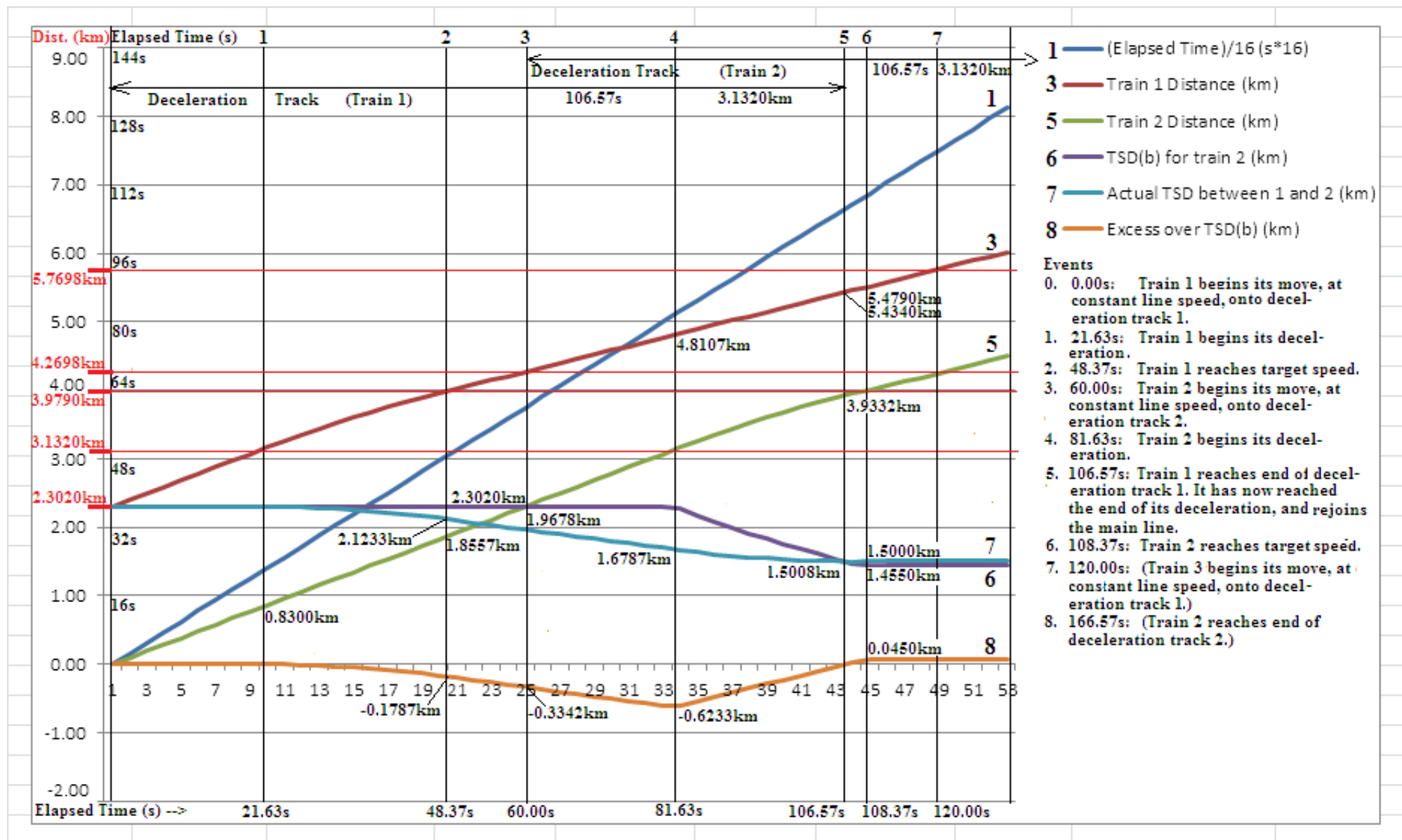
Deceleration Range for High Speed Capacity 60tph. Sweet-Speed 38.37m/s. Sour-Speed 21.63m/s.

But operating to Basic TSD standard.



Deceleration Speed Graphs at constant capacity 60tph, to a time scale unit of 4sec. HS range, so UHS switch,

but operated to Basic TSD standard.



Deceleration Distance Graphs at constant capacity 60tph, to a time scale unit of 16sec. HS range, so UHS switch,
but operated to Basic TSD standard.

21.63s Train 1 begins its actual deceleration.

Train 1 travels the buffer length, 830m, onto deceleration track 1 at full line speed. At this point, the switch has moved, behind train 1, to point to track 2, onto which train 2 will be routed, when it reaches the switch. In the same time, train 2 has unsurprisingly also travelled 830m, on the main line. The separation between the trains is still 2.3020km. Train 1 is the buffer length beyond the switch and train 2 is 1.4720km before it, still on the main line, but effectively on deceleration track 2, since that is where it will be routed.

Train 1 is thus no longer in the path of train 2, and may, finally, begin its actual deceleration.

48.37s Train 1 reaches the target line speed.

Train 1 travels the distance 0.8470km, and reaches the target speed of 25m/s. It will maintain that, as a constant speed, from this point on, for the time being, on deceleration track 1. In the same time, train 2 has travelled 1.0257km at constant overall line speed.

60.00s Train 2 begins its virtual deceleration.

Train 1 travels a further 0.2908km at constant target line speed. In the same time, train 2 has travelled 0.4463km at constant overall line speed, and reached the bifurcation of the main line.

81.63s Train 2 begins its actual deceleration.

Train 2 travels the buffer length, 830m, onto deceleration track 2, at overall line speed. At this point, the switch has moved, behind train 2, to point back to track 1, onto which train 3 will be routed, when it reaches the switch. In the same time, train 3 has unsurprisingly also travelled 830m, on the main line. The separation between the trains is still 2.3020km; train 2 is the buffer length, 830m, beyond the switch and train 3 is 1.4720km before it, still on the main line, but effectively on deceleration track 1, since that is where it will be routed.

Train 2 is no longer in the path of train 3, and may, finally, begin its actual deceleration. At this point it is also its minimum distance, 1.6787km, behind train 1.

106.57s Train 1 reached end of deceleration track (TSD = TSD(b) for train 2, precisely).

Train 1 has travelled 0.6233km at constant target line speed, since train 2 began its (actual) deceleration, and has thus cleared the deficit in separation distance at that location, and reached the point at which it is precisely TSD(b) (calculated for train 2's instantaneous speed of 25.90m/s) ahead of train 2, which has in the same time decelerated from 36.37m/s to 25.90m/s, and travelled a distance of 0.8012km.

Although the TSD is still decreasing, and will continue to decrease until train 2 reaches the target speed, its excess over the instantaneous TSD(b) for train 2 continues to increase until train 2 reaches the target speed. This (i.e. train 1's location) is precisely the point at which the deceleration tracks are no longer required, and therefore merge, to re-constitute the main line.

Train 1 has thus completed the overall deceleration process.

The length of the deceleration tracks (i.e. since the beginning of the entire process, when train 1 began its virtual deceleration) is thus $5.4340 - 2.3020 = 3.1320$ km. This is no longer, for Basic TSD standard cases, a constant value, as it is for normal High Speed, but is, very clearly, the same thing.

108.37s Train 2 reaches the target line speed.

Train 2 travels a further 0.0458km to reach the target speed. In the same time, train 1 travels almost the same distance, very slightly less, at 0.0450km.

The distance between trains 1 and 2 is now precisely 1.5km, and will remain at this value while their speeds remain the same.

166.57s Train 2 reaches end of deceleration track (TSD = TSD(b) for train 3, precisely).

Train 2 has thus completed the overall deceleration process.

Overtaking on a Same Speed Railway

The subject of overtaking was introduced in the section ‘Timetabling Considerations and Sweet Speeds’ in Volume 1 of the present article. The treatment there was taken very slowly, with plenty of commentary and elucidation, appropriate to an introduction to this fundamentally important and difficult, if you’ve never seen it before, topic. It was written shortly after I had discovered the solution, and closely follows the investigative route I had myself taken.

It is possible to derive the essential result – the station wait time necessary to synchronise the transit of the train over the virtual and physical station loops with the passage along the main line of the empty capacity slot which it has to join – by a more direct calculation, which actually yields a formula for the station wait time.

We **define** the (raw) Slot Stream Advance, SSA_{raw} , as the distance the empty slot has advanced at constant line speed v_l along the main line **beyond** that of the train which originally occupied it, before diverging to call at the station, when that train reaches the end of the virtual station loop, having decelerated to zero at the station and immediately re-accelerated to line speed at the point at which it reaches the end of the virtual station loop. (*Shorter sentences, please!*) The train has spent no time waiting at the station, the required wait time being what this calculation yields. We know that this quantity is equal to the length of the virtual station loop, since the empty slot travels twice as far at constant line speed as the train does in decelerating to zero and then re-accelerating straight-away up to line speed.

