Same Speed Railways

Introduction

This article was originally entitled ‘The Philosophy of High Speed Rail’, a pompous title, perhaps, but I couldn’t think of a better one at the time, and high speed railways is what it dealt with. Having written it, however, I realised that the underlying principles applied equally well to Low Speed Rail, which is a humorous concept only at first thought, but really is another example of a **Same Speed Railway**. I prefer ‘same speed’ to ‘uniform speed’, as that, to my ear, implies a railway route with the same line speed throughout, whereas what the concept actually involves is a railway where all trains travel at the same speed at any given location, but that speed may vary between different locations. For simplicity and familiarity I leave the original content as written, but then add a further section to generalise the theory.

The Purpose

What this article seeks to do is to define what HS rail is (as compared with other types), the circumstances in which it is appropriate to deploy it (thus what we intend to achieve by it), and recommendations on its deployment – a set of guidelines.

My article ‘Towards a High Speed **Network**’ seeks to make the case for developing a network plan for all the HS routes which will eventually be needed, as opposed to the free-standing, isolated approach which characterises the HS2 proposals (I contend, in the above article, that certain aspects of the HS2 plans **prove** that they were developed in isolation from all other routes). The title is a reasonable shorthand, but really there is no such thing as a high speed network; there is only the **railway network**, certain lines of which happen to be high speed, but all of which are intimately connected and work together. This may seem an obvious point, but I contend that many (I won’t say all because I don’t know them well enough) countries which have developed high speed lines have developed them as a separate, stand-alone system, and any interfaces with the existing (‘classic’) railway have been an afterthought. I think this is certainly the case with France and Germany. Such an approach works to an extent, and does clearly have some benefits, but I contend that it loses the very significant benefits of synergy, and results in an overall rail system, parts of which are good, but the rest of which is disregarded, shabby and starved of investment.

That the UK is so far behind much of the rest of the world in developing HS lines does give us the opportunity to learn from and avoid the mistakes that others have made (just as the rest of the world learnt from our mistakes, as the original developers of railways and so, for example, went for much more generous loading gauges). This is a sweet irony.

The Nature of a High Speed Railway

Fundamentally, a HS railway is essentially just a modern railway, a railway designed and built to the best of modern standards. With the superb alignments which modern construction techniques make possible, the quality of modern trackwork, the power and flexibility of modern signalling and control technologies (especially dynamic block working and automatic train control), and the performance of modern trains, it isn’t surprising that the trains can go a lot faster than on classic routes. But there’s a lot more to it than that.

A HS railway is not a mixed-traffic railway, it is a dedicated, express passenger railway. Certain types of freight traffic, with similar performance characteristics to passenger traffic, such as mail and parcels, can also be accommodated. Theoretically, one could envisage a HS line being used for general freight traffic overnight, when there is no passenger traffic, but in many if not most cases, the types of alignment suited to passenger trains, involving (in classic terms) quite severe gradients, make HS lines unsuitable for heavy freight.

A HS railway is, very importantly, a High Capacity railway. All the traffic shares similar performance characteristics, in particular all travelling within the same, narrow speed band (which varies, obviously, between different locations, but all traffic has essentially the same speed at any particular location). This is the essential condition for maximum throughput. Dynamic block and automatic train control further ensure that this maximum is at as high a value as possible.

HS trains never stop on the main line, or they **all** stop, at a particular location, i.e. a station. A (non-terminal) HS station generally consists of two island platforms, i.e. two platform faces in each direction. If all services stop at that station, then that's all there is to it, and the pointwork can be ordinary, fairly low speed. If not all services stop at the station, then the main line must continue unobstructed through the centre of the alignment, without adjacent platforms, (or in tunnel, underneath, or on viaduct, overhead,) and the stopping lines must diverge some distance either side of the station, using high speed pointwork, so that trains diverge at full line speed, then have adequate braking distance (mainly regenerative) to come to a stand at the station platform, then have adequate distance to accelerate back up to full line speed before rejoining the main line. (Note that this behaviour is **exactly** analogous to motorway driving: vehicles do not slow down on the motorway before diverging at a junction; they travel at full speed onto the slip road, and slow down there. Likewise they accelerate to full traffic speed before joining the motorway. 'Slip-lines' are, of course, rather longer than slip-roads.) The fundamental point is that nothing must prevent a train from travelling at full line speed anywhere on the main line. In certain locations (such as Nottingham), it is more convenient to have the HS station on a loop, away from the main line, so the main line bypasses the station completely.

The above explanation is valid in its essentials, and good enough for a high-level understanding, but it **is** very much simplified. The true situation is considerably more complicated and more subtle. Technology junkies are referred to appendix B, which contains the full story, and references to the original source articles, for those who **must** have the really hard stuff.

Some people think a HS line must have virtually no intermediate stations. This a misconception; the defining characteristic, as explained in the previous paragraph, is that a non-stop train is never **impeded** by the presence of stations, but can travel at uninterrupted line speed for as far as necessary. Nor is it impeded by other trains stopping at the stations, as these get out of its way in a timely fashion. All trains travel on the main line at full line speed; a train decelerates from line speed only when it has already left the main line, and is on a station line or loop, and accelerates on the station line or loop back up to line speed before rejoining the main line. Within reason, a HS line could have any number of intermediate stations, and some trains could stop at some or all of them without impeding those not stopping. (My proposals for HS3 envisage two categories of service, the HS Metro services to York and Preston, which stop at all intermediate stations, and the long-distance UHS – Ultra HS – services to the NE and Scotland, which travel non-stop to South Yorkshire or York.) Of course, the more intermediate stops a particular train makes, the lower is its overall end-to-end average speed, even though on the main line it travels at line speed. The long distance UHS services travel for long distances at line speed, without stopping, so their overall average speed is high. (And it is these long-distances-without-stopping UHS services that really make use of and justify the 250mph maximum.) In fact, given careful timetabling, a HS line can readily accommodate a mixture of stopping and non-stop services.

HS lines are built to the GC-standard loading gauge. This is a practical rather than an essential characteristic, and is only really relevant to the UK, where the loading gauges of the classic lines are so restricted (in some countries, everything is GC gauge or even better anyway). This means that two types of train run on (UK) HS lines – GC-gauge (or 'captive') trains, which can only run on HS lines (or, in a few cases, extensions therefrom of GC-gauge on classic lines), and Classic-Compatible trains, which are built to UK standard loading gauge, and can run on both HS and classic lines (even sharing the same, variable platforms with GC-gauge trains – see Appendix B of my 'Network' article for an explanation of variable platforms).

Some people think it ridiculous to have trains which cannot run on all lines, but are restricted to a relatively small part of the network. Other things being equal, this argument has some merit. but other things are not equal – the increase in capacity offered by GC-gauge is profound. In particular, GC-gauge readily accommodates double-deck trains, with plenty of room inside them. As a rule of thumb, a double-deck train offers the same passenger capacity in two thirds of the length of an equivalent single-decker, or alternately, a single-decker has to be half as long again to offer the same capacity. This offers very serious savings in platform length and thus station area. In any case, I think the above is a defeatist attitude, which accepts the restricted UK gauges for ever. With GC gauge for HS lines, we have an important and growing proportion of the overall network which can accept the high-capacity, including double-deck, GC-gauge trains.

When to Deploy HS Lines

There is only one fundamental, **deciding** reason to deploy a new HS line: when an existing classic route is overloaded and significant additional capacity is required. (Thus a HS line is always associated with a particular classic route.) The fact that trains can travel much faster on these lines is a **reinforcing** reason, not a deciding one. 'High Speed Railways' is in fact a misnomer, they really ought to be called High Capacity Railways, but the usage is now entrenched and it's pointless to try to change it.

HS-Antis and other romantic mediaevalists try to argue that upgrading the existing, classic line to expand its capacity would be cheaper and less disruptive. It is strange that anyone feels they can make this argument seriously, after the experience of the WCML upgrade, which was **monstrously** expensive, **hugely** disruptive over a **prolonged** period, and after all that didn't even deliver the goods, and needs further work now, a few years later. Of course, there are changes that could advantageously be made and should be made to classic routes – the odd flyover, discreet extra tracks here and there – but these are, however worthwhile in themselves, mere ameliorations, when what is needed is a quantum leap.

There is, however, a more fundamental argument against trying to increase the capacity of an existing classic line beyond its reasonable limit. These are all mixed-traffic routes with, usually, several intermediate stations. This very fact severely restricts the available capacity, as compared with what the same infrastructure could accommodate if the traffic were homogeneous, as on a metro, for example. The requirement is usually to increase capacity between the end points, typically between a major regional centre and a London terminus, Manchester – Euston, for example. Beyond the reasonable capacity limit of the route, the only way to get additional capacity out of the existing infrastructure is to increase the priority of one type of traffic at the expense of the others. In the extreme case this would be just an express service between the end points, serving nowhere in between, and no other traffic, and so no longer a mixed-traffic route. But even short of this extreme, it would involve severe degradation in the service offered to intermediate locations. Even the most simple-minded HS scheme, serving just the end points, significantly reduces the loading of the classic route, and allows a decent traffic mix, even improving the service to intermediate locations; thus the HS line benefits people who don't even use it.

Guidelines to HS Deployment

Guideline 1: No location should suffer a worse service as a consequence of a HS line opening. Self-evidently true, surely? Yet many places, most infamously Stoke-on-Trent, will suffer a worse service when HS2 phase 2 opens, according to current plans.