SSA_{raw} is expressed in capacity slots, distance or time, (using distance slots here as we’re talking about distances). The distance slot is that distance travelled at constant line speed during a time slot. The time slot, in seconds, = $3600/c_l$, where c_l is the line capacity in tph. The distance slot is therefore $3600v_l/c_l$ in metres, or $3.6v_l/c_l$ in km. Thus:

$$\begin{aligned} SSA_{\text{raw}} &= (v_l^2/2a_d + v_l^2/2a_a) / (3600v_l/c_l) \\ &= v_l c_l (1/a_a + 1/a_d) / 7200 \\ &= v_l c_l ((a_a + a_d)/a_a a_d) / 7200 \\ &= v_l c_l / 7200 a_j \quad \text{where } a_j = a_a a_d / (a_a + a_d) \text{ is the } \mathbf{Adjacency Coefficient}, \end{aligned}$$

which will be encountered in all sorts of surprising locations (seemingly where deceleration and acceleration rates appear together in identical and complementary roles; in any case it saves writing the expression out in full). For the standard values $a_d = 0.5\text{m/s}^2$, $a_a = 0.3\text{m/s}^2$, $a_j = \mathbf{0.1875 = 3/16}$.

SSA_{raw} is stated in capacity slots, (time or distance). What is actually needed is the next integer value immediately above it. This is the **corrected** SSA, SSA_{corr} .

$$SSA_{\text{corr}} = \text{Int}(v_l c_l / 7200 a_j) + 1$$

This gives the basic station wait time which, as a fraction of a single time slot, is too short to be any practical use. It allows insufficient time for any more than a very few passengers to join or leave the train. (It is, however, a very important theoretical result, since it **always does** allow overtaking, irrespective of line capacity or line speed – in my opinion a very surprising result.) We can always add the same, integer number of time slots to both SSA_{corr} and the basic station wait time, and maintain synchronisation between stopping train and capacity slot stream. So let’s add n extra time slots:

$$SSA_{\text{corr}} = \text{Int}(v_l c_l / 7200 a_j) + (1 + n)$$

However, very few values of n are actually available. There is also the very stringent requirement that the resulting configuration permits a valid, usable timetable. For this to be possible, two very similar conditions must apply simultaneously:

1. SSA_{corr} must be an integer sub-multiple of the line capacity c_l (tph) (so that the number of overtakings possible repeats an integer number of times per hour and thus that the overtaking pattern is the same every hour. (Another, very useful way of looking at this is that the slot stream divides into SSA_{corr} sub-streams each of which contains the same number of trains per hour, obeying a stopping pattern particular to that sub-stream, or is non-stop.) This is not difficult to achieve, but, in addition:
2. SSA_{corr} must also divide the **hour of time**, 60 minutes, into an integer number of equal, **useful** units. In much the same way that it is difficult, rigorously to **define** an elephant, but anybody can recognise one, the useful units of time are easily recognised. Effectively, you can work them out instantly in your head, and don't need to perform a calculation. 4tph in a sub-stream is an excellent value, giving a repeat interval of 15 minutes. 8tph is equally good, giving a repeat interval of 7½ minutes – it doesn't have to be an integer number of minutes, or even seconds. 9tph, on the other hand is not a good frequency, even though its repeat interval, at 6⅔ minutes is an integer number of seconds – 400.

Very, very few line speeds, (amazingly few, in fact,) give results which satisfy this second condition. I call them the Sweet-Speeds. (A Sweet-Speed is a property of a particular line capacity, and is unique to that capacity. It is the maximum speed at which a Same Speed Railway can be operated, at that capacity.)

The examples following show different values for n .

Having derived SSA_{corr} , the station wait time is immediately available – equal to $SSA_{corr} - SSA_{raw}$, the amount (of **time**, naturally) by which SSA_{raw} must be increased to make it an integer (plus as may extra time slots as required. Here's how:

$$SSA_{corr} - SSA_{raw} = [\text{Int}(v_l c_l / 7200 a_j) + (1 + n)] - v_l c_l / 7200 a_j \quad \text{slots (distance or time).}$$

For the station wait time we need time, so multiply the above result by the slot time, $3600/c_l$.

The following illustrates the method, for the most common applications.

1. For capacity 32tph and line speed 90.80m/s / 326.87kph / 203.11mph (switch type UPS):

$SSA_{raw} = 2.1522\text{slots}$, we want $SSA_{corr} = 4$ (so $n = 1$), and $SSA_{corr} - SSA_{raw} = 1.8478\text{slots}$

Multiply by the time slot, 112.5s, so the **station wait time = 207.875s**, the by-now very familiar value.

2. For capacity 32tph and line speed 63.89m/s / 230kph / 142.95mph (switch type UPS):

(Note that this is **not** the sweet speed.)

$SSA_{raw} = 1.5144\text{slots}$, we want $SSA_{corr} = 4$ (so $n = 2$), and $SSA_{corr} - SSA_{raw} = 2.4855\text{slots}$

Multiply by the time slot, 112.5s, so the **station wait time = 279.630s**, also a now familiar value.

3. For capacity 48tph and line speed 53.38m/s / 192.15kph / 119.40mph (switch type HV):

$SSA_{raw} = 1.8980\text{slots}$, we want $SSA_{corr} = 6$ (so $n = 5$), and $SSA_{corr} - SSA_{raw} = 4.1020\text{slots}$

Multiply by the time slot, 75s, so the **station wait time = 307.653s**.

4. For capacity 64tph and line speed 33.70m/s / 121.31kph / 75.38mph (switch type GV):

$SSA_{\text{raw}} = 1.5976\text{slots}$, we want $SSA_{\text{corr}} = 4$ (so $n = 3$), and $SSA_{\text{corr}} - SSA_{\text{raw}} = 2.4024\text{slots}$

Multiply by the time slot, 56.25s, so the **station wait time = 135.133s**.

(For the avoidance of confusion, case 4 above is typically used for a **semi-metro**, i.e. one which allows overtaking. The core section of a metro route, passing in tunnel underneath the metropolis, would almost invariably be operated as a **pure metro**, with all trains stopping at every station, and with no overtaking. The results for a pure metro are significantly different. It would normally transform itself into a semi-metro on leaving the central, tunnel section.)

Timetabling of Adjacent Stations on a Same Speed Railway

Adjacent Stations, in this particular usage, (as it has been named in the ‘Same Speed Railways’ article for many years, since the concept was first recognised and investigated,) are a pair of successive stations, whose intervening distance is insufficient for a train, which calls at both, to reach line speed between them. In other words, the Inter-Station (I-S) distance is less than the sum of the distance required to accelerate from zero up to line speed and then decelerate from line speed back down to zero, even without any section at constant line speed in between. Consider the general case of an arbitrary line speed v_l , with trains accelerating at constant rate a_a from zero up to an arbitrary Inter-Station (I-S) maximum speed $v_{\max I-S}$, and then instantaneously switching to decelerating at constant rate a_d back down to zero, assumed to be over the distance between successive stations, it is readily shown that this is a pair of adjacent stations if $v_{\max I-S} < v_l$. Define the **Adjacency Coefficient** $a_j = a_a a_d / (a_a + a_d)$. (The notation deliberately highlights that the Adjacency Coefficient has the characteristics of a (constant) pseudo-acceleration rate.)

The following results are straightforward (albeit **dreadfully** tedious) to derive:

$$\begin{aligned} v_{\max I-S} &= a_a t_a = a_d t_d, \therefore t_a = v_{\max I-S} / a_a \text{ and } t_d = v_{\max I-S} / a_d \\ t_{I-S} &= t_a + t_d = v_{\max I-S} \{1/a_a + 1/a_d\} \\ &= v_{\max I-S} / \{a_a a_d / (a_a + a_d)\} \\ \therefore v_{\max I-S} &= a_j t_{I-S} \\ s_{I-S} &= \int v_{\max I-S} dt = a_j t_{I-S}^2 / 2 \\ &= s_a + s_d = v_{\max I-S}^2 / 2a_a + v_{\max I-S}^2 / 2a_d \\ &= v_{\max I-S}^2 / 2a_j \end{aligned}$$

What we actually want is these results re-stated in terms of the Inter-Station distance, s_{I-S} , as the independent variable, since we always know the value for this. Thus:

$$\begin{aligned} v_{\max I-S} &= v(2a_j) \times v(s_{I-S}) \\ t_{I-S} &= v(2/a_j) \times v(s_{I-S}) \quad [\text{cf } s_{I-S} = a_j t_{I-S}^2 / 2 \text{ above}] \end{aligned}$$

With the usual values $a_a = 0.3 \text{ m/s}^2$ and $a_d = 0.5 \text{ m/s}^2$, this gives the value $a_j = 0.1875$ (3 / 16 is the easy way to remember it). Hence, for the normal line speed of 327kph, we have:

$$\begin{aligned} v_{\max I-S} &= 0.612372 \times v(s_{I-S}) \\ t_{I-S} &= 3.265986 \times v(s_{I-S}) \quad [\text{cf } s_{I-S} = a_j t_{I-S}^2 / 2 \text{ above}] \end{aligned}$$

So, for the borderline case, we have a pair of adjacent stations if:

$$\begin{aligned} v_{\max I-S} &< 90.796879 \text{ m/s} \\ \text{and thus } s_{I-S} &< 21.9842 \text{ km,} \\ \text{and thus } t_{I-S} &< 484.250 \text{ s} \end{aligned}$$

Spot on, in both cases, which serves as a refreshing sanity check. I made the point in an earlier aside about the dreadful tediousness of these calculations. They are also distressingly error prone. A successful

sanity check is therefore very welcome. (Adjacent Stations are far more common on High Speed railways than for other speed ranges, so all numerical examples will be for HS 32tph.)

Provided that the actual line speed $v_l \leq v_{\max I-S}$, then the two stations are **not**, in the present sense, adjacent, and the theory expounded in the previous section of the present article applies (likewise the treatment in volume 1 of this article applies unchanged, particularly the sections ‘Timetabling Considerations and Sweet-Speeds’, ‘Timetable Diagrams’ and ‘Stations on the Main Line: HS-Metros, Pure Metros and Semi-Metros’.) The above results serve merely to confirm that this is indeed the normal case. But if the maximum I-S speed above is less than the actual line speed, then we do have a case of adjacent stations, and special techniques are required to accommodate it.

The first point to make is the somewhat redundant one that there would be no problem, if **every** train, i.e. if **all capacity substreams**, called at both those stations. This is not quite the equivalent of the *reductio ad absurdum* that there would be no problem of overtaking at a station, if all trains stopped there. It **could** be the case, albeit implausibly, that every train called at a particular pair of adjacent stations, but there was overtaking at other, non-adjacent locations.

The fundamental problem of timetabling is to ensure that services of different characteristics conform to some underlying, universal time standard. This is what the Capacity Slot Model, within the context of the Same Speed Model, provides. The way it does this is by ultra-precise determination of the station wait times. When there are no adjacent stations involved, then the same station wait time applies in all cases which also have the same line speed, (with a different, same value for other stations in another section with a different line speed, if there are such). When adjacent stations are involved, then the total wait time for the section between the two adjacent stations must be distributed between the two stations, probably equally, but possibly not (I’m not yet sure), but, either way, by quantities peculiar to that particular configuration.

The technique used to analyse adjacent stations for overtaking is entirely similar to that of section ‘Timetabling Considerations and Sweet-Speeds’ in volume 1. We note that the capacity slot given up by a stopping train travels precisely twice as far, at constant line speed, on the main line, as the train itself does, at varying speeds, including zero, on the (virtual) station loop. The only difference is that the calculations are carried out for both stations, as a pair. (Obviously so, as there is no station between them, and therefore no waiting time to adjust. That can only be done at the two stations themselves.) We know the distance that the empty slot has travelled beyond the end of the (virtual) station loop of the second station, and, for stopping trains to be able to re-join the capacity slot stream, the slot stream advance, the distance of the empty slot beyond the end of the (virtual) station loop, expressed in slots, must be an integer (the **corrected SSA**).

Dealing with a pair of Adjacent Stations, we define the **extended (virtual) station loop** to run from the point, on the main line, of start of deceleration before the first station of the pair, to the point of end of acceleration, back up to line speed, following the second station of the pair, calling at both stations. The physical distance is the same travelling between those two points via the station platforms, or via the main line, avoiding the platforms. A high-level description of what happens is that a train stopping at both stations is fully synchronised, in position and speed, with the capacity slot stream before the start of the extended station loop and following its end, but not in between. This is true also for the normal case with just one station, but for an adjacent pair, the train actually (usually) re-joins the main line temporarily between the stations, during which time it will **not** be synchronised with the capacity slot stream. This will happen provided that the distance between the stations is not too much shorter than the minimum

distance between non-adjacent stations, specifically that the maximum Inter-Station speed exceeds the Turnout Limit Speed of the switches. If this is not the case then there must be continuous quadruple track between the stations, since the main line cannot be re-joined other than at the TLS (this is part of how the Capacity Slot Stream operates, and not itself relevant in the present context – refer to the ‘Same Speed Railways’ article if you really want the details).

Therefore, taking the quadruple track aspect into account (with numerical values for UHS 32tph):

• v_l	$> v_{\max I-S}$	$> v_t$	90.797	$> v_{\max I-S}$	$> 63.889\text{m/s}$
• $a_j v_l$	$> t_{I-S}$	$> a_j v_t$	17.024	$> t_{I-S}$	$> 11.979\text{s}$
• $v_l^2/2a_j$	$> s_{I-S}$	$> v_t^2/2a_j$	21.9842	$> s_{I-S}$	$> 10.8848\text{km}$

We can say that, if the specific value of interest is **outside** the specified range then **either** (to the left) we don’t have adjacent stations at all **or** (to the right) **continuous** quadruple track exists between the stations. (The point of bothering with this seemingly esoteric detail is that if continuous quadruple track exists between the adjacent stations then we **know** that there can be no possibility of the stopping train interfering with non-stop, passing trains, since they are on separate tracks all the way between the stations. But for trains within the above range, the possibility of interference must be considered, shortly.)

This may sound alarming or even dangerous at first acquaintance. Be assured that there **is** a suitable empty slot available, which the train uses, but it has insufficient time (or need) to reach its standard position in the slot, before quitting it for the second station. Whether or not the main line is used between the stations does not affect the calculations; the significant fact is that the train is properly re-synchronised with the slot stream following the second station, whatever happens between them.

That may perhaps sound clear enough in principle, but to understand it properly needs an actual example. This is the section of HS3 between Durham Relly Mill and Consett. The Inter-Station distance is 20km, clearly less than the minimum distance for non-adjacency of 21.9842km. For this Inter-Station distance:

$v_{\max I-S} = 86.602540\text{m/s}$	$t_{I-S} = 461.880215\text{s}$	$t_a = 288.675134\text{s}$	$t_d = 173.205081\text{s}$
		$s_a = 12.5\text{km}$	$s_d = 7.5\text{km}$
Peak Inter-Station speed = 86.60m/s = 311.77kph = 193.77mph Slot Length = 9.7428km (*)			

For the route overall:

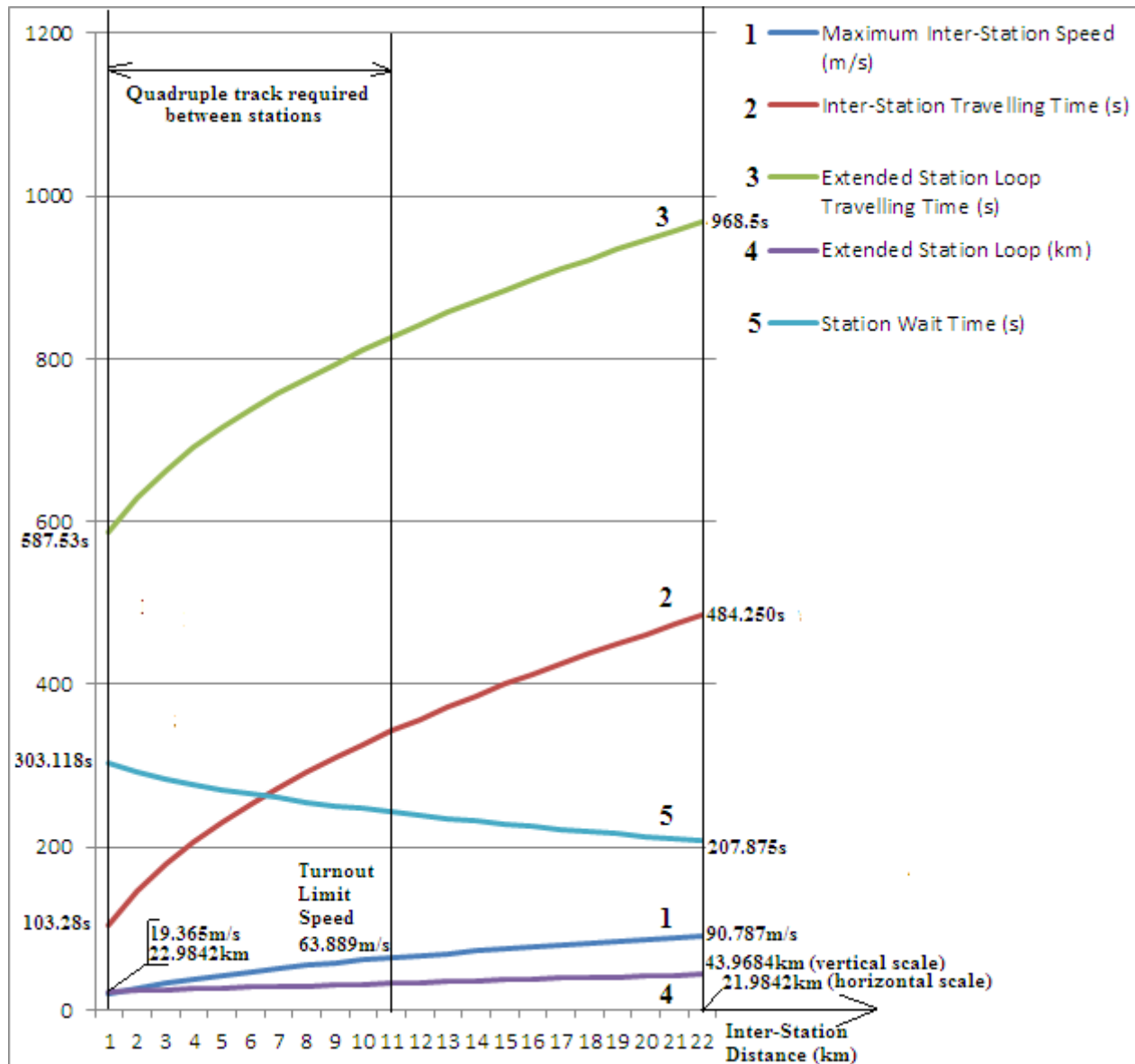
Line Capacity = 32tph.	Slot Time = 112.5s.
Line Speed = 90.80 m/s = 326.87kph = 203.11mph.	Slot Length = 10.2146km

Deceleration time to Durham.	= 181.59s	= 1.6142slots
Acceleration time from Durham	= 288.675s	= 2.5660 slots
Deceleration time to Consett	= 173.205s	= 1.5396 slots
Acceleration time from Consett	= 302.66s	= 2.6903 slots
Deceleration distance to Durham	= 8.2441km	= 0.8071 slots
Acceleration distance from Durham	= 12.5km	= 1.2830 slots (*)
Deceleration distance to Consett	= 7.5km	= 0.7698 slots (*)
Acceleration distance from Consett	= 13.7401km	= 1.3451 slots
Extended Station loop travelling time	= 946.13s	= 8.4100 slots
Extended Station loop distance	= 41.9842km	= 4.2050 slots (NB includes * values, above)

The alert reader may well feel outraged at the above trick (as it appears) of switching to a local value of distance slot between the stations (indicated by the apostrophe). I agree, it does look like a bit of sharp practice, computationally speaking. But in fact, that is what is required to ensure that the relationship of distance travelled by the empty slot (at constant line speed,) remains precisely twice that of the distance travelled by the train, decelerating down to zero then, without pause, re-accelerating back up to line speed. (This is dealt with at some length, including proof, in Volume 1 of the present article. Remember that the time slot stays constant throughout.

By the time the stopping train has travelled the length of the **extended station loop**, i.e from the start of deceleration before Durham to the end of acceleration after Consett, its empty slot, which it gave up on entering the first loop, has travelled 8.4100 slots (time or distance) on the main line. It is thus 4.2050 slots beyond the end of the extended station loop. We know that the required slot stream advance is 4 slots; two station stops are involved here, so two slot stream advances or 8 slots in total. This implies that the train must wait for a total of $(8 - 4.2050 =) 3.7950$ slots in total, or 1.8975 time slots, = 213.47s, at each station.

Note that the above is all that is required to derive the station wait times. The following graph illustrates how the several properties, in particular the station wait time, vary with the Inter-Station distance.



Adjacent Stations: Dependencies on Inter-Station Distance.

A surprising quirk in the above diagram, is that it looks very much as though it takes twice as long to accelerate up to 90,797m/s as it does to reach the turnout limit speed of 63.889m/s. But quirk is all that it is: the results are indeed close – but no cigar, nor, more importantly, significance.

The following illustration is just that, an illustration, to elucidate what actually, physically happens during overtaking. As always with such illustrations, maximum utilisation is assumed. Less than maximum is always significantly easier.

We now follow the progress of slot stream and stopping train.

Time Slot 1:

The empty slot advances 1 slot along the main line.

The train decelerates for 1 time slot. It thus has 0.6142 time slots of deceleration still to do.

Time Slot 2:

The empty slot advances a further 1 slot along the main line. It is now 2 slots beyond the start of the extended station loop.

The train completes its deceleration in 0.6142 time slots and reaches Durham Relly Mill station, where it waits for 0.3858 time slots. It thus has 1.5117 time slots still to wait there.

Time Slot 3:

The empty slot advances a further slot along the main line. It is now 3 slots beyond start start of the extended station loop.

The train waits for 1 time slot at Relly Mill. It thus has 0.5117 time slots still to wait there.

Time Slot 4:

The empty slot advances a further 1 slot along the main line. It is now 4 slots beyond the start of the extended station loop.

The slot containing the next stopping train for that particular platform has arrived at the start of the extended station loop.

The train completes its wait at Relly Mill in 0.5117 time slots, and then performs 0.4883 time slots of acceleration. It thus has 2.0777 time slots of acceleration still to do.

Time Slot 5:

The empty slot advances a further 1 slot along the main line. It is now 5 slots beyond the start of the extended station loop, and also 0.7950 slots beyond its end.

The second empty slot advances 1 slot along the main line.

The train performs 1 time slot of acceleration. It thus has 1.0777 time slots of acceleration still to do.

The second train decelerates for 1 time slot. It thus has 0.6142 time slots of deceleration still to do.

Time Slot 6:

The empty slot advances a further 1 slot along the main line. It is now 6 slots beyond the start of the extended station loop, and also 1.7950 slots beyond its end.

The second empty slot advances a further 1 slot along the main line. It is now 2 slots beyond the start of the extended station loop.

The train performs 1 time slot of acceleration. It thus has 0.0777 time slots of acceleration still to do.

The second train completes its deceleration in 0.6142 time slots and reaches Durham Relly Mill station, where it waits for 0.3858 time slots. It thus has 1.5117 time slots still to wait there.

Time Slot 7:

The empty slot advances a further 1 slot along the main line. It is now 7 slots beyond the start of the extended station loop, and also 2.7950 slots beyond its end.

The second empty slot advances a further 1 slot along the main line. It is now 3 slots beyond the start of the extended station loop.

The train completes its acceleration in 0.0777 time slots, then performs 0.9223 time slots of deceleration. It thus has 0.6173 time slots of deceleration still to do.

The second train waits for 1 time slot at Relly Mill. It thus has 0.5117 time slots still to wait there.

Time Slot 8:

The empty slot advances a further 1 slot along the main line. It is now 8 slots beyond the start of the extended station loop, and also 3.7950 slots beyond its end.

The second empty slot advances a further 1 slot along the main line. It is now 4 slots beyond the start of the extended station loop.

The slot containing the next-but-one stopping train for that particular platform has arrived at the start of the extended station loop.

The train completes its deceleration in 0.6173 time slots and reaches Consett station, where it waits for 0.3827 time slots. It thus has 1.5148 time slots still to wait there.

The second train completes its wait at Relly Mill in 0.5117 time slots, and then performs 0.4883 time slots of acceleration. It thus has 2.0777 time slots of acceleration still to do.

Time Slot 9:

The empty slot advances a further 1 slot along the main line. It is now 9 slots beyond the start of the extended station loop, and also 4.7950 slots beyond its end.

The second empty slot advances a further 1 slot along the main line. It is now 5 slots beyond the start of the extended station loop, and also 0.7950 slots beyond its end.

The third empty slot advances 1 slot along the main line.

The train waits at Consett for a further 1 time slot. It thus has 0.5148 time slots still to wait there.

The second train performs 1 time slot of acceleration. It thus has 1.0777 time slots of acceleration still to do.

The third train decelerates for 1 time slot. It thus has 0.6142 time slots of deceleration still to do.