The problem arises because express services on a classic trunk route between a major regional centre and London (Manchester – Euston, for example, again) typically have a number of stops at the regional end, to pick up traffic from lesser but still important locations in the originating region (the 'secondaries', say – in the present example Stockport, Macclesfield and Stoke-on-Trent), then a long non-stop (or just one or two stops) run to London. The bulk of the traffic is from the first station (Manchester Piccadilly). A HS line links the endpoints of the associated classic route, and would reasonably be expected to take over all the end-to-end traffic from the classic route. It may serve other intermediate locations, but will not directly serve the secondaries, which thus could face a worse service than previously. The way to solve this dilemma is to run a classic-compatible service along the initial section of the classic route, serving all the secondaries (and ideally a few more secondary-type locations, to help fill it), and then to leave the classic route and join the HS route at an intermediate junction. In the present example, my proposal is to run a classic-compatible service Manchester Piccadilly – Stockport – Macclesfield – Stoke-on-Trent – Stone – Stafford – Rugeley Trent Valley (for Walsall and Cannock) – <Lichfield HS junction> – Birmingham Interchange – Calvert – Old Oak Common – Euston. This also has the serious advantage of freeing up slots on the classic route (over the entire section beyond the intermediate junction with the HS route, but most importantly on the approach to London, where capacity is most likely to be under pressure). If the traffic is no longer sufficient to fill the classic-compatible train adequately, use a shorter formation. We thus have:

Guideline 2: There should be at least one intermediate junction to the HS route from the associated classic route, to allow classic compatible services to run serving those regional secondary locations served by the original classic express service, but not served directly by the HS route (and perhaps additional secondary locations on the classic route), joining the HS route at this intermediate junction, then high speed thereafter. This intermediate junction can also take other classic-compatible services from locations beyond the associated classic route (in the Manchester – Euston example, services such as from Preston and Liverpool). Indeed **all** services on the associated classic route from before the HS junction (which may originate on other classic routes which join it) are candidates to become classic-compatible services, freeing up slots on the classic route beyond that junction.

Guideline 3: Terminal HS stations in locations of high traffic demand are a very bad idea, as they need to be disproportionately large to provide the necessary capacity (trains terminating, being serviced in situ, then forming a service in the reverse location make prolonged demands on platforms). This applies especially to London locations. It is far better for such locations to have through stations (of the standard double-island model, with all services stopping), with the HS route subsequently branching to serve several terminal destinations, each individually needing only moderate capacity. A prime example of this is the proposed Euston Cross, with services travelling on to HS1 and Kent / East Sussex, and terminating at Maidstone, Hastings or Dover. This also provides excellent inter-regional facilities.

Guideline 4: Services on the associated classic route change, as soon as the HS route opens, to the Regional Metro pattern. This consists of two groups of (passenger) services, semi-fast and stopping. The semi-fast services are regular interval, over the whole or portions of the route, stopping at all traffic sources of reasonable size (i.e. towns / large villages or parkway-type locations with a sizeable drive-in area). The stopping services are generally hourly, stopping everywhere on a particular section of the route, and connecting into or out of the semi-fast service at each end. At all appropriate stations served by both HS and semi-fast regional metro trains, the regional metro trains being timetabled to make interchange connections into and out of the HS trains, and have similar frequencies. (Note that the HS trains mentioned in the preceding sentence could of course be classic compatibles, running on the initial section of the classic route.)

Low Speed Railways

The fundamental characteristic of high speed railways, that all trains travel at the same line speed, anywhere on the main line, has nothing to do with high speed as such, but is the defining characteristic of a Same Speed Railway, whatever that speed actually is.

Obviously, for dedicated express passenger railways, the speed should be as high as practicable, taking into account the characteristics of the traffic (such as the average length of travel without stopping, thus whether ultra-high speed is appropriate or whether a lower maximum would give the same benefits, with a significant saving in construction costs). But the same principle would also apply very advantageously to a dedicated freight route – a Low Speed Railway. A same speed railway could thus be any railway where the traffic is all of the same type, specifically with the same performance characteristics and thus capable of travelling at the same speed. Low speed railways could indeed carry passenger traffic, but this would have to travel at the same line speed as the freight traffic – if this is in the range 50-70mph then that needn't be a problem on a secondary passenger route.

It is important to stress that a low speed railway is not simply a freight line as currently understood. Those aspects of same speed railways which **enable** all traffic to have the same speed – specifically the location and type of pointwork – apply in just the same way as already explained for high speed lines. If a low speed line has a passenger service, then the station platforms must be on passing loops – there are never platforms adjacent to the main line – and the stopping lines must diverge some distance either side of the station, to allow stopping trains to diverge at full line speed and then decelerate to come to a stand at the station platform, likewise to accelerate back up to full line speed before rejoining the main line. The pointwork required is obviously less demanding than for the high speed case, and the stopping lines shorter. Note that exactly the same considerations apply to freight trains, diverging into sidings in goods yards, but also that the stopping distances for freight trains will be considerably longer than those for passenger trains decelerating from the same line speeds.

For all that, a lot of classic routes for which freight is the dominant traffic could readily be enhanced to (low) same speed standards at moderate cost. Examples which spring to mind include Felixstowe – Peterborough – Leicester – Nuneaton – West Midlands (multiple destinations), GN/GE line Peterborough – Spalding – Lincoln – Doncaster and the Settle and Carlisle line north of Skipton (followed by the GSW route on to Glasgow).

It is worth repeating yet again that the whole purpose and justification of same speed lines is to maximise capacity, whatever the speed. Other technologies, principally signalling and control, determine the actual maximum value, but it is the fact that all traffic has the same speed which enables a maximum to be achieved at all.

Appendix A – Theoretical Maximum Line Capacity

Assume a same speed line with the characteristics:

λ = (maximum) length of train
δ = (minimum) permissible distance between trains
ν = line speed – speed of every train
Train Envelope = Length of train + separation distance to following train
= $\lambda + \delta$

A given train, and thus a train envelope, travels a distance νt in time t,
so the number of trains passing a given point in time t is $\nu t / (\lambda + \delta)$,
thus the capacity of the line is $\nu / (\lambda + \delta)$ trains per unit of time.

For traditional fixed-block working, δ has a constant value, so the capacity is linearly proportional to line speed. This is actually correct, but of course in traditional working, a maximum speed was selected, and the block length determined as the braking distance of a typical train travelling at that maximum (or possibly vice versa). In this situation, maximum capacity is indeed achieved when all trains travel at that maximum speed.

I am not sufficiently familiar with the technical details of dynamic block working to know how the distance to be maintained between trains is determined, but for illustrative purposes I assume it is proportional to the square of the speed, so that it relates to the kinetic energy of the train, which seems plausible. So take $\delta = \kappa\nu^2$. So the line capacity c is:
$c = \nu / (\lambda + \kappa\nu^2)$

$$
\begin{aligned}
\partial c/\partial \nu &= \{(\lambda + \kappa\nu^2) - \nu(2\kappa\nu)\} / (\lambda + \kappa\nu^2)^2 \\
&= (\lambda - \kappa\nu^2) / (\lambda + \kappa\nu^2)^2 \\
&= 0 \text{ when } \nu^2 = \lambda/\kappa
\end{aligned}
$$

So maximum capacity $c_{max} = \sqrt{(\lambda/\kappa)} / (\lambda + \kappa(\lambda/\kappa)) = 1/(2\sqrt{(\kappa\lambda)})$

$$
\begin{aligned}
[\text{Also } \partial^2 c/\partial \nu^2 &= \{-2\kappa\nu(\lambda + \kappa\nu^2)^2 - (\lambda - \kappa\nu^2)(4\kappa\nu(\lambda + \kappa\nu^2))\} / ((\lambda + \kappa\nu^2)^2)^2 \\
&= \{-2\kappa\nu(\lambda + \kappa\nu^2) - 4\kappa\nu\,(\lambda - \kappa\nu^2)\} / (\lambda + \kappa\nu^2)^3 \\
&= \{2\kappa^2\nu^3 - 6\kappa\nu\lambda\} / (\lambda + \kappa\nu^2)^3 \\
&= 2\kappa\nu(\kappa\nu^2 - 3\lambda) / (\lambda + \kappa\nu^2)^3 \\
&= 2\kappa\nu\{(\lambda + \kappa\nu^2) - 4\lambda\} / (\lambda + \kappa\nu^2)^3
\end{aligned}
$$

When $\nu^2 = \lambda/\kappa$ then $\partial^2 c/\partial \nu^2 = -\sqrt{(\kappa\lambda)} / (2\lambda^2)$, i.e. $\partial^2 c/\partial \nu^2 < 0$ so the above extreme value is indeed a maximum. (I would have been profoundly shocked had it turned out to be a minimum!)]

(I admit I didn't actually **remember** the formula for differentiating a quotient – not at any time in the last 50 years! – but the excellent website 'Paul's Online Math Notes', at
http://tutorial.math.lamar.edu/Classes/CalcI/ProductQuotientRule.aspx
reminded me.)

This appendix is purely theoretical, but does usefully demonstrate that there does exist an actual maximum capacity at an actual optimum speed. Appendix B, following, (a much later addition,) gives the real life stuff, for technology junkies.

Appendix B – Actual High Speed Line Capacity

I am indebted to Piers Connor of PRC Rail Consulting Ltd. for the information in this appendix. PRC Rail Consulting Ltd. publishes a series of occasional articles ‘Technical Web Pages’, at:

http://www.railway-technical.com/prcrailpage.shtml

Two articles are particularly relevant. The first is ‘(Rules for) High Speed Line Capacity’ v3, 26 August 2011, at:

http://www.railway-technical.com/Infopaper%203%20High%20Speed%20Line%20Capacity%20v3.pdf

which is in the Technical Web Pages series, and treats the subject as a series of 10 rules.

The article: ‘High Speed Railway Capacity’ (not sure of the date, but probably 2014) at:

http://www.railway-technical.com/High%20Speed%20Railway%20Capacity%20v13%20conf.pdf

seems more of a working paper, and serves as the background to a presentation at Birmingham University in December 2014, at:

http://www.railway-technical.com/HSR%20Presentation%20Piers%20Connor%20v1.pdf

All of these are thoroughly interesting, and contain essential information; technology junkies will, I am sure, love them.