Time Slot 10:

The empty slot advances a further 1 slot along the main line. It is now 10 slots beyond the start of the extended station loop, and also 5.7950 slots beyond its end.

The second empty slot advances a further 1 slot along the main line. It is now 6 slots beyond the start of the extended station loop, and also 1.7950 slots beyond its end.

The third empty slot advances a further 1 slot along the main line/. It is now 2 slots beyond the start of the extended station loop.

The train completes its wait at Consett in 0.5148 time slots. It departs and performs 0.4852 time slots of acceleration. It thus has 2.2050 time slots of acceleration still to do.

The second train performs 1 time slot of acceleration. It thus has 0.0777 time slots of acceleration still to do.

The third train completes its deceleration in 0.6142 time slots and reaches Durham Relly Mill station, where it waits for 0.3858 time slots. It thus has 1.5117 time slots still to wait there.

Time Slot 11:

The empty slot advances a further 1 slot along the main line. It is now 11 slots beyond the start of the

extended station loop, and also 6.7950 slots beyond its end.

The second empty slot advances a further 1 slot along the main line. It is now 7 slots beyond the start of the extended station loop, and also 2.7950 slots beyond its end.

The third empty slot advances a further 1 slot along the main line/. It is now 3 slots beyond the start of the extended station loop.

The train accelerates for 1 time slot. It thus has 1.2050 time slots of acceleration still to do.

The second train completes its acceleration in 0.0777 time slots and switches to deceleration. It performs 0.9223 time slots of deceleration. It thus has 0.6173 time slots of deceleration still to do.

The third train waits for 1 time slot at Relly Mill. It thus has 0.5117 time slots still to wait there.

Time Slot 12:

The empty slot advances a further 1 slot along the main line. It is now 12 slots beyond the start of the extended station loop, and also 7.7950 slots beyond its end.

The second empty slot advances a further 1 slot along the main line. It is now 8 slots beyond the start of the extended station loop, and also 3.7950 slots beyond its end.

The third empty slot advances a further 1 slot along the main line/. It is now 4 slots beyond the start of the extended station loop.

The train accelerates for 1 time slot. It thus has 0.2050 time slots of acceleration still to do.

The second train completes its deceleration in 0.6173 time slots and reaches Consett station, where it waits for 0.3827 time slots. It thus has 1.5148 time slots still to wait there.

The third train completes its wait at Relly Mill in 0.5117 time slots, and then performs 0.4883 time slots of acceleration. It thus has 2.0777 time slots of acceleration still to do.

During Time Slot 13:

The empty slot advances a further 0.2050 slots along the main line. It is now 12.2050 slots beyond the start of the extended station loop, and precisely 8 slots beyond its end. The (corrected) Slot Stream Advance is **8 slots exactly beyond the end of the extended station loop.**

The second empty slot advances a further 0.2050 slots along the main line. 8.2050 slots beyond the start of the extended station loop and therefore **4 slots precisely beyond its end.**

The third empty slot advances a further 0.2050 slots along the main line. It is now 4.2050 slots beyond the start of the extended station loop, **and therefore coincides precisely with the end of the extended station loop.**

The train performs its final 0.2050 time slots of acceleration. **It therefore coincides precisely with the end of the extended station loop and is travelling at full (normal) line speed.**

(The second train waits for 0.2050 time slots at Consett station – but so what?)

The third train performs 0.2050 time slots of acceleration – but so what?)

Note that the train, on re-joining the main line, takes over the empty slot given up by the **second** stopping train following it – of course it does! That given up by the stopping train immediately following has already been taken over by the one immediately preceding!

It is frankly acknowledged that this stuff is mind-numbingly difficult to envisage. To become a believer, you simply have to watch it happen, (over and over again, if that's what it takes,) which the above exposition seeks to demonstrate. If you still find it horrible to read, console yourself with imagining how dreadful it must have been to write. And it was, mainly because of the **vast** amount of checking it takes to eradicate (most of, but never all,) the errors – mainly trivial but still infuriating – which inevitably creep into hugely detailed stuff like this.

It is possible to imagine (and, regrettably, even to find,) sets of three adjacent stations, where the distances between first and second, and also between second and third, are less than required for a stopping train to reach line speed between station stops. A similar approach is required, aggregating the three stations into a single extended station loop, calculating the overall stations wait time and distributing it between them. There is in fact nothing to prevent any number of adjacent stations being too close together. This situation is most likely to occur when converting a conventional line to the Same Speed, standards, particularly a busy suburban route. In fact, for the whole stretch of the South Western route out of Waterloo, every single damn station between Wimbledon and West Byfleet (eleven of them - count the buggers!) is adjacent to its predecessor and successor save for the single exception of the bit between Surbiton and Esher!!! And, as you might expect, the inter-station distances are all different!

Timetabling of Propinquant Junctions on a Same Speed Railway

The first problem is how to describe it.

A Propinquant Junction is one which is sufficiently close to a station to interfere with the normal behaviour of trains departing from or arriving at the station. That much is clear. Only one junction is involved. If only one station is involved, then the propinquancy applies to that side of the junction; the other side is normal. But, theoretically at least, there could be two stations involved, one each side. I've yet to encounter such a case, but Sod's Law threatens, always. I'll work out what to call it if it happens.

Assuming, unless and until that eventuality actually happens, just one station, then the distance between station and junction is insufficient for a train departing from the station to reach line speed before having to begin the deceleration down to turnout limit speed at the junction. Alternatively, if the train is approaching the station, to call there, the distance is insufficient for it to re-accelerate back up to line speed, after the junction, before having to begin its deceleration for its station stop. The precise location of a junction is invariably taken to be coterminous with the associated switch points. (A Route Junction may in certain cases not be a Track Junction, if the track divergence is some distance away. So, to be absolutely precise, we're talking about the distance between the reference point of the station – generally the prescribed location at the platform of the front of a train when calling at the station – and the switch points of the relevant junction. But I'll still call it just a 'junction', for convenience.)

The main line, on which the junction is located is, obviously, that unaffected by divergence or convergence, along which the junction can be crossed, in both directions, at full line speed.

If the junction is crossed by trains departing from a station (on whichever side of the junction), it is termed Propinquant Accelerating, and the trains complete their acceleration up to line speed following the junction. If the junction is crossed by trains calling at the station (on whichever side of the junction), it is termed Propinquant Decelerating, and, obviously, the trains complete their deceleration to zero at the station, eventually.

If the junction is crossed by trains leaving the main line and joining the branch, then the junction is termed Propinquant Diverging. If the junction is crossed by trains leaving the branch and joining the main line, it is termed Propinquant Converging.

If the junction is crossed by trains departing from a station on the main line, it is termed Propinquant Diverging/Accelerating. If by trains from the main line calling at a station on the branch, it is termed Propinquant Diverging/Decelerating. If by trains from the branch calling at a station on the main line, it is termed Propinquant Converging/Decelerating. If by trains departing from a station on the branch, it is termed Propinquant Converging/Accelerating.

There are thus four types of Propinquant Junction. As with Adjacent Stations, Propinquant Junctions are far more common on High Speed railways than for other speed ranges, so all numerical examples will be for HS 32tph.

It may be obvious, but still needs to be stressed, that propinquant junctions are always route junctions. They are quite different from the (far more common) station loop junctions, which enable overtaking at the station. The branch diverging is generally a separate route (often to one of the several distinct termini of a High Speed route). But it is theoretically possible for the branch to reappear back on the main line. This would be where it serves a station which is some distance from the main line, and accessed by a

station loop **branch**. (The pre-eminent example of a station loop branch is Nottingham, but this doesn't actually involve a propinquant junction.)

There is one other, important distinction to deal with, concerning the train-length and movable switch parts effects. Every switch type has a Turnout Limit Speed, which is a property of the switch. This is the maximum speed at which the switch may be crossed when diverging from or converging onto the main line. Strictly, this is the maximum speed at which the **movable parts** of the switch may be crossed. This interacts with the train length (taken as 400m) and the direction in which the switch is crossed, thus whether the switch points are **directing** or **trailing**. (These two lengths are part of the buffer component of the Train Separation Distance.) How these two lengths, travelled at turnout limit speed, are distributed between the two adjacent sections of route, either side of the switch, is critically important in the scheduling of Same Speed railways. (**Everything** is either critically important, or unimportant in the scheduling of Same Speed railways. There are no **degrees** of importance. Importance is binary; it is important or it isn't.)

For Propinquant Diverging/Accelerating Junctions, (where the switch points are directing,) both lengths are assigned to the normal, non-propinquant section following the junction.

For Propinquant Converging/Accelerating Junctions, (where the switch points are trailing,) the movable switch parts length is assigned to the propinquant section, preceding the junction, and the train length to the normal, non-propinquant section following the junction.