The reason I refer to both papers titled ‘High Speed Rail Capacity’, is that they use different bases; the 2011 article considers a top speed of 300kph, whereas the later paper, which covers the same ground but in rather more detail, takes a top speed of 360kph. I need both, since my projected routes are either Ultra High Speed, UHS – HS2, HS3 and HS4 over most of the distance – for which 360kph is a good top speed, or HS Metro, where trains stop at all stations, for which 300kph is entirely satisfactory. In the present context, I quote only the results that I need. Readers should refer to the original source articles for derivations and justifications.

Top Speed 300kph Values

Top Speed	300kph = 187.5mph
Train/Platform Length	400m
Average Acceleration	0.3m/s^2
Deceleration	0.5m/s^2
Acceleration Distance	11.5km = 7.2miles
Acceleration Time	276 sec = 4min 36sec
Service Brake Distance	7.2km = 4.5miles
Service Brake Time	170 sec = 2min 50sec
Buffer Zone	100m
Train Separation Distance	7.7km = 4.8miles
Turnout Limit Speed	160kph = 100mph

Top Speed 360kph Values

Top Speed	360kph = 225mph
Train/Platform Length	400m
Average Acceleration	$0.3m/s^2$
Deceleration	$0.5\ m/s^2$
Acceleration Distance	16.67km = 10.42miles
Acceleration Time	333 sec = 5min 33sec
Service Brake Distance	10km = 6.25miles
Service Brake Time	200 sec = 3min 20sec
Buffer Zone	300m
Train Separation Distance	12.3km = 7.7miles
Turnout Limit Speed	230kph = 143.75mph

Top Speed 400kph Values

Top Speed	400kph = 250mph
Acceleration Time	370sec = 6min 10sec
Service Brake Time	222sec = 3min 42sec

Interpretation and Consequences of the above Values

The above values are direct quotes from the two articles. (The two values quoted for a top speed of 400kph are rather a throwaway line from the first article, designed, I interpret, to indicate how preposterous they are.)

The top speed of 300kph is current practice in Europe and Japan. The values quoted are validated by many years' experience. The top-speed of 360 is the initial projected value for HS2. 400kph is of course the headline projected value for HS2, which Piers Connor does not believe will ever be implemented, because 'it is not energy efficient for the distances between HS2 stations'. (He thinks the same for 360kph, but goes along with it for the purposes of argument.) I maintain a principled agnostic position in the matter.

The train separation distance is the minimum distance which must be maintained between the front of one train and the front of the following train; it is composed of the braking distance, the buffer zone and the train length and perhaps a response time (included in the second article but not the first).

The turnout limit speed is the maximum speed allowed when diverging or joining at a junction. The value of 160kph presumably represents the actual limits on existing lines. The value of 230kph reflects technological progress, and is the best available now (or was at 2014), for new installations. This is where the simplified explanation of HS operation given earlier (trains diverge from the main line at full line speed) really falls down; there are no points currently available which would allow for a turnout speed of 360kph or even 300kph, nor likely to be in the near future. What this means in practice is that diverging trains have to slow down on the main line, to reduce their speed to the turnout limit by the point at which the turnout is reached. (Likewise trains joining the main line accelerate up to the turnout limit, by the time they reach the junction, and then continue to accelerate on the main line until they reach full line speed.) This critically determines the locations of the turnout / rejoin junctions either side of a station.

The turnout limit speed also means that the statement that stopping trains do not delay non-stop ones because they get out of their way in a timely fashion is no longer true. A stopping train slowing down before the diverging junction for a station stop does in fact delay an immediately following non-stop train. There are two ways that this can be accommodated, either the non-stop train must maintain an increased distance behind the stopper, so that, maintaining full line speed, it only reaches the separation distance at the point where the slowing stopper has just diverged from the main line, or, alternatively that the non-stop train also slows down by a similar amount when approaching the junction, to maintain the separation distance behind the stopper, until the point at which the stopper has just left the main line, whereupon it can accelerate back up to the full line speed. This latter strategy has an amusing side effect: since the separation distance reduces as the speed reduces, the two trains can become progressively closer, until the stopper diverges.

Either way, the mere presence of the junction imposes a penalty. Either the trains must be kept further apart, so there is a penalty in line capacity overall, but not in journey time for the non-stop train, or, in the case of both trains slowing, the bunching effect means that there is less of a capacity penalty(*), but the non-stop train also suffers a penalty in extended journey time. Since the main point of HS lines is that they are, more importantly, high-capacity lines, the latter strategy, of slowing and bunching is to be preferred. (I understand that this strategy has been in use for many years on metro systems, to maximise capacity at a junction by ‘speed control’.)

(*) In fact there is no capacity penalty whatever. See the following section ‘Capacity vs Line Speed’. However, the above argument is a little simplistic, and the other alternative, of increasing the separation distance between trains, requires further consideration. This is dealt with in the section ‘The Effect of Junctions – Revisited’ (p.18).

Of course, the above penalty applies only where some trains are non-stop (UHS) over most of the distance. For HS Metro routes – all except HS2, HS3 and HS4 – where every train stops at every station, there is simply no problem, nor any need for fancy point work at stations (this is still required at genuine route junctions – the only ones left)..

Derivation of Necessary and Useful Results

For those of us who last used calculus regularly some time ago (50 years in my case) a small crib is in order.

If s, v, a and t are distance, speed, acceleration and time, then:

$v = ds/dt$ $\quad$ $a = dv/dt$ $\quad$ so, assuming constant acceleration a:

$v = \int a\, dt = at$ $\quad$ $s = \int v\, dt = at^2/2$ $\quad$ thus, for **definite** integrals:

$v - v_0 = a(t - t_0)$ so $v = v_0 + a(t - t_0)$

$$s - s_0 = \int v\, dt = \int [v_0 + a(t - t_0)]\, dt = v_0(t - t_0) + a(t^2 - t_0^2)/2 - at_0(t - t_0)$$
$$= (t - t_0)[v_0 + a(t + t_0)/2 - at_0] = (t - t_0)[v_0 + a(t - t_0)/2]$$
$$s = s_0 + v_0(t - t_0) + a(t - t_0)^2/2$$

Notice how involved it gets when we are dealing with definite integrals, and a double integration (acceleration to speed, speed to distance) is involved. (It took me a **long** while to work it out!) If $s_0 = v_0 =$

$t_0 = 0$ then it simplifies enormously, which is why it is very much easier to calculate the results to or from a standstill, and take differences to obtain intermediate results. (The above formula for s will calculate **directly** the deceleration distance to the junction; s_0 and t_0 are zero, but v_0 **isn't** – and don't forget a<0!).

Converting the speeds of interest to metres/sec:

300kph = 83.33m/s
160kph = 44.44m/s
360kph = 100m/s
230kph = 63.89m/s

First of all I will consider line capacity as a function of speed. Be warned that this is a very simple-minded approach, (refer back to Piers Connor's original articles for a full account of the many variables involved,) but the results are, I feel, very interesting and quite surprising.

Train Separation Distance (I call this 'Train Envelope' in appendix A) = Service Brake Distance + Train Length + Buffer

Service Brake Distance = distance required to come to a standstill (under normal, not emergency braking conditions)

$v = at$, so $t = v/a$. $s = at^2/2, = v^2/2a = v^2$, taking a as constant at $0.5m/s^2$.

So Train Envelope = $v^2 + 400 + 100 = v^2 + 500$

Capacity = number of trains passing a given point per unit time, = speed / envelope

Thus $c = v/(v^2 + 500)$ trains per second, $= 3600v/(v^2 + 500)$ tph

$$dc/dv = [(v^2 + 500)*3600 – 3600v*2v]/(v^2 + 500)^2 = 3600(500 – v^2)/(v^2 + 500)^2$$
$$= 0 \text{ when } v^2 = 500 \text{ i.e. when } v = 22.36\text{meters/sec}$$

$$d^2c/dv^2 = [-(v^2 + 500)^2 * 7200v – 3600(500 – v^2) * 2(v^2 + 500) * 2v]/ (v^2 + 500)^4$$
$$= 7200v[-(v^4 + 1000v^2 + 250000) + 2(v^2 – 500) *(v^2 + 500)]/(v^2 + 500)^4$$
$$= 7200v[-v^4 – 1000v^2 – 250000 + 2(v^4 – 250000)]/(v^2 + 500)^4$$
$$= 7200v(v^4 – 1000v^2 – 750000)/(v^2 + 500)^4$$

When $v^2 = 500$ then $v^4 – 1000v^2 – 750000 = 250000 – 500000 – 750000 = -1000000$, in other words the second derivative of the capacity is negative at the extremum value of v, confirming that this extremum is in fact a maximum.

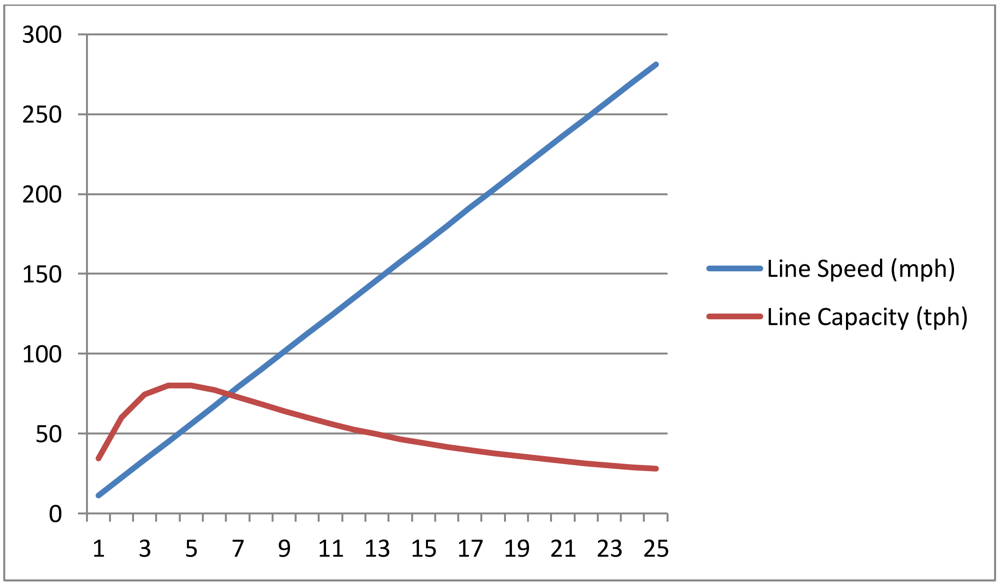
Thus the maximum capacity of the line is 80.496tph at a line speed of 22.36 meters/sec, = 80.5kph = 50mph.

This is, as stressed, a very simplistic argument, nonetheless, it is rather surprising that the maximum capacity occurs at such an astonishingly low speed. On the other hand, it may not be surprising that the maximum occurs in the speed range of the typical metro system.

It does, I regret to say, make the argument that HS railways are, more importantly, high capacity railways, look rather sick. They **are** of course high capacity, but only in comparison with mixed-traffic railways; they derive their capacity benefit from the traffic being homogeneous in its performance characteristics, just like a metro. (A heterogeneous traffic mix is lethal for capacity.) One may thus say that a HS railway has the maximum available capacity **for a particular line speed**, but one must now always add the latter qualification. Of course, whereas maximum capacity at a line speed of 50mph is just fine for metros, there would be very few takers for long distance travel at that speed, no matter how many trains per hour the line could carry. The maximum speed to aim for is thus a business decision, not a technical one.

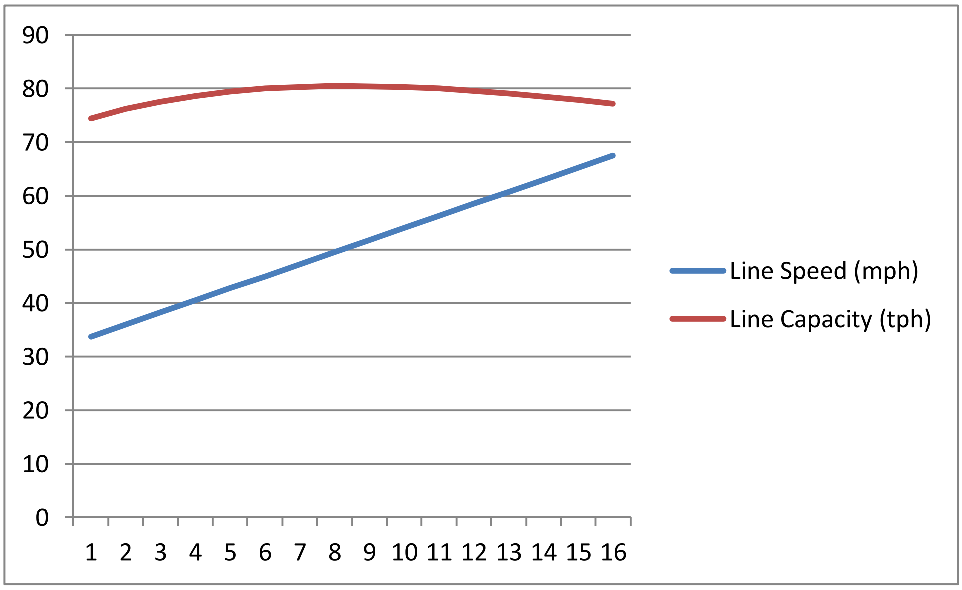
Capacity vs Line Speed

Line Speed (Meters/sec)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph)
5	18	11.25	34.28571429
10	36	22.5	60
15	54	33.75	74.48275862
20	72	45	80
25	90	56.25	80
30	108	67.5	77.14285714
35	126	78.75	73.04347826
40	144	90	68.57142857
45	162	101.25	64.15841584
50	180	112.5	60
55	198	123.75	56.17021277
60	216	135	52.68292683
65	234	146.25	49.52380952
70	252	157.5	46.66666667
75	270	168.75	44.08163265
80	288	180	41.73913043
85	306	191.25	39.61165049
90	324	202.5	37.6744186
95	342	213.75	35.90551181
100	360	225	34.28571429
105	378	236.25	32.79826464
110	396	247.5	31.42857143
115	414	258.75	30.16393443
120	432	270	28.99328859
125	450	281.25	27.90697674



Capacity values have been derived in a spreadsheet, and plotted on a line chart, both for the full range, above, and at a larger scale around the extremum, below.

Line Speed (Meters/sec)	Line Speed (kph)	Line Speed (mph)	Line Capacity (tph)
15	54	33.75	74.48275862
16	57.6	36	76.19047619
17	61.2	38.25	77.56653992
18	64.8	40.5	78.6407767
19	68.4	42.75	79.44250871
20	72	45	80
21	75.6	47.25	80.34006376
22	79.2	49.5	80.48780488
23	82.8	51.75	80.4664723
24	86.4	54	80.29739777
25	90	56.25	80
26	93.6	58.5	79.59183673
27	97.2	60.75	79.08868999
28	100.8	63	78.5046729
29	104.4	65.25	77.85234899
30	108	67.5	77.14285714



The Effect of Junctions

Two distinct cases need to be considered, the pure route junction, where routes diverge (for different destinations) or converge, and the double junctions required either side of a station, where some services are non-stop.

Consider first the diverging case:

The diverging train must decelerate to the turnout speed limit, by the time that it reaches the junction. (As noted earlier, the calculation is most easily performed by taking the decelerations to zero, then taking the differences.) Thus, for line speed 300kph and turnout limit speed 160kph:

1. 160kph to zero:
   $v = 0$ $v_0 = 44.44$ $a = -0.5$ s0 = 0 $t_0 = 0$ so:
   $44.44 = 0.5t$ so $t = 88.88$sec
   $s = 0.5t^2/2 = 88.88^2/4 = 1975$metres
2. 300kph to zero:
   $v = 0$ $v_0 = 83.33$ $a = -0.5$ $s_0 = 0$ $t_0 = 0$ so:
   $83.33 = 0.5t$ so $t = 166.67$sec
   $s = 0.5t^2/2 = 166.67^2/4 = 6945$metres

so the diverging train decelerates from 300kph to 160kph at the junction in a distance of (6945 – 1975) = 4970metres, 5km, say, (3.11miles,) and in a time of (166.67 – 88.88) = 78, say 80, secs.

Now consider the converging case (imagine that the train accelerates from a standstill, reaching the turnout / turnin speed at the junction):

1. Zero to 160kph:
   $v = 44.44$ $v_0 = 0$ $a = 0.3$ s0 = 0 $t_0 = 0$ so:
   $44.44 = 0.3t$ so $t = 148$sec
   $s = 0.3t^2/2 = 0.15 * 148^2 = 3292$metres
2. Zero to 300kph:
   $v = 83.33$ $v_0 = 0$ $a = 0.3$ $s_0 = 0$ $t_0 = 0$ so:
   $83.33 = 0.3t$ so $t = 277.77$sec = 278sec
   $s = 0.3t^2/2 = 0.15 * 277.77^2 = 11573$metres

so the converging train accelerates from 160kph at the junction to 300kph in a distance of (11573 – 3292) = 8281metres, 8.3km, say, (5.18miles,) and in a time of (278 – 148) = 130 secs.

Assume that trains on the main line likewise decelerate to / accelerate from the turnout speed at the junction, since we know that this will have no adverse effect on line capacity, whereas the alternative, of maintaining line speed and thus increasing distances between trains very definitely would reduce capacity. Assume also that they do decelerate to / accelerate from that speed **exactly** (whereas in fact the actual speed would be very slightly higher, since the trains become bunched slightly closer together approaching the diverging junction and become slightly more widely separated accelerating away from the converging junction – but this is a second-order effect and, hell! this is engineering, not particle physics, and the effect is merely mentioned to reassure the reader that it hasn't been forgotten). Thus for both the diverging train and the straight-on train, there is a time penalty (the same in both cases) as compared with travelling the same distance at line speed.

Both trains thus decelerate from 300kph to 160kph in a distance of 4970metres and a time of 78secs. The time required to travel 4970 metres at 300kph is 4970/83.3 = 60secs. So there is a time penalty of 18secs during deceleration. The both likewise accelerate from 160kph back to 300kph after the junction in a distance of 8281metres and a time of 130secs. The time required to travel 8281metres at 300kph is 8281/83.33 = 99.4secs, say 100secs. So there is a time penalty of 30secs during acceleration. One must not forget the length of the train itself – 400metres. The train must travel (at least) 400metres at the turnout speed in actually crossing the junction, thus a time of 400/44.44 = 9 secs, which, if travelled at line speed would take 400/83.33 = 4.8, say 5 secs. The crossing of the junction itself thus imposes an extra time penalty of 4secs (!). The total time penalty imposed on a diverging or converging train by a junction is thus 52secs.

The situation around a station requires very little further calculation. The total distance between the beginning of the deceleration before the station and completion of acceleration after it is the sum of the deceleration distance to a full stop at the station (6945metres in 167secs) and the acceleration distance from stationary after it (11573metres in 278secs). Thus we have a total distance on the main line affected by the presence of the station of 6945 + 11573 = 18518metres = 18.5km, and a total deceleration / acceleration time of 445secs.This distance travelled at 300kph would take 18518/83.33 = 222secs, so the penalty time for stopping at the station is 445 – 222 = 223secs = 3min43secs, plus whatever the waiting time is at the station, ideally about 3 minutes, so the total time penalty of a station stop is 7minutes, let's say.