For Propinquant Diverging/Decelerating Junctions, (where the switch points are directing,) both lengths are assigned to the propinquant section, following the junction.

For Propinquant Converging/Decelerating Junctions, (where the switch points are trailing,) the movable switch parts length is assigned to the normal section preceding the junction, and the train length to the propinquant section, following the junction.

These four types are now illustrated by Speed Profiles for the relevant line sections. The notation is:

v_l = line speed

v_t = turnout limit speed

v_q = maximum intermediate speed in propinquant section.

s' , t' = distance and time of the notional origin of acceleration up to the maximum, or notional destination of deceleration from the maximum.

s_q , t_q = distance and passing time of the maximum intermediate speed

s_{q1} , t_{q1} = distance and passing time of the turnout limit speed passing location nearest the station

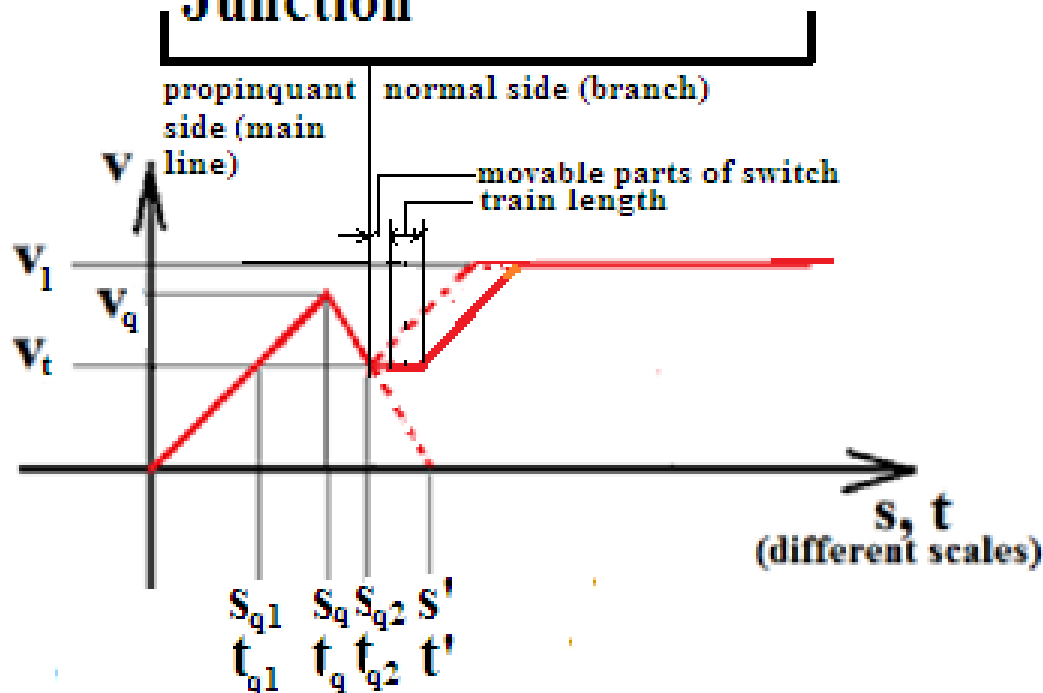
s_{q2} , t_{q2} = distance and passing time (if for switch points) or arrival time (if at station) of the far end of the propinquant section.

s_{q2}^* , t_{q2}^* are the actual distance between junction and station, and the actual time taken to traverse it.

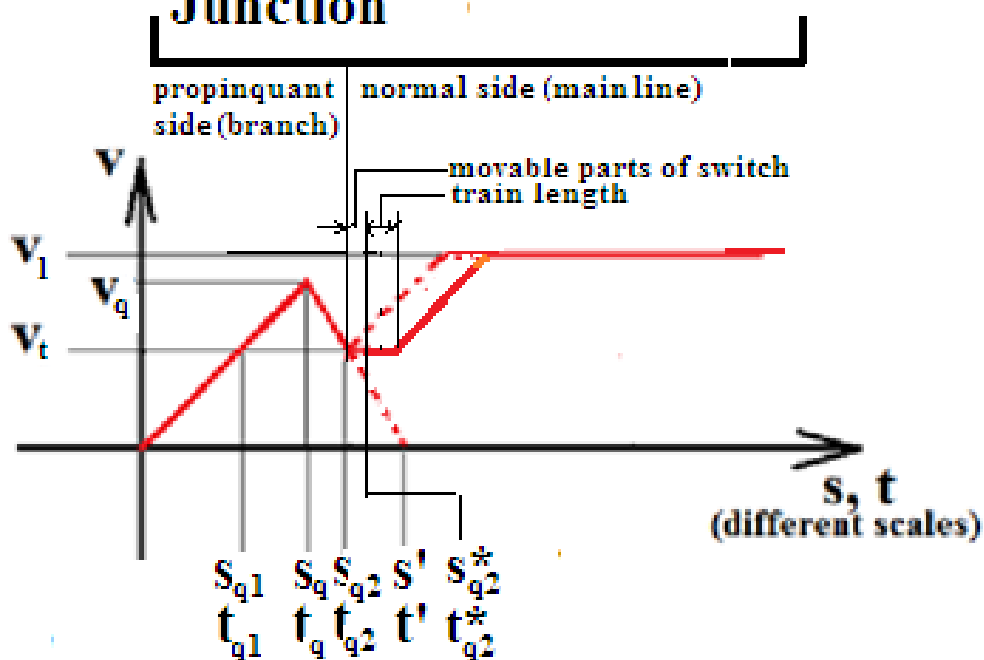
They are as for s_{q2} , t_{q2} , but including the assignment of the train length and movable switch parts effects, distance and time. (These effects are grossly exaggerated in the diagrams, of course; if they were to the same scale as everything else, they would hardly be noticeable.)

For all time and distance quantities, the origin of measurement is the station. This makes the values, for deceleration to the station technically negative, but we ignore this.

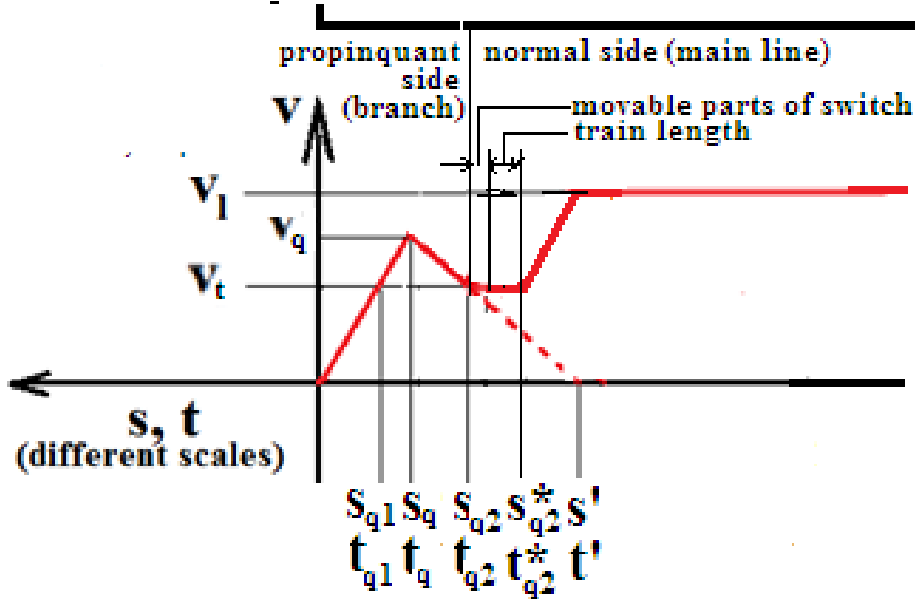
Propinquant Diverging/Accelerating Junction