For a train not stopping at the station, the time it would need to cover 18.5km at full speed would be 222secs. Suppose it travelled between the two station junctions at the turnoff speed of 160kph, instead of accelerating and decelerating, then the distance between junctions (1975 + 3292) = 5267metres would take 5267/44.44 = 118secs. Its total time travelling the entire 18.5km is thus (78 + 118 + 130) = 326secs as compared with 222secs at full line speed. The station penalty for a non-stop train is thus 326 – 222 = 104secs, say 2.5minutes. (Suppose the distance between station junctions were travelled at 300kph rather than 160kph, then this would take 5267/83.33 = 63 secs, as opposed to 118 secs. So the maximum benefit available to the non-stop train of accelerating after the diverging junction and decelerating before the converging one is less than 1 minute – a lot less – so keeping to the turnout speed between the junctions is actually a no-brainer.)

Summarising these results, and also repeating them for the case of a top speed of 360kph (using the same formulae; Piers Connor has some refinements, such as multiple deceleration ranges, but these are ignored here):

Line Speed (kph)	300	360
Turnout Limit Speed (kph)	160	230
Decelerating Distance Total (metres)	6945	10000
Decelerating Time Total (secs)	167	200
Decelerating Distance on Station Loop (metres)	1975	4082
Decelerating Time on Station Loop (secs)	89	128
Decelerating Distance on Main Line (metres)	4969	5918
Decelerating Time on Main Line (secs)	78	72
Accelerating Distance Total (metres)	11573	16667
Acceleration Time Total (secs)	278	333
Accerating Distance on Station Loop (metres)	3292	6803
Accelerating Time on Station Loop (secs)	148	213
Accelerating Distance on Main Line (metres)	8281	9864
Accelerationg Time on Main Line (secs)	130	110
Time to travel across junction itself (secs)	9	6
Single Junction Time Penalty, All Trains (seconds)	53	26
Station Time Penalty, Non-Stop Trains (minutes)	2.5	2.5
Station Time Penalty, Stopping Trains (minutes)	7	7.5

The values for a line speed of 360kph need some comment (they certainly worried me when I first saw them!).The slightly lower acceleration and deceleration times on the main line (as compared with 300kph) is due to the fact that there is actually less to do there – the turnout speed limit is only 130kph less than the line limit, as compared with 140kph in the slower case – and correspondingly a lot more to do on the station loop. The single junction time penalty (which is not big anyway) is only half the value of the slower case, again reflecting that less deceleration/acceleration is done on the main line.

Adjacent Stations

Consider two stations, one after the other, and a train that stops at both, followed by one that is non-stop. For line speed 300kph, (360kph,) the total deceleration / acceleration distance for the first station stop is, as usual, 6945 + 11573 = 18528metres, (10000 + 16667 = 26667metres,) in a time (excluding station wait time) of 167 + 278 = 445secs, (200 + 333 = 533secs). After this, the stopping train is travelling at full line speed. Providing the two stations are at least 18528metres, (26667metres,) apart then that is the full story; the behaviour around the second station is identical to that around the first.

But 18.5km, 11.6miles, (26.7km, 16.7miles,) is a significant distance, and it could well be the case that two stations exist closer together than that. This needs further consideration. In this case, the train stopping at both would accelerate away from the first station to some intermediate speed, less than full line speed, then immediately switch to deceleration for the second station. In this situation, I strongly recommend that the train do **not** rejoin the main line, even though it would have accelerated beyond the turnout speed limit, unless the stations were closer together than 1975 + 3292 = 5267metres, 3.3miles, (4082 + 6803 = 10885metres, 6.8miles,) which does seem improbably close, since there is no benefit in its doing so, and it might obstruct a non-stop train on the main line. Instead, the station loops should continue between the stations, so we have a 4-track section, maximally 23806metres, 14.9miles, (37552metres, 23.5miles,) in length. (The calculation is **twice** 18528 less the deceleration distance on the main line before the first station and the acceleration distance on the main line after the second.)

If s_a is the distance (<18258metres) between the stations, and v_a the maximum speed reached between them, and if s_1, s_2 are the acceleration / decelerating distances and t_1, t_2 the corresponding times, then:

$v_a = 0.3t_1 = 0.5t_2$, thus $t_2 = 0.6t_1$

$s_a = s_1 + s_2 = 0.3t_1^2/2 + 0.5t_2^2/2$

$t_2 = 0.6t_1$ so $s_1 = 0.3t_1^2/2$, $s_2=0.5(0.6t_1)^2/2 = 0.18t_1^2/2$

So $s_a = s_1 + s_1 = t_1^2(0.3 + 0.18)/2 = 0.24t_1^2$

Thus $t_1^2 = 4.167s_a$ so $t_1 = 2.04\sqrt{s_a}$ and $t2 = 0.6t1 = 1.225\sqrt{s_a}$

So $v_a = 0.3t_1 = 0.6124\sqrt{s_a}$

Results have been generated by spreadsheet (for line speeds 300kph and 360kph – only the final column is different) over the whole relevant range. Getting the above equations right was extraordinarily troublesome and error prone, and the penultimate column in the spreadsheet is there primarily to satisfy the reader (and me!!) that the results **are** correct. (The quirky unit chosen – tenths of a km, deci-km? – is dictated by the requirements of the line chart.) I-S in the column headers means Inter-Station. The final column gives the overall time penalty for the double station stop, which includes deceleration before the first station and acceleration after the second.

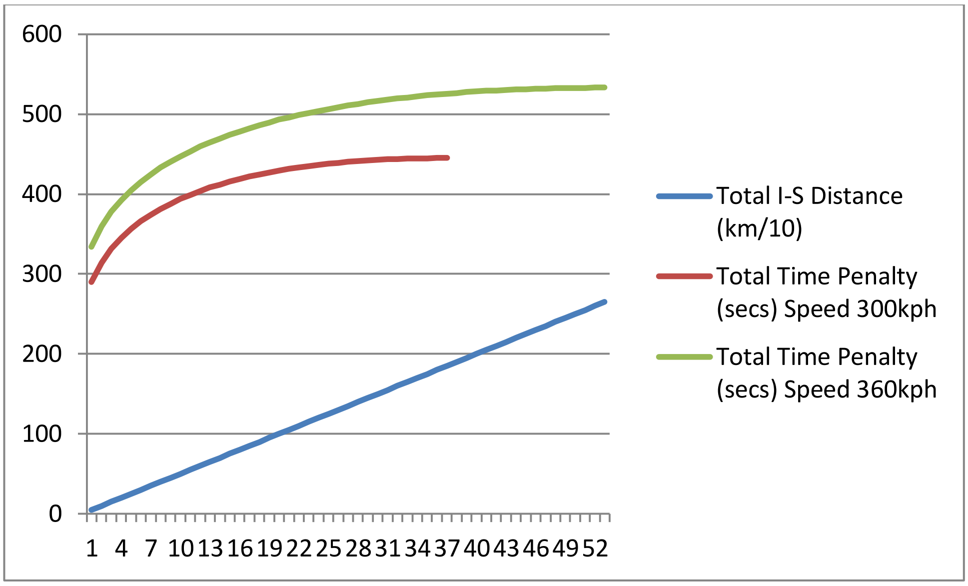
Note that this figure excludes the station wait times. As the total distance between stations approaches 18528metres, the overall time penalty converges on the total for two separate stops, i.e. 2 * 223secs, (2 * 267secs).

Distance Apart (metres)	Intermediate Speed (metres/sec)	Accel'n Time (secs)	Accel'n Distance (metres)	Decel'n Time (secs)	Decel'n Distance (metres)	Total I-S Time (secs)	Total I-S Distance (km/10)	Total Time Penalty (secs) Speed 300kph
500	13.6936803	45.64537	312.525	27.386	187.5	73.0315	5.00025	289.806071
1000	19.3657884	64.5523	625.05	38.73	375	103.282	10.0005	314.0561688
1500	23.71815	79.0601	937.575	47.434	562.5	126.494	15.0008	331.2677596
2000	27.3873606	91.29074	1250.1	54.772	750	146.063	20.001	344.835951
2500	30.62	102.0662	1562.625	61.237	937.5	163.303	25.0013	356.0758095
3000	33.5425294	111.8079	1875.15	67.082	1125	178.89	30.0015	365.6617811
3500	36.2300726	120.7663	2187.675	72.457	1312.5	193.223	35.0018	373.9945178
4000	38.7315768	129.1046	2500.2	77.46	1500	206.564	40.002	381.3350665
4500	41.0810409	136.9361	2812.725	82.158	1687.5	219.095	45.0023	387.8647509
5000	43.3032193	144.3433	3125.25	86.603	1875	230.946	50.0025	393.7155919
5500	45.4167995	151.3886	3437.775	90.83	2062.5	242.218	55.0028	398.9872543
6000	47.4363	158.1202	3750.3	94.868	2250	252.989	60.003	403.757168
6500	49.3732665	164.5767	4062.825	98.742	2437.5	263.319	65.0033	408.0869088
7000	51.23706	170.7893	4375.35	102.47	2625	273.259	70.0035	412.0264023
7500	53.0353957	176.7838	4687.875	106.07	2812.5	282.85	75.0038	415.6167941
8000	54.7747212	182.5815	5000.4	109.54	3000	292.126	80.004	418.8924708
8500	56.4604903	188.2007	5312.925	112.92	3187.5	301.117	85.0043	421.8825193
9000	58.0973652	193.6569	5625.45	116.19	3375	309.846	90.0045	424.6118041
9500	59.6893686	198.9636	5937.975	119.37	3562.5	318.337	95.0048	427.101779
10000	61.24	204.1323	6250.5	122.47	3750	326.607	100.005	429.3711077
10500	62.7523265	209.1734	6563.025	125.5	3937.5	334.672	105.005	431.4361476
11000	64.2290539	214.0958	6875.55	128.45	4125	342.548	110.006	433.3113293
11500	65.6725836	218.9075	7188.075	131.34	4312.5	350.247	115.006	435.0094606
12000	67.0850588	223.6157	7500.6	134.16	4500	357.78	120.006	436.541971
12500	68.4684015	228.2269	7813.125	136.93	4687.5	365.158	125.006	437.9191107
13000	69.824343	232.7466	8125.65	139.64	4875	372.389	130.007	439.1501143
13500	71.15445	237.1803	8438.175	142.3	5062.5	379.483	135.007	440.2433363
14000	72.4601452	241.5326	8750.7	144.91	5250	386.446	140.007	441.2063644
14500	73.7427252	245.8079	9063.225	147.48	5437.5	393.287	145.007	442.0461142
15000	75.0033759	250.01	9375.75	150	5625	400.01	150.008	442.7689102
15500	76.2431851	254.1427	9688.275	152.48	5812.5	406.622	155.008	443.3805536
16000	77.4631536	258.2092	10000.8	154.92	6000	413.129	160.008	443.8863817
16500	78.6642043	262.2127	10313.33	157.32	6187.5	419.534	165.008	444.2913174
17000	79.8471911	266.156	10625.85	159.69	6375	425.843	170.009	444.5999138
17500	81.0129051	270.0417	10938.38	162.02	6562.5	432.06	175.009	444.8163912
18000	82.1620818	273.8722	11250.9	164.32	6750	438.189	180.009	444.9446705
18500	83.2954054	277.65	11563.43	166.58	6937.5	444.233	185.009	444.9884022

Distance Apart (metres)	Intermediate Speed (metres/sec)	Accel'n Time (secs)	Accel'n Distance (metres)	Decel'n Time (secs)	Decel'n Distance (metres)	Total I-S Time (secs)	Total I-S Distance (km/10)	Total Time Penalty (secs) Speed 360kph
500	13.6936803	45.64537	312.525	27.386	187.5	73.0315	5.00025	334.36125
1000	19.3657884	64.5523	625.05	38.73	375	103.282	10.0005	359.6116378
1500	23.71815	79.0601	937.575	47.434	562.5	126.494	15.0008	377.8235186
2000	27.3873606	91.29074	1250.1	54.772	750	146.063	20.001	392.3920001
2500	30.62	102.0662	1562.625	61.237	937.5	163.303	25.0013	404.6321486
3000	33.5425294	111.8079	1875.15	67.082	1125	178.89	30.0015	415.2184102
3500	36.2300726	120.7663	2187.675	72.457	1312.5	193.223	35.0018	424.551437
4000	38.7315768	129.1046	2500.2	77.46	1500	206.564	40.002	432.8922757
4500	41.0810409	136.9361	2812.725	82.158	1687.5	219.095	45.0023	440.4222501
5000	43.3032193	144.3433	3125.25	86.603	1875	230.946	50.0025	447.2733811
5500	45.4167995	151.3886	3437.775	90.83	2062.5	242.218	55.0028	453.5453335
6000	47.4363	158.1202	3750.3	94.868	2250	252.989	60.003	459.3155372
6500	49.3732665	164.5767	4062.825	98.742	2437.5	263.319	65.0033	464.6455681
7000	51.23706	170.7893	4375.35	102.47	2625	273.259	70.0035	469.5853516
7500	53.0353957	176.7838	4687.875	106.07	2812.5	282.85	75.0038	474.1760334
8000	54.7747212	182.5815	5000.4	109.54	3000	292.126	80.004	478.4520002
8500	56.4604903	188.2007	5312.925	112.92	3187.5	301.117	85.0043	482.4423387
9000	58.0973652	193.6569	5625.45	116.19	3375	309.846	90.0045	486.1719135
9500	59.6893686	198.9636	5937.975	119.37	3562.5	318.337	95.0048	489.6621784
10000	61.24	204.1323	6250.5	122.47	3750	326.607	100.005	492.9317972
10500	62.7523265	209.1734	6563.025	125.5	3937.5	334.672	105.005	495.997127
11000	64.2290539	214.0958	6875.55	128.45	4125	342.548	110.006	498.8725987
11500	65.6725836	218.9075	7188.075	131.34	4312.5	350.247	115.006	501.5710201
12000	67.0850588	223.6157	7500.6	134.16	4500	357.78	120.006	504.1038205
12500	68.4684015	228.2269	7813.125	136.93	4687.5	365.158	125.006	506.4812502
13000	69.824343	232.7466	8125.65	139.64	4875	372.389	130.007	508.7125438
13500	71.15445	237.1803	8438.175	142.3	5062.5	379.483	135.007	510.8060559
14000	72.4601452	241.5326	8750.7	144.91	5250	386.446	140.007	512.769374
14500	73.7427252	245.8079	9063.225	147.48	5437.5	393.287	145.007	514.6094138
15000	75.0033759	250.01	9375.75	150	5625	400.01	150.008	516.3324998
15500	76.2431851	254.1427	9688.275	152.48	5812.5	406.622	155.008	517.9444333
16000	77.4631536	258.2092	10000.8	154.92	6000	413.129	160.008	519.4505513
16500	78.6642043	262.2127	10313.33	157.32	6187.5	419.534	165.008	520.8557771
17000	79.8471911	266.156	10625.85	159.69	6375	425.843	170.009	522.1646635
17500	81.0129051	270.0417	10938.38	162.02	6562.5	432.06	175.009	523.3814309
18000	82.1620818	273.8722	11250.9	164.32	6750	438.189	180.009	524.5100002
18500	83.2954054	277.65	11563.43	166.58	6937.5	444.233	185.009	525.554022
19000	84.4135146	281.377	11875.95	168.82	7125	450.196	190.01	526.5169015
19500	85.517006	285.0553	12188.48	171.03	7312.5	456.082	195.01	527.4018216
20000	86.6064386	288.6867	12501	173.21	7500	461.892	200.01	528.2117621
20500	87.6823362	292.273	12813.53	175.36	7687.5	467.63	205.01	528.949518
21000	88.7451912	295.8158	13126.05	177.48	7875	473.298	210.011	529.6177146
21500	89.7954667	299.3167	13438.58	179.58	8062.5	478.9	215.011	530.2188218
22000	90.8335991	302.7771	13751.1	181.66	8250	484.436	220.011	530.755167
22500	91.86	306.1985	14063.63	183.71	8437.5	489.91	225.011	531.2289458
23000	92.8750584	309.582	14376.15	185.74	8625	495.324	230.012	531.6422324
23500	93.8791423	312.9289	14688.68	187.75	8812.5	500.679	235.012	531.9969892

24000	94.8726	316.2404	15001.2	189.74	9000	505.977	240.012	532.2950745
24500	95.8557621	319.5176	15313.73	191.7	9187.5	511.221	245.012	532.5382503
25000	96.828942	322.7615	15626.25	193.65	9375	516.411	250.013	532.7281892
25500	97.7924377	325.9732	15938.78	195.58	9562.5	521.549	255.013	532.8664806
26000	98.7465329	329.1535	16251.3	197.48	9750	526.638	260.013	532.9546362
26500	99.6914973	332.3033	16563.83	199.37	9937.5	531.677	265.013	532.9940957

Depicting the last two columns on a line chart:



Adjacent Junctions

A similar effect to adjacent stations occurs when there are adjacent junctions. There aren't many practical instances of this, but they **do** exist. It is thus important that they be analysed.

We are of course speaking of **divergent / convergent** junctions. Nuthall South and North junctions on the HS3 main line, for example, are straight-ahead; the junctions have no effect on the through trains. (I am anticipating slightly; see the next section, 'The Effect of Junctions – Revisited' for the justification of this.) Likewise Awsworth and Strelley junctions have no effect on through traffic passing between HS7 and the Nottingham loop of HS3.

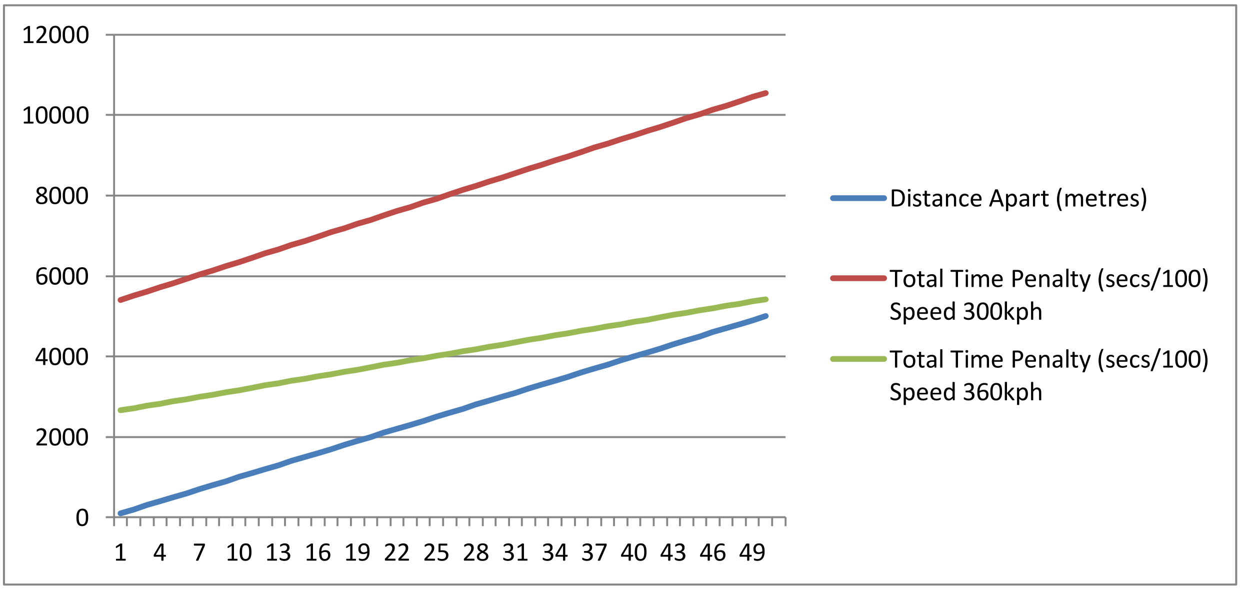
So the situation is where a junction diverging from one route, after a short stretch of intermediate track, converges on another route. Trains decelerate from line speed to the turnout limit speed on the first route, before the divergent junction, and accelerate back up to the lime speed on the second route, following the convergent junction. Between the two they probably maintain the turnout limit speed – it is unlikely to be worth trying to go any faster over this short distance.