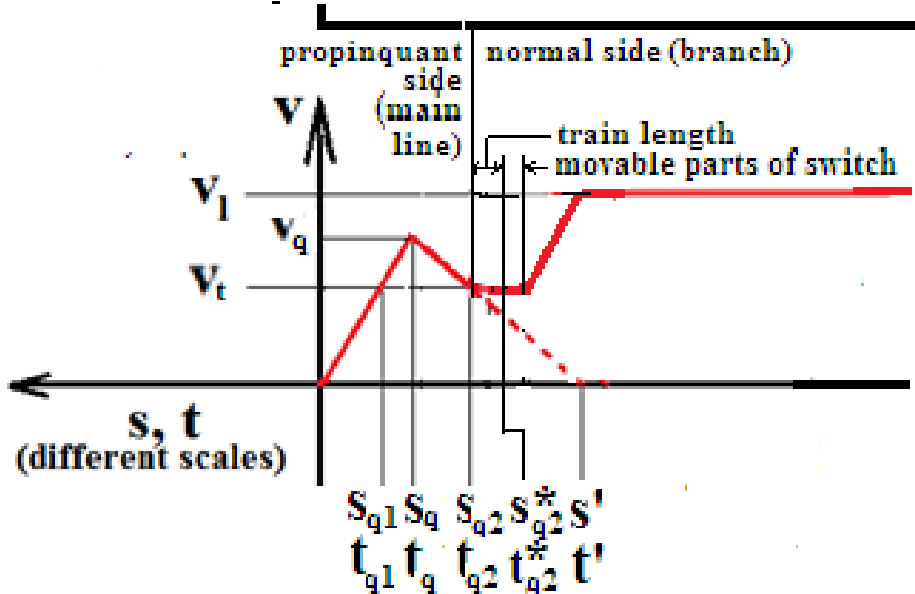
Propinquant Converging/Accelerating Junction



Propinquant Diverging/Decelerating Junction



Propinquant Converging/Decelerating Junction



The problem is: knowing v_l , v_t and s_{q2}^* (and thus also s_{q2} , since its difference from s_{q2}^* is known for each propinquant junction type,) determine v_q and t_{q2} , (and thus also t_{q2}^* , since its difference from t_{q2} is known for each propinquant junction type). Note that the precise location of the junction is at the switch points, a precise distance of s_{q2}^* from the station. Note finally that a propinquant converging junction on one track is invariably accompanied by a propinquant diverging junction on the other track, in the other direction, but that the reverse is not necessarily true, the decelerating distances being smaller than the accelerating ones.

Subtle stuff, eh?

This topic is surprisingly straightforward to analyse. The Adjacency Coefficient, $a_j = a_a a_d / (a_a + a_d)$, introduced in the previous section on Adjacent Stations, once again proves its worth.

1. Propinquant Accelerating Junctions:

The quantities v_t , s_{q2} are known in advance. Therefore so are: $t_{q1} = v_t/a_a$ and $s_{q1} = v_t^2/2a_a$.

$$t' - t_{q2} = v_t/a_d \quad \therefore t_{q2} = t' - v_t/a_d \text{ is the result we seek.}$$

$$s' - s_{q2} = v_t^2/2a_d \quad \therefore s' = s_{q2} + v_t^2/2a_d \text{ is also known.}$$

$$t_q = v_q/a_a \quad t' - t_q = v_q/a_d \quad \therefore t' = v_q(1/a_a + 1/a_d), = v_q/a_j$$

$$s_q = v_q^2/2a_a \quad s' - s_q = v_q^2/2a_d \quad \therefore s' = v_q^2(1/a_a + 1/a_d)/2, = v_q^2/2a_j$$

$$\text{But } s' = s_{q2} + v_t^2/2a_d \quad \therefore v_q^2 = 2a_j(s_{q2} + v_t^2/2a_d)$$

$$\therefore v_q = \sqrt{2a_j(s_{q2} + v_t^2/2a_d)} = \sqrt{3s_{q2}/8 + 1530.671} \text{ m/s}$$

$$t_{q2} = t' - v_t/a_d = v_q/a_j - v_t/a_d$$

$$\text{So } t_{q2} = \sqrt{2(s_{q2} + v_t^2/2a_d)/a_j} - v_t/a_d = \sqrt{32s_{q2}/3 + 43539.095} - 127.778 \text{ s}$$

$$\text{Also } t_q = v_q/a_a \quad t' - t_q = v_q/a_d$$

$$s_q = v_q^2/2a_a \quad s' - s_q = v_q^2/2a_d$$

2. Propinquant Decelerating Junctions:

The analysis is identical to case 1, but with a_a and a_d switched. The results are:

$$v_q = \sqrt{2a_j(s_{q2} + v_t^2/2a_a)} = \sqrt{3s_{q2}/8 + 2551.119} \text{ m/s}$$

$$t_{q2} = \sqrt{2(s_{q2} + v_t^2/2a_a)/a_j} - v_t/a_a = \sqrt{32s_{q2}/3 + 72565.158} - 212.963 \text{ s}$$

$$\text{Also } t_q = v_q/a_d \quad t' - t_q = v_q/a_a$$

$$s_q = v_q^2/2a_d \quad s' - s_q = v_q^2/2a_a$$

s_{q2}^* and t_{q2}^* follow automatically for each propinquant junction type. Summarising (for the usual HS 32tph case):

Propinquant Diverging/Accelerating:	$s_{q2}^* = s_{q2}$	$t_{q2}^* = t_{q2}$
Propinquant Converging/Accelerating:	$s_{q2}^* = s_{q2} + 194.5 \text{ (m)}$	$t_{q2}^* = t_{q2} + 3.044 \text{ (s)}$
Propinquant Diverging/Decelerating:	$s_{q2}^* = s_{q2} + 594.5 \text{ (m)}$	$t_{q2}^* = t_{q2} + 9.305 \text{ (s)}$
Propinquant Converging/Decelerating:	$s_{q2}^* = s_{q2} + 400 \text{ (m)}$	$t_{q2}^* = t_{q2} + 6.261 \text{ (s)}$

Also, for this usual case:

A junction is Propinquant Diverging (accelerating or decelerating) if $6.8030 < s_{q2} < 17.9024\text{km}$

A junction is Propinquant Converging (accelerating or decelerating) if $4.0818 < s_{q2} < 15.7757\text{km}$

The relevant value is determined by whether the junction's switch points are directing (thus diverging) or trailing (thus converging), in the direction of normal travel. Any junction above the relevant limit is a normal junction. Remember that the known value is s_{q2}^* , so make the necessary adjustment for s_{q2} .

6.8030km is the distance required to accelerate from zero to turnout limit speed, 230kph, and 4.0818km is the distance required to decelerate from turnout limit speed to zero. If there is insufficient distance for the train to reach turnout limit speed, propinquency is moot. If there is insufficient distance to decelerate to zero, then deceleration must commence before the junction is reached. Either way, the junction is a normal one.

Note that this section is not at all concerned with adjusting station wait times to maintain slot stream synchronisation. There is no need. Propinquant junctions affect identically all trains passing over them.

Sub-Stream Synchronisation

This concerns merging services from different routes onto a **recipient** route, along which they travel for some distance, either diverging again later, or staying with that route for the rest of their journey. One technique is always possible: if the routes converge at a station, where all trains stop, then they can be restarted from that station in their new, merged sequence. That is the crude, blunderbuss approach, and is, indeed, always possible, provided that sufficient free capacity is available. A subtler approach is however available, and that is the subject of this section.

When a service joins a Same Speed route, either from another Same Speed route, or from a conventional railway, it must be synchronised with the route it joins. This has three separate aspects, Line Capacity, Slot Synchronisation and Sub-stream Synchronisation.

It is obvious that if Same Speed routes are to be merged, they must be operating to the same line capacity, so that their properties, specifically time slot, slot stream advance / number of capacity sub-streams, station wait time and line speed are identical. It is not essential, but it is a great simplification, if the service joining the recipient route is confined to one sub-stream of that route, which is not used by any other service.

Slot Synchronisation means that each slot abuts the back end of the preceding slot and the front end of the following slot, with no overlapping and no gaps. This is of course merely stating a truism; it is a fundamental property of the Capacity Slot Model. What it means in practice, which leads directly to the means by which it is enforced, is that the front end of each slot, or the front end of the train occupying the standard position in the slot, (if that is easier to imagine,) passes a specific location (the **Timing Point**) in a regular, repeating time sequence. Remember that the capacity slot stream is a purely virtual concept, with no physical embodiment. Trains **could** join a route at any time whatever, but with likely disastrous consequences. But calculating the times at which they would pass the timing point indicates the change which must be made to their scheduling to enable them to join the route safely. The capacity slot stream is, I believe, the successor to the moving block concept, the holy grail of generations of railwaymen.

Capacity slot sub-streams are an integral component of the means of overtaking at stations, thus enabling a mixture of stopping and non-stop services (over particular sections of route). Each sub-stream defines a particular calling sequence of stations and, strictly, individual platform faces, called at by all trains within that sub-stream, and at which they may be overtaken by trains in other sub-streams. Trains (potentially) run in each sub-stream at a time interval equal to the slot time multiplied by the number of sub-streams. Trains joining a recipient route must join one or more specific sub-streams and conform in all cases with the timing of their relevant stream. What this means is that the overall sequence of trains on the main line consists of one train **or** one empty slot from each sub-stream in turn, in fixed order, the pattern repeating indefinitely.

Several stream-merging configurations are possible. The underlying methods of synchronisation are essentially the same, but differ in detail and appearance. The most straightforward case is where a single service, already running on a Same Speed Railway, joins (a single sub-stream of) the recipient route. The receiving sub-stream would normally be empty, but this is not essential. It is of course essential that sufficient empty slots must exist in the receiving sub-stream to accommodate the joining traffic. A timing point is selected, on the recipient route, beyond the joining point, at which existing and joining traffic will be behaving similarly, specifically travelling at the same speed, generally line speed, but not essentially so, or the point of arrival at a station at which both services call. The standard journey time calculation

will give the arrival or passing time at the timing point, for each service. (The timing point at the **other** – starting – end of each of the two section over which the times are measured is the junction point at which the joining service joins, for the main line, the recipient route, and the departure time or nearest previous calling station **before** the junction point for the joining service.) The essential synchronisation condition is that the difference between these two times must be an integer multiple of the slot time. By default, it wouldn't be, so it must be **forced** to be; see below. (What this means is that both trains occupy the identical position within their own individual slot. They do not arrive at the **same** time because they are – of course – in different slots.) If the difference in the two times is, additionally, an integer multiple of the number of capacity sub-streams, then the two services will merge into the same sub-stream. If it isn't, they won't. (So, for example, if the slot time is 112.5s and the number of sub-streams is 4, and the timing difference is any integer multiple of 112.5s, thus 112.5, 225.0, 337.5, 450.0 and so on and, in addition, it is an integer multiple of 4 (thus 450 only, of the above since $450 = 4 * 112.5$), then the two services merge into the same sub-stream. Otherwise they will be 1, 2 or 3 (= 1, looking in the other direction,) sub-streams apart.

So how to ensure an integer number of time slots? By precisely the same method used to enable overtaking at a station. The difference in times at the timing point is precisely equivalent to the Slot Stream Advance (SSA), and an appropriate quantity of time is added as wait time at the nearest previous stopping station before the merging junction, to ensure that the SSA is an exact integer multiple of the slot time. (Effectively this means that the joining service is synchronised with the receiving route from the point at which it departs from that nearest preceding station.)

The above explanation applies to the joining service being already on a Same Speed Railway. If it is, in fact, a conventional branch line, and so does not conform to Same Speed standards, then the same method is used, adjusting the wait time at the nearest, preceding, calling station to ensure an integral SSA, and the line becomes Same Speed, and synchronised with the receiving route, from that point onwards.

But what if, for either case, there **is** no preceding station? In that case the time must be measured from start of journey, so the control has to be applied to the time instant at which the journey is obliged to begin – much less convenient, but perfectly possible.

Note that it doesn't matter however many quantities ($SSA * \text{Time Slot}$) there are in the time difference value; they all simply cancel out and have no effect on the wait-time adjustment required at the nearest, preceding, calling station. Note that the above quantity ($SSA * \text{Time Slot}$) is the **Station-Calling Time Penalty** – the additional time required to call at an intermediate station, as compared with travelling through or past it at full line speed, without stopping. Strictly speaking, this is for a normal station call' not requiring any extra wait time adjustment for Slot-Stream Synchronisation, as in this present section, or for Adjacent Stations, or Propinquant Junctions, in the previous two sections.

The other important stream-merging configuration is where a branch diverges from the main line of the Same Speed Railway, but subsequently re-joins it. This is invariably a station branch, where the station is either located some distance from the main line, but still part of the High Speed route, (these are almost invariably encountered on High Speed lines,) or where the train diverges onto another High Speed route, calling at one or more stations thereon, but then leaves that route and re-joins the original one or where that route as a whole merges with the recipient route. Very obviously, the distance travelled between diverging and re-joining is quite different from that travelled on the main line between those points, so the diverging train must be re-synchronised on re-joining. This sounds complicated, but is, in my opinion, only conceptually so; it's actually easier to calculate than the notionally straightforward case above.

This is, I know, all fairly confusing, certainly at first sight, so here is a real example, of the supposedly more difficult station branch case.

In the HS3 route from London to Scotland and the North East, via the East Midlands and West Yorkshire, Nottingham is served by a station on a branch from the main line, diverging at Stanford Junction, near Loughborough and re-joining it at Nuthall South Junction, to the north west of Nottingham. The distance between Stanford and Nuthall South junctions is 6.38km longer via Nottingham, compared with the direct route along the main line. The timing point chosen is Huthwaite Junction, 18km north of Nuthall South Junction, because all trains (other than those diverging there) are travelling at full line speed, whereas Nottingham trains have to accelerate up to line speed after re-joining the main line at Nuthall South, and have done so well before reaching Huthwaite (where none of them diverge).

Note the differences from the normal case of overtaking at stations on the main line. There, stopping trains diverge onto a station loop, which serves the platforms. Non-stop, overtaking trains remain on the main line, which bypasses the station platforms, but still passes through or very close to the station, such that the distance travelled is the same via the station loop or on the main line throughout. On the Nottingham station loop branch there is no overtaking – of course not; that's being performed on the main line, some distance away. The standard station wait time is determined by overtaking, in the standard case of stations on the main line, and it may not be varied from this standard. To do so would damage the capacity sub-stream structure – they would no longer be the same distance apart.

(It was only as a result of the analysis of the Nottingham branch that I came to realise that the station wait time is a property of the line capacity, and may not be arbitrarily varied. The earlier section on Adjacent Stations does not contradict this; it is a special case, where the two stations are treated as a single entity, and the adjustment of the wait times is contained internally, with no external effect. But when a particular section of line has no overtaking, such as the Nottingham branch, where all trains call at Nottingham, the station wait time may be varied, provided all trains observe the same value, and this has no effect **within that particular section of line**, though it will have effects outside that section. That is a critical perception, in dealing with this case.)

The essential condition for trains travelling via Nottingham to re-join the main line is that they re-join their own sub-stream (in which they were travelling before diverging onto the Nottingham branch) in the correct locations, such that, examining the trains in that sub-stream after the Nottingham branch has re-joined, (and after the trains have had time to accelerate back up to line speed,) it is impossible to tell, from their distance apart or their speed, whether they travelled via Nottingham or directly along the main line. This is another epiphany: once formulated it is so damn obvious that it couldn't be any other way.

Actually it **could** – they could join a **different** sub-stream if there were space available there and a good – timing – reason to do so. Also the question is itself virtual; since a sub-stream is defined as a calling sequence at a set of stations and platforms, we would **know** that all the trains had called at Nottingham. (Though on further reflection I am not, yet, certain about that.) It is, after all, perfectly okay for non-stop trains to share a sub-stream with stoppers, provided they actually diverge from the main line, and thus leave the sub-stream, before any of those stations have been reached. In practice, this happens a lot.

As it happens, Sub-Stream Synchronisation automatically takes care of Slot Synchronisation also, since all trains in a sub-stream pass the timing point at exactly the same time relative to the start of slot, since all trains in a sub-stream (in fact all trains within every sub-stream, once travelling at steady line speed,) occupy the same position within their slot.

To the numerical values, then. Travelling directly, non-stop from Pancras Cross, a train reaches the timing point of Huthwaite Junction (distance 214km) in 2508.237seconds. Travelling via Nottingham (total distance 220.38km), stopping there but with zero station wait time, it takes 2850.072 seconds. The time interval between trains in the same sub-stream is 450seconds (7½min). The passing time at Huthwaite for Nottingham trains must therefore be $2508.237 + 450 = 2958.237\text{s}$. This is achieved by a station wait time at Nottingham of $2958.237 - 2850.072 = 108.165\text{s}$. This is too short – less than 2 minutes – while the next possibility, of extending it by a further 450s to 558.165s – over 9 minutes – is too long. This is unfortunate, so we have to have recourse to the possibility mentioned in passing above, of joining a different sub-stream, and, as it happens, all the Nottingham trains (in sub-stream 4) will leave the main line again at Wales Junction, 25km north of Huthwaite, and over this section sub-stream 2 is completely empty (its trains from St Pancras having diverged onto classic routes at Leicester or earlier, and further trains not joining until York), so that is available for an extra wait time of 225s, thus a total wait time at Nottingham of 333.165s, 5½min, which is longer than the standard wait time for this capacity and speed, (287.878, 3½min,) but still perfectly reasonable. This wait time also ensures that the trains, immediately on departing Nottingham are travelling in sub-stream 2.

I freely admit that, in building timetables, it can get rather tricky keeping track of which sub-stream a particular service is travelling in, for a particular section, if sub-streams get switched, automatically, by the synchronisation process. But that's life.

(And, as has been mentioned previously, if you find the preceding section a fundamental pain to read, imagine what it was like to write!)