Thus if s_j is the distance between the two junctions. The train decelerates from line speed (300kph) to turnout limit speed (160kph) in a distance of 4969metres and a time of 78secs (for 360kph/230kph the values are 5918metres in 72 secs). It then travels the distance s_j metres, plus a further 400metres beyond the second junction, to ensure that the entire train has cleared it, at 160kph, thus in a time of $(s_j + 400)/44.44$ secs (for 360kph/230kph the time is (sj + 400)/63.89). Finally, it accelerates back up to line speed in a distance 8281metres in 130secs (for 360kph/230kph the values are 9864metres in 110 secs). Thus we have a total deceleration/steady/ acceleration distance of 4969 + (s_j + 400) + 8281 = (13650 + s_j) metres, in a time of 72 + (s_j + 400)/44.44+ 130 = (211 + s_j/44.44) secs. (For 360kph/230kph, the values are (16182 + s_j) metres in (188 + s_j/63.89) secs.) Travelled at full line speed, that distance would take (13650 + s_j)/83.33 secs, so the time penalty for the double junction is [(211 + s_j/44.44) – (13650 + s_j)/83.33], = (107 + 95.22*s_j) secs. (For 360kph/230kph, the time penalty is (27 + 176.93*s_j).)

The results are:

Distance Apart (metres)	Total Accel'n/ Steady/ Decel'n Distance (metres) Speed 300kph	Total Accel'n/ Steady/ Decel'n Time (secs) Speed 300kph	Total Time Penalty (secs/100) Speed 300kph	Total Accel'n/ Steady/ Decel'n Distance (metres) Speed 360kph	Total Accel'n/ Steady/ Decel'n Time (secs) Speed 3600kph	Total Time Penalty (secs/100) Speed 360kph
100	13750	213	5405	16282	190	2657
200	13850	216	5510	16382	191	2713
300	13950	218	5615	16482	193	2770
400	14050	220	5720	16582	194	2826
500	14150	222	5825	16682	196	2883
600	14250	225	5930	16782	197	2939
700	14350	227	6035	16882	199	2996
800	14450	229	6140	16982	201	3052
900	14550	231	6245	17082	202	3109
1000	14650	234	6350	17182	204	3165
1100	14750	236	6455	17282	205	3222
1200	14850	238	6560	17382	207	3278
1300	14950	240	6665	17482	208	3335
1400	15050	243	6770	17582	210	3391
1500	15150	245	6875	17682	211	3448
1600	15250	247	6980	17782	213	3504
1700	15350	249	7085	17882	215	3561
1800	15450	252	7190	17982	216	3617
1900	15550	254	7295	18082	218	3674
2000	15650	256	7400	18182	219	3730
2100	15750	258	7505	18282	221	3787
2200	15850	261	7610	18382	222	3843
2300	15950	263	7715	18482	224	3900
2400	16050	265	7820	18582	226	3956
2500	16150	267	7925	18682	227	4013
2600	16250	270	8030	18782	229	4069
2700	16350	272	8135	18882	230	4126
2800	16450	274	8241	18982	232	4183
2900	16550	276	8346	19082	233	4239
3000	16650	279	8451	19182	235	4296
3100	16750	281	8556	19282	237	4352
3200	16850	283	8661	19382	238	4409
3300	16950	285	8766	19482	240	4465

3400	17050	288	8871	19582	241	4522
3500	17150	290	8976	19682	243	4578
3600	17250	292	9081	19782	244	4635
3700	17350	294	9186	19882	246	4691
3800	17450	297	9291	19982	247	4748
3900	17550	299	9396	20082	249	4804
4000	17650	301	9501	20182	251	4861
4100	17750	303	9606	20282	252	4917
4200	17850	306	9711	20382	254	4974
4300	17950	308	9816	20482	255	5030
4400	18050	310	9921	20582	257	5087
4500	18150	312	10026	20682	258	5143
4600	18250	315	10131	20782	260	5200
4700	18350	317	10236	20882	262	5256
4800	18450	319	10341	20982	263	5313
4900	18550	321	10446	21082	265	5369
5000	18650	324	10551	21182	266	5426



The odd time units, hundredths of a second, are clearly to get the best spread on the line chart. Despite much trying, I haven’t managed a better display for the table.

Note how surprisingly small these double-junction time penalties are, and how slowly they increase with distance between the junctions. For the section between Strelley and Nuthall South Junctions, the distance apart is a mere 300metres, and the time penalty all of 28secs. Even for the section between Awsworth and Nuthall North junctions, where the distance apart is 3.8km, the time penalty is still only 47secs. (I’m taking line speed 360kph for both.)

There are, in fact, only five instances of adjacent junctions in the entire HS network. They might as well be listed. The line speed is 360kph in all cases.

Junction Pairs	Distance Apart (metres)	Time Penalty (secs)
Strelley / Nuthall South	300	28
Awsworth / Nuthall North	3800	47
Gosforth East and West	700	30
Kenyon South and West	1100	32
Kenyon West and North	900	31

The Effect of Junctions – Revisited

When first considering the effect of junction, in the section ‘Interpretation and Consequences of the above Values’ (p.10), it was pointed out that two approaches are available to handle the impact of a diverging train on a following, non-diverging one (actually, the argument applies whether or not the following train also diverges):

1. the distance between trains could be increased, so that the following train, maintaining line speed, only reaches separation distance behind the diverging train at the point where that train has just diverged at the junction, or
2. the following train could also decelerate, maintaining separation distance (which of course decreases as the speed decreases, so the two trains become closer together).

The second option was chosen because it has no effect on line capacity, (whereas the first definitely reduces it,) although it thereby imposes a time penalty on the following train (which the first doesn’t). However, no proper consideration was given to precisely how the following train decelerates. It was implicitly assumed that the following train begins its deceleration at the same point as the diverging train, (which is of course a known location, determined by the need for the diverging train to decelerate to the turnout limit speed at the junction,) and itself decelerate to the turnout limit speed at the junction. If the junction is a route junction (thus the diverging line has a different destination) then the train accelerates back up to line speed immediately after the junction. But if the junction is at the start of a station loop, then the train maintains that speed until the following junction, where the station loop rejoins the main line, and only then re-accelerates, (because there is no practical benefit in trying to go faster between the ends of the station loop).

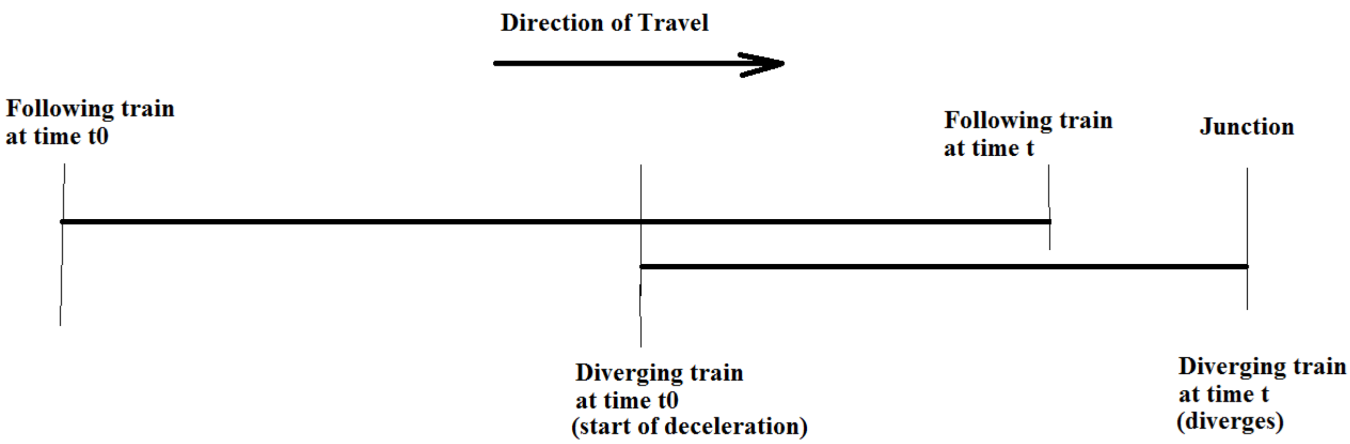
But this behaviour means that the following train approaches the diverging train more closely than the prescribed separation distance, which is a bad thing. So we need to consider this aspect with more rigour. Note that, in the preceding sections, the results derived are all valid, unaffected by the present issue, **except** for those which derive a time penalty imposed on the following train, either by the single junction per se, or by the two junctions and station loop.

All of this stuff assumes that the trains are under a very much state-of-the-art automatic train control system (much of it just wouldn’t be possible otherwise). It may therefore be assumed that the following train ‘knows’ its precise distance behind the preceding train at all times, and thus is able to maintain this at exactly the separation distance (or any other required value). This is what Moving Block is all about. So, the moment the diverging train began its deceleration before the junction, the following train would recognise that its distance behind that train was below the required separation distance, and begin its own deceleration. And the next train would do the same, and the next after that. In fact, the start of deceleration of the diverging train would cause every following train on the line to decelerate in lockstep with it. This is of course assuming that the line is being operated to maximum capacity at the chosen line speed. The situation is clearly ludicrous. So let's see what option 1 above would offer.

Train separation distance is composed of several components, as described earlier (p.9), of which only the braking distance is variable (taken as $at^2/2$, $= v^2/2a$, where v is the line speed, t is the total stopping time and a is the acceleration, assumed constant with the value $0.5m/s^2$, thus simply $= v^2$). We thus take the basic train separation distance = $(v^2 + 500)$ metres, in line with the 300kph line speed (the treatment for 360kph involves a few extra subtleties, ignored here). We define an Enhanced Train Separation Distance, s_e, with an extra component, to ensure that a following train only approaches a diverging train at the basic separation distance, when the diverging train has just actually diverged at the junction. Thus the non-diverging train maintains line speed, (and, incidentally, the simplified treatment, whereby 'stopping trains do not obstruct non-stop trains because they get out of their way in a timely fashion' actually becomes true!).

The diverging train decelerates to the turnout limit speed in a distance $s = v_l t + at^2/2$, and $v_t = v_l + at$ where v_l is the line speed and v_t the turnout limit speed (both of which are known), t is the deceleration time and a the acceleration (negative value, of course). (This is from the definite integral formulae derived on p.10.) So $t = (v_t – v_l)/a$ and $s = v_l(v_t – v_l)/a + a((v_t – v_l)^2/a^2)/2$.

Thus, in time $t = (v_l – v_t)/a$ secs, the diverging train decelerates from v_l to v_t in a distance
$s = v_l(v_t – v_l)/a + a((v_t – v_l)^2/a^2)/2 = (v_l^2 - v_t^2)/2a$.
In the same time, the following train travels a distance $v_l t = v_l(v_l - v_t)/a$ at line speed v_l.




```
|==========================================================|===========|
           |                                                    |
           |                              |====================================|
           |                                     |              |          `
     vl(vl – vt)/a                         (vl² - vt²)/2a        sb
```

In the above line diagram, s_b, the basic train separation distance, $= v_l^2/2a$ + const, (const being the constant stuff included, the train length and buffer zone,) and s_e, the extended train separation distance, is the distance between the trains at time t0, as s_b is the distance between them at time t. Thus:

$s_e = v_l(v_l – v_t)/a + s_b - (v_l^2 - v_t^2)/2a = v_l(v_l – v_t)/a + (v_l^2/2a + \text{const}) – (v_l^2 - v_t^2)/2a$

So $s_e = [v_l^2 + (v_l – v_t)^2]/2a + \text{const}$ and $s_b = v_l^2/2a + \text{const}$ so $s_e – s_b = (v_l – v_t)^2/2a$

Thus, to allow the following train to proceed at line speed all the way, we need an **extra** distance between trains of $(v_l - v_t)^2/2a$.

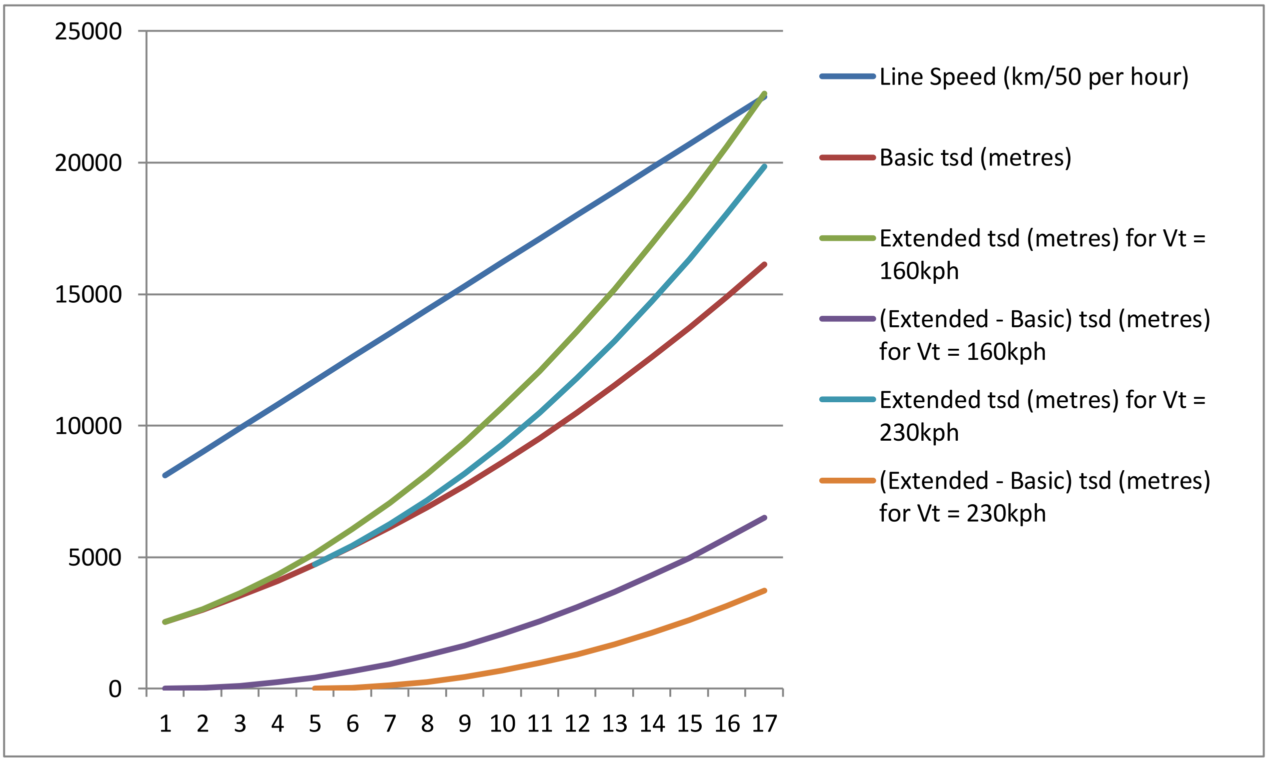
These are completely general results. Applying the particular values of interest:

1. v_l = 300kph (v_t = 160kph) s_e = 8455 + 500 s_b = 6943 + 500 $(s_e - s_b)$ = 1512
2. v_l = 360kph (v_t = 230kph) s_e = 11304 + 500 s_b = 10000 + 500 $(s_e - s_b)$ = 1304

It really is surprising to find that the extra distance required between trains is lower for the higher line speed. But as with previous superficially surprising effects, it is a consequence of the difference between line speed and turnout limit being smaller at the higher line speed.

Basic and Extended tsds have been derived in a spreadsheet, and plotted on a line chart. My apologies for the truly weird unit used for line speed (50ths of a km per hour, i.e. the number of 20metre units per hour!) – it is of course purely to get this variable to use the full area of the chart – otherwise it's stuck right at the bottom, with a gradient of near zero.

Line Speed (meters/sec)	Line Speed (km/50 per hour)	Basic tsd (metres)	Extended tsd (metres) for Vt = 160kph	(Extended - Basic) tsd (metres) for Vt = 160kph	Extended tsd (metres) for Vt = 230kph	(Extended - Basic) tsd (metres) for Vt = 230kph
45	8100	2525	2525	0		
50	9000	3000	3031	31		
55	9900	3525	3637	112		
60	10800	4100	4342	242		
65	11700	4725	5148	423	4726	1
70	12600	5400	6053	653	5437	37
75	13500	6125	7059	934	6248	123
80	14400	6900	8165	1265	7160	260
85	15300	7725	9370	1645	8171	446
90	16200	8600	10676	2076	9282	682
95	17100	9525	12081	2556	10493	968
100	18000	10500	13587	3087	11804	1304
105	18900	11525	15193	3668	13215	1690
110	19800	12600	16898	4298	14726	2126
115	20700	13725	18704	4979	16337	2612
120	21600	14900	20609	5709	18048	3148
125	22500	16125	22615	6490	19859	3734



Capacity = speed / train separation distance

So, for line speed 300kps, taking basic tsd s_b, capacity = 40tph; taking extended tsd se, capacity = 33.5tph

Or, for line speed 360kps, taking basic tsd s_b, capacity = 34tph; taking extended tsd se, capacity = 30.5tph

These actually look very reasonable values. They are of course based purely on keeping the trains a safe distance apart. There are other constraints to be considered also. Nonetheless, I believe that these are very useful results, and that this approach, adopting the extended train separation distance, is very much to be preferred, especially as it means that there is now **no time penalty at all** for non-diverging / non-stop trains at route junctions or at station loops.

Stations on the Main Line

The previous sections considered in detail the effect of stations, where some trains are non-stop and need to be able to overtake those stopping at the station. However, most of the routes I've designed are for HS Metro services, where every train stops at every station. These routes are all designed to have a top speed of 300kph.

Any train takes 6945m and 167s to stop at the station. All stations have two platform faces in each direction. The next train immediately following arrives 167s later, and takes the other platform face. The first train must have left the station within a further 167s, for a third train to be able to take its platform face.

So the only consideration for HS Metro lines is that the maximum station stop must not exceed 2*167 secs, i.e. 334s. This is over 5.5 minutes, and really should not provide any problem whatever.

Termini

Piers Connor's articles consider carefully the capacity constraints imposed at terminal stations. I avoid this problem entirely by not having any terminals, not in London, at any rate, where the proposed redevelopment of Euston as a terminal is, in my considered opinion, outright, unmitigated lunacy.

Instead, all my proposed HS routes (that serve London) pass through London, branching subsequently to serve multiple destinations, which do indeed have terminal stations, but not in general required to take more that 4tph, for which 2 at a pinch or 3 platforms comfortably will suffice.